

6678/01

Edexcel GCE

Mechanics M2

Silver Level S4

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Green)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
62	54	46	38	30	23

1. A particle P moves along the x -axis. At time t seconds the velocity of P is v m s⁻¹ in the positive x -direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O . Find the distance of P from O when P is moving with minimum velocity. (8)
-

2. A particle P of mass m is moving in a straight line on a smooth horizontal surface with speed $4u$. The particle P collides directly with a particle Q of mass $3m$ which is at rest on the surface. The coefficient of restitution between P and Q is e . The direction of motion of P is reversed by the collision.

Show that $e > \frac{1}{3}$.

(8)

- 3.

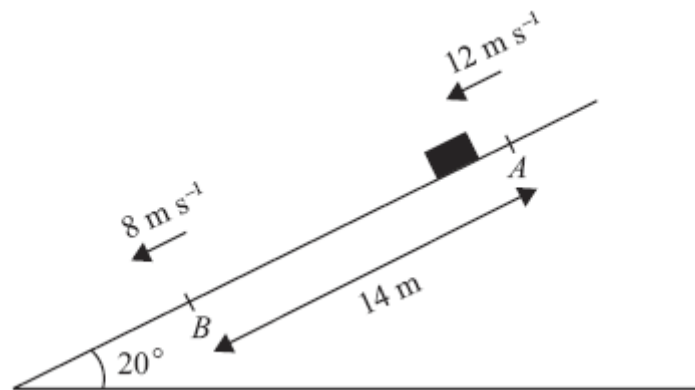


Figure 1

A package of mass 3.5 kg is sliding down a ramp. The package is modelled as a particle and the ramp as a rough plane inclined at an angle of 20° to the horizontal. The package slides down a line of greatest slope of the plane from a point A to a point B , where $AB = 14$ m. At A the package has speed 12 m s⁻¹ and at B the package has speed 8 m s⁻¹, as shown in Figure 1.

Find

- (a) the total energy lost by the package in travelling from A to B , (5)
- (b) the coefficient of friction between the package and the ramp. (5)
-

4.

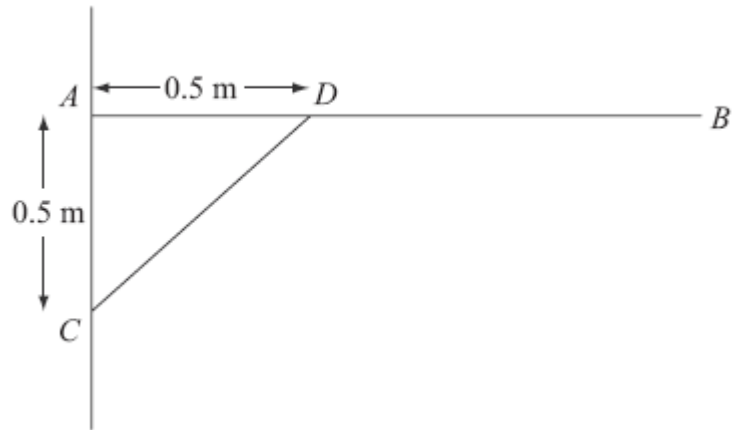


Figure 1

A uniform rod AB , of length 1.5 m and mass 3 kg, is smoothly hinged to a vertical wall at A . The rod is held in equilibrium in a horizontal position by a light strut CD as shown in Figure 1. The rod and the strut lie in the same vertical plane, which is perpendicular to the wall. The end C of the strut is freely jointed to the wall at a point 0.5 m vertically below A . The end D is freely jointed to the rod so that AD is 0.5 m.

(a) Find the thrust in CD .

(4)

(b) Find the magnitude and direction of the force exerted on the rod AB at A .

(7)

5.

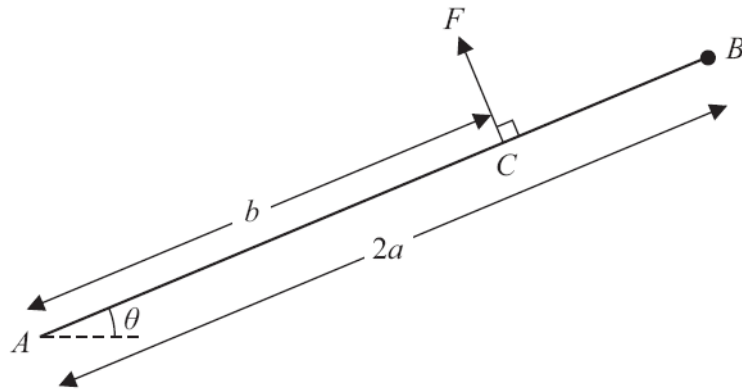


Figure 3

A uniform rod AB , of mass m and length $2a$, is freely hinged to a fixed point A . A particle of mass m is attached to the rod at B . The rod is held in equilibrium at an angle θ to the horizontal by a force of magnitude F acting at the point C on the rod, where $AC = b$, as shown in Figure 3. The force at C acts at right angles to AB and in the vertical plane containing AB .

