Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2
Silver Level S3

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 7 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
64	56	48	40	32	24

1. A cyclist starts from rest and moves along a straight horizontal road. The combined mass of the cyclist and his cycle is 120 kg. The resistance to motion is modelled as a constant force of magnitude 32 N. The rate at which the cyclist works is 384 W. The cyclist accelerates until he reaches a constant speed of v m s^{-1}.

Find

(*a*) the value of v,

(3)

(*b*) the acceleration of the cyclist at the instant when the speed is 9 m s^{-1}.

(3)

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2. At time $t = 0$ a particle P leaves the origin O and moves along the x-axis. At time t seconds the velocity of P is v m s^{-1}, where

$$v = 8t - t^2 .$$

(*a*) Find the maximum value of v.

(4)

(*b*) Find the time taken for P to return to O.

(5)

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3. A particle P moves along a straight line in such a way that at time t seconds its velocity v m s^{-1} is given by

$$v = \frac{1}{2}t^2 - 3t + 4$$

Find

(*a*) the times when P is at rest,

(4)

(*b*) the total distance travelled by P between $t = 0$ and $t = 4$.

(5)

4.

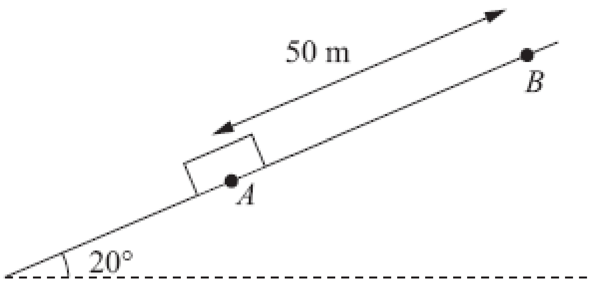

Figure 1

A box of mass 30 kg is held at rest at point *A* on a rough inclined plane. The plane is inclined at 20° to the horizontal. Point *B* is 50 m from *A* up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from *A* to *B* by a force acting parallel to *AB* and then held at rest at *B*. The coefficient of friction between the box and the plane is $\frac{1}{4}$. Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

(*a*) find the work done in dragging the box from *A* to *B*.

(6)

The box is released from rest at the point *B* and slides down the slope. Using the work-energy principle, or otherwise,

(*b*) find the speed of the box as it reaches *A*.

(5)

5.

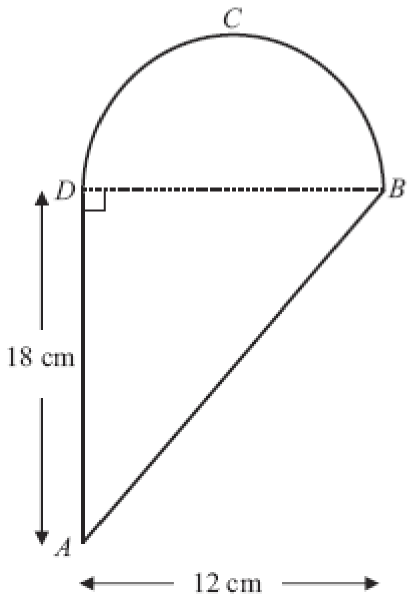

Figure 2

A uniform lamina *ABCD* is made by joining a uniform triangular lamina *ABD* to a uniform semi-circular lamina *DBC*, of the same material, along the edge *BD*, as shown in Figure 2. Triangle *ABD* is right-angled at *D* and *AD* = 18 cm. The semi-circle has diameter *BD* and *BD* = 12 cm.

(*a*) Show that, to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *AD* is 4.69 cm.

(4)

Given that the centre of mass of a uniform semicircular lamina, radius *r*, is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

(*b*) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina *ABCD* from *BD*.

(4)

The lamina is freely suspended from *B* and hangs in equilibrium.

(*c*) Find, to the nearest degree, the angle which *BD* makes with the vertical.