(a) Show that $F = \frac{3amg \cos \theta}{b}$. (4)

(b) Find, in terms of a , b , g , m and θ ,

(i) the horizontal component of the force acting on the rod at A ,

(ii) the vertical component of the force acting on the rod at A .

(5)

Given that the force acting on the rod at A acts along the rod,

(c) find the value of $\frac{a}{b}$. (4)

6. A car of mass 1200 kg pulls a trailer of mass 400 kg up a straight road which is inclined to the horizontal at an angle α , where $\sin \alpha = \frac{1}{14}$. The trailer is attached to the car by a light inextensible towbar which is parallel to the road. The car's engine works at a constant rate of 60 kW. The non-gravitational resistances to motion are constant and of magnitude 1000 N on the car and 200 N on the trailer.

At a given instant, the car is moving at 10 m s^{-1} . Find

(a) the acceleration of the car at this instant, (5)

(b) the tension in the towbar at this instant. (4)

The towbar breaks when the car is moving at 12 m s^{-1} .

(c) Find, using the work-energy principle, the further distance that the trailer travels before coming instantaneously to rest. (5)

7. [The centre of mass of a semi-circular lamina of radius r is $\frac{4r}{3\pi}$ from the centre.]

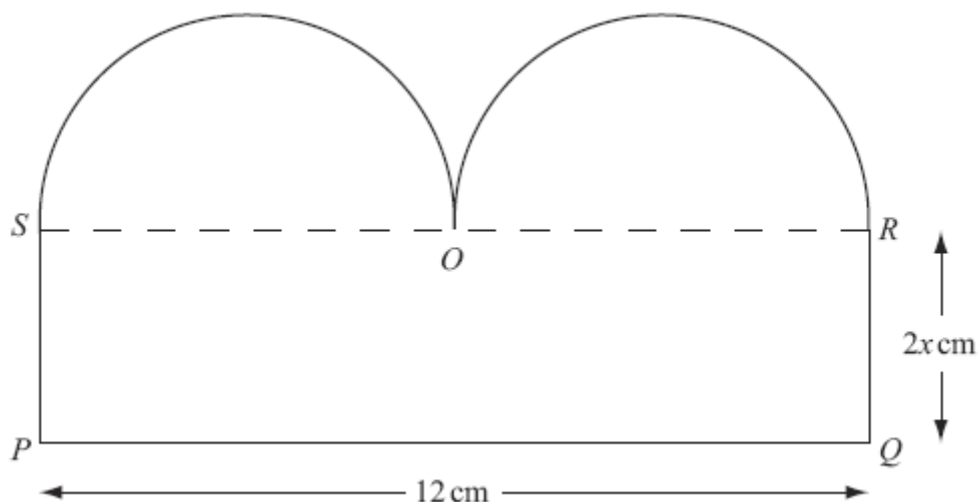


Figure 3

A template T consists of a uniform plane lamina $PQROS$, as shown in Figure 3. The lamina is bounded by two semicircles, with diameters SO and OR , and by the sides SP , PQ and QR of the rectangle $PQRS$. The point O is the mid-point of SR , $PQ = 12$ cm and $QR = 2x$ cm.

- (a) Show that the centre of mass of T is a distance $\frac{4|2x^2 - 3|}{8x + 3\pi}$ cm from SR . (7)

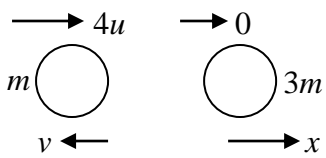
The template T is freely suspended from the point P and hangs in equilibrium.