(4)

6. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

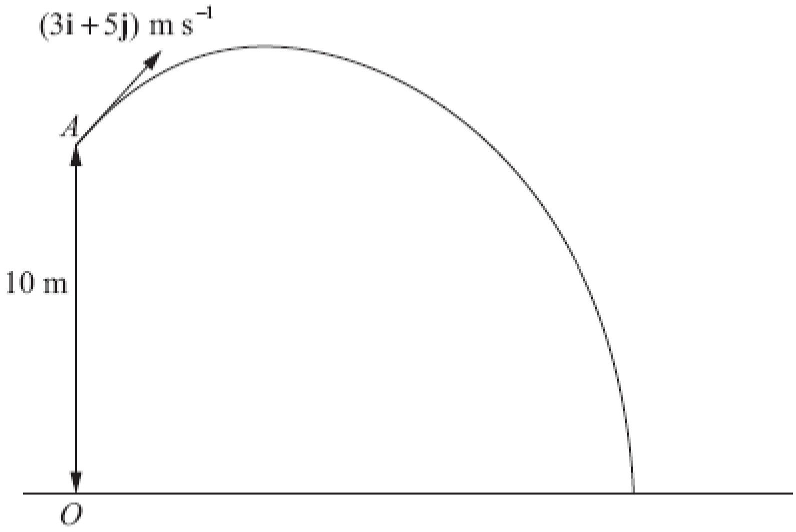

Figure 3

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after T seconds.

(*a*) For $0 \leq t \leq T$, show that, with respect to O, the position vector, $\mathbf{r}$ metres, of P at time t seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$$

(3)

(*b*) Find the value of T.

(3)

(*c*) Find the velocity of P at time t seconds $(0 \leq t \leq T)$.

(2)

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(*d*) Find the time taken for P to move from A to B.

(2)

(*e*) Find the speed of P as it passes through B.

(2)

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7. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

(*a*) Find, in terms of e and u,

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(*b*) find the range of possible values of e,

(6)

(*c*) determine whether there will be a second collision between A and B.

(3)

TOTAL FOR PAPER: 75 MARKS

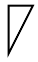
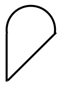
END

Question Number	Scheme	Marks
1. (a)	Constant speed ⇒ Driving force = resistance , $F = 32$ $P = F \times v = 32v = 384$ $v = 12$ (ms^{-1})	**B1** **M1** **A1** **(3)**
(b)	$P = F \times v \Rightarrow 384 = F \times 9$, $F = \frac{384}{9}$ Their $F - 32 = 120a$, $a = 0.089(\text{ms}^{-2})$	**M1** **M1** **A1** **(3)** **[6]**
2. (a)	$\frac{dv}{dt} = 8 - 2t$ $8 - 2t = 0$ Max $v = 8 \times 4 - 4^2 = 16\,(\text{ms}^{-1})$	**M1** **M1** **M1A1** **(4)**
(b)	$\int 8t - t^2 dt = 4t^2 - \frac{1}{3}t^3 (+c)$ (t=0, displacement = 0 ⇒c = 0) $4T^2 - \frac{1}{3}T^3 = 0$ $T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$ $T = 12$ (seconds)	**M1A1** **DM1** **DM1** **A1** **(5)** **[9]**