Given that $x = 2$ and that θ is the angle that PQ makes with the horizontal,

- (b) show that $\tan \theta = \frac{48 + 9\pi}{22 + 6\pi}$. (4)

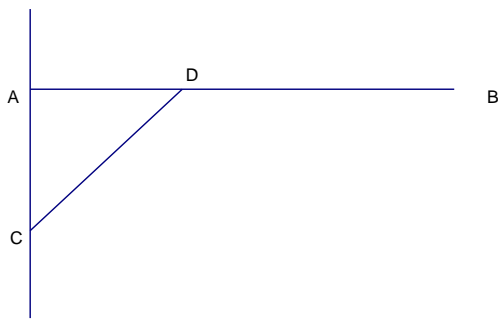
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\frac{dv}{dt} = 6t - 4$ $6t - 4 = 0 \Rightarrow t = \frac{2}{3}$ $s = \int 3t^2 - 4t + 3 dt = t^3 - 2t^2 + 3t (+c)$ $t = \frac{2}{3} \Rightarrow s = -\frac{16}{27} + 2 \text{ so distance is } \frac{38}{27} \text{ m}$	<p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>M1 A1</p> <p>[8]</p>
2.	 $4mu = 3mx - mv$ $4ue = x + v$ $4u = 3(4ue - v) - v$ $4u = 12ue - 4v$ $v = (3e - 1)u$ $v > 0 \Rightarrow 3e > 1$ $\therefore e > \frac{1}{3} \quad **$	<p>M1 A1</p> <p>M1 A1</p> <p>DM1 A1</p> <p>DM1</p> <p>A1</p> <p>[8]</p>
3.	<p>(a) $\Delta KE = \frac{1}{2} \times 3.5(12^2 - 8^2) (=140)$ or KE at A, B correct separately</p> <p>$\Delta PE = 3.5 \times 9.8 \times 14 \sin 20^\circ (\approx 164.238)$ or PE at A, B correct separately</p> <p>$\Delta E = \Delta KE + \Delta PE \approx 304, 300$</p> <p>(b) Using Work-Energy</p> $F_r = \mu \times 3.5g \cos 20^\circ$ $304.238 \dots = F_r \times 14 \quad \text{ft their (a), } F_r$ $304.238 \dots = \mu 3.5g \cos 20^\circ \times 14$ $\mu \approx 0.674, 0.67$	<p>B1</p> <p>M1 A1</p> <p>DM1 A1</p> <p>(5)</p> <p>M1 A1</p> <p>M1 A1 ft</p> <p>A1</p> <p>(5)</p> <p>(10 marks)</p>

4

(a)



Taking moments about A:

$$3g \times 0.75 = \frac{T}{\sqrt{2}} \times 0.5$$

$$T = 3\sqrt{2}g \times \frac{7.5}{5} = \frac{9\sqrt{2}g}{2} (= 62.4N)$$

M1A1A1

A1

(4)

(b)

$$\leftarrow \pm H = \frac{T}{\sqrt{2}} (= \frac{9g}{2} \approx 44.1N)$$

$$\uparrow \pm V + \frac{T}{\sqrt{2}} = 3g \quad (\Rightarrow V = 3g - \frac{9g}{2} = \frac{-3g}{2} \approx -14.7N)$$

$$\Rightarrow |R| = \sqrt{81+9} \times \frac{g}{2} \approx 46.5(N)$$

at angle $\tan^{-1} \frac{1}{3} = 18.4^\circ$ (0.322 radians) below the line of BA

161.6° (2.82 radians) below the line of AB
(108.4° or 1.89 radians to upward vertical)

B1

M1A1

M1A1

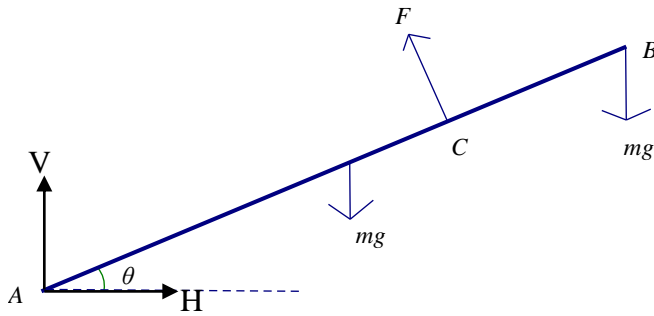
M1A1

(7)

[11]

5

(a)



Moments about A:

$$bF = a \cos \theta mg + 2a \cos \theta mg (= 3a \cos \theta mg)$$

$$F = \frac{3amg \cos \theta}{b} \quad \text{*Answer given*}$$

(b)

$$\rightarrow: H = F \sin \theta = \frac{3amg \cos \theta \sin \theta}{b}$$

$$\uparrow: 2mg = \pm V + F \cos \theta$$

$$\pm V = 2mg - \frac{3amg \cos \theta}{b} \times \cos \theta \left(= 2mg - \frac{3amg \cos^2 \theta}{b} \right)$$