Question Number	Scheme	Marks
3.		
(a)	$\frac{1}{2}t^2 - 3t + 4 = 0$	M1
	$t^2 - 6t + 8 = 0$	
	$(t-2)(t-4) = 0$	DM1
	$t = 2 \text{ s } \textit{ or } \ 4 \text{ s}$	A1 A1
		(4)
(b)	$\int \frac{1}{2}t^2 - 3t + 4\text{d}t$	M1
	$= \frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\left(+C\right)$	A1
	$s = \int_0^2 \frac{1}{2}t^2 - 3t + 4 \text{ d}t - \int_2^4 \frac{1}{2}t^2 - 3t + 4 \text{ d}t$	DM1
	$= \left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_0^2 - \left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_2^4$	
	$= \frac{8}{6} - 6 + 8 - (\frac{64}{6} - 24 + 16 - (\frac{8}{6} - 6 + 8))$	A1
	$= \frac{10}{3} - \frac{8}{3} + \frac{10}{3}$	
	$= 4$	A1
		(5)
		[9]
4		
(a)	Work done against friction = 50 × μR	**M1**
	= 50 × ¼ × 30cos 20° × 9.8	**A1**
	Gain in GPE = 30 × 9.8 × 50 sin 20°	**M1 A1**
	Total work done = WD against Friction + gain in GPE	**DM1**
	= 8480(J), 8500(J)	**A1**
		(6)
(b)	Loss in GPE = WD against friction + gain in KE 3 terms	**M1**
	30×9.8×50sin20° = 50×¼×30×9.8×cos20° +½×30× v^2 -1 ee	**A2 1,0**
	½ v^2 = 50×9.8×(sin20° − ¼cos20°) ,	**DM1**
	v =10.2 m s^{-1} .	**A1**
		(5)
		[11]

Question Number	Scheme	Marks

5 (a)

MR	 108	 18π	 $108+18\pi$
x_i $(\rightarrow)$ from *AD*	4	6	$\bar{x}$
y_i $(\downarrow)$ from *BD*	6	$-\frac{8}{\pi}$	$\bar{y}$

B1

B1

$AD(\rightarrow)$: $108(4) + 18\pi(6) = (108+18\pi)\bar{x}$ — M1

$\bar{x} = \dfrac{432 + 108\pi}{108 + 18\pi} = 4.68731... = \underline{4.69}$ (cm) (3 sf) **AG** — A1

(4)

(b)

y_i $(\downarrow)$ from *BD*	6	$-\dfrac{8}{\pi}$	$\bar{y}$

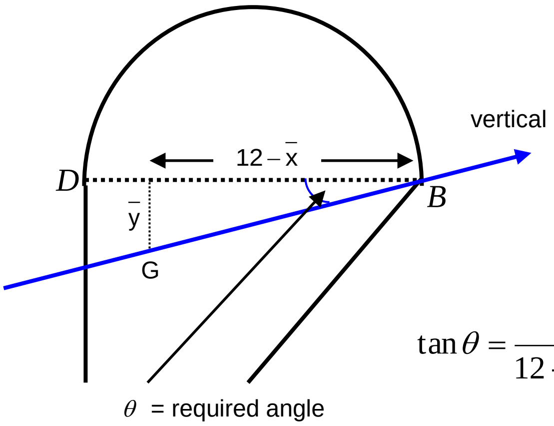
B1 oe

$BD(\downarrow)$: $108(6) + 18\pi(-\frac{8}{\pi}) = (108+18\pi)\bar{y}$ — M1, A1ft

$\bar{y} = \dfrac{504}{108 + 18\pi} = 3.06292... = 3.06$ (cm) (3 sf) — A1

(4)

(c)



M1

$\tan\theta = \dfrac{\bar{y}}{12 - 4.68731..}$ — dM1

$= \dfrac{3.06392..}{12 - 4.68731..}$ — A1

$\theta = 22.72641... = \underline{23}$ (nearest degree) — A1

(4)

[12]

Question Number	Scheme	Marks
6		
(a)	Using $s = ut + \frac{1}{2}at^2$ Method must be clear	**M1**
	r = (3t)**i** + (10 + 5t – 4.9t^2) **j** Answer given	**A1 A1**
		(3)
(b)	**j** component = 0 : 1 0 + 5t – 4.9t^2	**M1**
	quadratic formula: $t = \frac{5 \pm \sqrt{25+196}}{9.8} = \frac{5 \pm \sqrt{221}}{9.8}$	**D1**
	T = 2.03(s), 2.0 (s) positive solution only.	**A1**
		(3)
(c)	Differentiating the position vector (or working from first principles)	**M1**
	v = 3**i** + (5 – 9.8t)**j** (ms^{-1})	**A1**
		(2)
(d)	At B the **j** component of the velocity is the negative of the **I** component:	**M1**
	5 – 9.8t = –3, 8 = 9.8t, t = 0.82	**A1**
		(2)
(e)	**v** = 3**i** – 3**j**, speed = $\sqrt{3^2+3^2}$ = √18 = 4.24 (m s^{-1})	**M1 A1**
		(2)
		[12]