(c)

$$\frac{2mg - \frac{3amg \cos^2 \theta}{b}}{\frac{3amg \cos \theta \sin \theta}{b}} = \tan \theta$$

$$\frac{2b - 3a \cos^2 \theta}{3a \cos \theta \sin \theta} = \frac{\sin \theta}{\cos \theta}$$

$$\Rightarrow 2b - 3a \cos^2 \theta = 3a \sin^2 \theta \Rightarrow 2b = 3a, \quad \frac{a}{b} = \frac{2}{3}$$

M1

A2

A1

(4)

M1

A1

M1

A1

A1

(5)

M1

A1

DM1

A1

(4)

[13]

Question Number	Scheme	Marks
6. (a) (b) (c)	$F = \frac{60000}{10} = 6000$ $F - 1200g \sin \alpha - 400g \sin \alpha - 1000 - 200 = 1600a$ $a = 2.3 \text{ (m s}^{-2}\text{)}$ $T - 400g \sin \alpha - 200 = 400 \times 2.3$ $T = 1400$ $200d = \frac{1}{2} 400.12^2 - 400gd \sin \alpha$ $d = 60 \text{ (m)}$	B1 M1 A1 A1 A1 M1 A1 ft A1 ft A1 M1 A1 A1 DM1 A1 (5) (4) (5) 14 marks

Question Number	Scheme	Marks
7. (a)	<div style="display: flex; justify-content: space-around;"> Rectangle Semicircles Template, T </div> $24x + 9\pi$ $24x$ 4.5π 4.5π	B2
	x $\frac{4 \times 3}{3\pi}$ $\frac{4 \times 3}{3\pi}$ \bar{x} $24x^2 - 4.5\pi \times \left(\frac{4 \times 3}{3\pi}\right) - 4.5\pi \times \left(\frac{4 \times 3}{3\pi}\right) = (24x + 9\pi)\bar{x}$ $\text{distance} = \bar{x} = \frac{4 2x^2 - 3 }{(8x + 3\pi)} **$	B2 M1 A1 A1 (7)
(b)	<p>When $x = 2$, $\bar{x} = \frac{20}{16 + 3\pi}$</p> $\tan \theta = \frac{6}{4 - \bar{x} } = \frac{6}{4 - \frac{20}{16 + 3\pi}}$ $= \frac{48 + 9\pi}{22 + 6\pi}$	B1 M1 A1 A1 (4) [11]

Examiner reports

Question 1

The majority of candidates offered confident responses to this opening question. Most of them successfully integrated the given velocity to find out the displacement of the particle. When it came to finding out the time of minimum velocity, most candidates used calculus again to find acceleration and made it equal to zero but some preferred to complete the square or use the expression for the turning point of a parabola. A few candidates attempted to find the time when the velocity of the particle was zero, believing this to be the minimum. A common error was find the minimum velocity and substitute this, rather than the time, into the displacement equation.

Question 2

Most candidates understood the principles of conservation of momentum and Newton's Experimental Law. However, many lost a mark here because they did not pay sufficient attention to the direction of motion of the particles after the collision, leading to inconsistent signs between their two equations. Even if they had indicated directions on a diagram, this was not always consistent with their equations. It was also common to see substitution into "template equations " rather than understanding the equations.

Candidates usually went on to eliminate v_p , v_q or u successfully, although some candidates offered no attempt at further work on their simultaneous equations.

The final 2 marks were very often lost because many failed to realise that the final direction of motion of P was the key to a solution. Several candidates did not attempt to obtain the stated result or wrote down the stated result without justification. Those candidates who had obtained an equation involving e but not v_p in the previous part of the question were often unable to make any further progress. For those who did attempt it, it was common for the direction of v_p to be inconsistent with their diagram. This confusion over the direction of the velocities led some candidates to make untrue statements about the value of e and to become very creative in their proofs. Use of $v_p = 0$ rather than using an inequality was often seen.

Question 3

(a) Many candidates lost marks here. This was usually because they found only the loss in kinetic energy rather than using both the kinetic energy and the potential energy terms

(b) Those candidates who chose to use their answer to part (a) and use the work-energy principle tended to be successful in finding a value for μ , but a significant number did not realise the connection between parts (a) and (b). Those who chose to use constant acceleration and then $F=ma$ generally found μ correctly.

The most common errors were to omit the weight component in their equation for $F=ma$ or to confuse the two methods and include extra energy terms in their energy equation. Many candidates lost marks through giving answers to inappropriate accuracy following the use of an approximate value for g .