Question Number	Scheme	Marks
7	$\longrightarrow u$ $\quad\longrightarrow 0$	
(a)	A $\quad\bullet\ B$	
	m $\quad 3m$	
	$v\longleftarrow$ $\quad\longrightarrow w$	
	$mu = -mv + 3mw$	M1
	$u = -v + 3w$	A1
	$eu = w + v$	M1
		A1
	$w = \frac{u}{4}(1+e)$	M1 A1
	$v = -w + eu = \frac{u}{4}(3e-1)$	A1
	$\longrightarrow \frac{u}{4}(1+e)$ $\quad\longrightarrow 0$	
(b)	B $\quad\bullet\ C$	
	$3m$ $\quad 4m$	
	$Y\longleftarrow$ $\quad\longrightarrow X$	
	$3mw = 4mX - 3mY$	M1
		A1ft
	$2ew = X + Y$	B1ft
	$7Y = W(8e-3)$	DM1
	Or $2ue(1+e) - \frac{3u}{4}(1+e) = 7Y$	
	$\rightarrow e > \frac{3}{8}$	A1
	$Y > 0 \rightarrow \frac{3}{8} < e \le \frac{1}{2}$	A1
(c)	$\frac{u}{28}(1+e)(8e-3) > \frac{u}{4}(3e-1)$	M1
	$2e^2 - 4e + 1 > 0$	
	$e = \frac{4 \pm \sqrt{16-8}}{4} = 1.707,\ 0.293$	M1
	$2e^2 - 4e + 1 < 0$ for	
	$\frac{3}{8} < e \le \frac{1}{2}$ so no second collision.	A1

Examiner reports

Question 1

This proved to be a friendly starter for most candidates. Part (a) most candidates obtained the correct answer, although there was often no clear statement that the driving force must be equal to the resistance. In part (b) there were many completely correct solutions. The most common error was to omit the resistance when writing down the equation of motion, often when a candidate had not drawn a diagram of forces.

Question 2

This question provided the opportunity for candidates to show that they could both differentiate the velocity function to find the acceleration and integrate it to find the displacement. In general both were done successfully, although as usual there were candidates who incorrectly attempted to solve the problem using constant acceleration formulae.

Although the majority of candidates used differentiation in part (a), there was also a large number who treated it by completing the square, and they were often successful in this approach. A number of candidates produced a table of discrete time values and corresponding speeds of the particle. Unfortunately they rarely scored full marks for their effort as the supporting statement about the symmetry of a quadratic function was usually missing. The most common error among candidates using differentiation was to stop when they had found the time and not go on to find the speed. In part (b) it would have been reassuring to have seen more candidates - even the successful ones - giving a more rigorous treatment of the constant of integration. Algebraic errors in solving the equation $4T^2 - \frac{1}{3}T^3 = 0$ were surprisingly common.

Question 3

(a) Almost all candidates found the times when *P* is at rest correctly.

(b) Most candidates recognised the need to integrate to find displacement, but few understood the hint available from their answer to (a) which should have told them that the total distance travelled would not be the same as the displacement.

Question 4

In part (a), some candidates interpreted the question as requiring just the work done against friction. Another frequent mistake was finding the correct frictional and gravitational forces but then failing to multiply by the distance. Some candidates double counted by including both the increase in gravitational potential energy and the work done against the weight of the box. The final answer was often given as 8481 J, which is inappropriate following the use of an approximate value for *g*.

In part (b) the solution was often correct. Some candidates using the work-energy principle did make errors through double counting, and sometimes made a sign error by attempting to use their answer from (a). The alternative method of using **F** = *ma* and *suvat* was usually successful provided the candidate did not omit the friction.