Question 4

In part (a) candidates who realised the thrust in the strut acted in the direction CD were generally successful in finding its magnitude. The overwhelming majority applied the most simple method of taking moments about A , although much longer alternative methods were often seen. A significant number, however, appeared to be confused by the use of the word thrust and many took this to be a vertical force acting at D .

For part (b) many candidates were able to find the horizontal and vertical components of the reaction at A and then the correct magnitude of this force. Most went on to find a direction but a significant number were unable to describe this direction properly. It was surprising to see relatively few diagrams and yet a diagram would have shown direction clearly.

The confusion over the direction of the thrust led to many errors in the vertical component of the reaction. Some candidates falsely assumed that the reaction at A acted vertically/horizontally. The candidates who took the force at D to be vertical were unable to complete this part as there was no horizontal force present.

Question 5

In part (a) the majority of candidates took moments and combined their results correctly. Candidates who overlooked the weight of the rod frequently fiddled the given result rather than look for their error. This commonly led to further difficulties in part (b).

In part (b) most candidates made sensible attempts to resolve horizontally and vertically following the request for horizontal and vertical components of the force at A . Candidates who resolved parallel and perpendicular to the rod at this stage usually found the subsequent algebra beyond them.

Some candidates offered no attempt to part (c), but following part (b), the most common approach was to use $\tan \theta$. Some candidates were not able to work through the resulting algebra and trigonometry to reach a conclusion. A number of very elegant solutions, involving moments about B or C or resolving perpendicular to the rod, did not depend on part (b) at all.

Question 6

In part (a), students who considered the system as a whole were often more successful than those who considered the car and trailer separately. The most common error was to omit one or more of the forces acting, often the weight of one of the objects, or one or both of the resistances.

Although many correct answers were seen in part (b), some candidates struggled to write down a correct equation for the motion of the car or the trailer alone. It was common to see the acceleration ignored, or the weight or resistance missing from the equation.

Part (c) was answered well. The majority of candidates now accept that if the question specifies that they must use the work-energy principle then they will earn no marks for an alternative approach. The most common error is still to see candidates double counting the change in potential energy by considering both the gain in GPE and the work done against the weight. Some candidates forgot to include the work done against the resistance. There did seem to be some confusion about what happens when the towbar breaks. Although the question clearly asks for the additional distance travelled by the trailer, some candidates considered both vehicles as if they were still attached, and some considered only the car.

Question 7

(a) The method was understood by most candidates and there was no problem in forming a moments equation for the centre of mass. Common errors included simplifying $\frac{4 \times 3}{3\pi}$ to 4π rather than $\frac{4}{\pi}$, using the area of a circle rather than a semicircle, and the use of 6 for the radius of the semicircle. From a correct table, accuracy marks were often lost in the moments equation because of a sign error. In general, those candidates who set out the masses and distances in a table tended to make fewer errors.

Many candidates made it more difficult to obtain the given answer by taking their measurements from PQ and attempting to subtract their result from $2x$, although this was often successfully completed. One advantage of this approach was that they were less likely to make a sign error in their moments equation.

Candidates very rarely justified the modulus sign at all, with most candidates simply writing the final answer after their last line of working. Students who had a negative coefficient for the x^2 term in the numerator were more likely to deal with this.

(b) The fact that the answer was given did guide some candidates to the correct result, indicating that they clearly appreciated the ‘show that’ nature of the question. Most candidates substituted $x = 2$ correctly into the given result and went on to find the tangent of an angle. Many candidates did identify the correct triangle although some went to great lengths to find the distance of the centre of mass from SP as an expression in x , not realising that it could be found by symmetry. Furthermore, they often did not then realise that their expression cancelled to 6.

Many of those who made progress with this part found the angle to the vertical, with quite a few unconvincingly converting to the given result or simply leaving it as the reciprocal.

Statistics for M2 Practice Paper Silver 4

Qu	Max Score	Modal score	Mean %	Mean average scored by candidates achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	8		82.4	6.59		7.33	5.96	5.04	4.34	3.76	2.75
2	8		74.1	5.93	7.27	6.54	5.69	5.09	4.36	3.70	2.09
3	10		69.9	6.99		8.39	6.98	5.96	5.10	3.90	2.83
4	11		51.3	5.64		7.58	4.91	3.47	2.31	1.67	0.91
5	13	13	63.6	8.27	11.32	9.89	8.00	6.46	5.21	3.90	2.09
6	14		57.8	8.09	11.12	9.58	7.12	5.51	4.14	3.00	1.97
7	11		65.0	7.15		8.06	5.83	4.25	3.62	2.15	1.51
	75		64.9	48.66		57.37	44.49	35.78	29.08	22.08	14.15