Question 5

The majority of candidates applied the correct mechanical principles to solve this problem. Most were able to find the relative masses and the centres of mass of the semi-circle and the triangle and obtain a correct moments equation. Many candidates did not show sufficient working to demonstrate that their equation led to the given result in part (a).

In part (b) the most common error was to fail to realise that the two centres of mass were on opposite sides of the line *BD* and they hence had a sign error in their expression. Those who decided to take moments about a line through *A*, perpendicular to *AD* avoided this problem.

Candidates were generally able to use the given result to find the centre of mass of the semi-circle, although it was quite common to see it written incorrectly as 8π.

A clear diagram tended to lead candidates to identify the correct angle in part (c) and the correct method for finding it.

Question 6

Solutions in part (a) often lacked a clear method. Candidates should be reminded of the need for detail when deriving a given answer. Candidates showed a poor knowledge of vector analysis and little understanding of the use of a displacement vector with a position vector. There were plenty of fudges to include $10\mathbf{j}$, only rarely was $\mathbf{r} = \mathbf{r}_0 + \mathbf{s}$ used. Many candidates considered the horizontal and vertical components separately. The horizontal component was easily found but the candidates found it difficult to justify the 10 in the vertical. Many, incorrectly, attempted to equate the vertical displacement to 10 without any reference to initial conditions. The best solutions used integration, with the 10 being found by using the initial conditions to find the constant of integration.

The best solutions in part (b) were where candidates equated the **j** component of their position vector to 0 and solved the resulting quadratic equation. Many started again and found the vertical displacement equation from scratch leaving a greater scope for error. A common error was to equate the **j** component from (a) to 10, failing to realise that the 10 was already included in the equation. As usual, there were a few unnecessarily long methods involving calculation of the time to reach the maximum height and then the time from there to the ground. Some candidates lost the final mark due to ‘over accurate’ answers following the use of a decimal approximation for *g*.

In part (c) some candidates clearly differentiated the result from part (a), and others derived the velocity from the initial information. There was evidence of confusion on some candidates who found the speed or velocity at a particular time, rather than a general expression for the velocity. Part (d) surprisingly, many candidates had difficulty here , commonly equating their **j** component to +3 rather than –3, often despite having a correct diagram. Others did not connect “45° below the horizontal” with equal horizontal and vertical components of velocity.

In part (e) many candidates had success here despite earlier problems, with most finding the modulus of a vector of the form $3\mathbf{i} + n\mathbf{j}$. Candidates should be encouraged to read all parts of questions as later parts do not always rely on success in earlier ones.

Question 7

Candidates made errors with inconsistent signs, or signs which did not reflect what they had shown in their diagrams. Several candidates did not start out with the direction of motion of *A* reversed after the collision, and only a few of these went on to give the correct speed of *A* after the collision. There were several algebraic errors in solving the simultaneous equations, often because of a lack of brackets after a minus sign, for example, errors such as $v = eu - \frac{u}{4}(1+e) = eu - \frac{u}{4} + \frac{eu}{4}$.

In Q7(b) many candidates scored the first three marks here for forming correct equations, although there were still errors due to inconsistent signs. Many also went on to solve for the speed of *B* after the second collision, but they often reached the negative of the correct answer because they did not consider the change in the direction of motion. It should be noted that it is much simpler to work through the equations for the second collision using v_B rather than substituting $\frac{u}{4}(1+e)$. Those candidates who had worked through correctly usually concluded that $\frac{3}{8}$ was the lower bound for the set of possible values of *e*, but very few candidates realised that the consequence of the coefficient of restitution between *B* and *C* being 2*e* was that the upper bound would be $\frac{1}{2}$.

In Q7(c) very few candidates offered a complete solution to this part of the question. Of those who attempted it, most appeared to understand the condition for a second collision between *A* and *B* to occur. Some did form a correct inequality in *e*, and a few then went on to consider the critical values of *e*. The majority of candidates made no attempt to use an algebraic approach; they reached their conclusion on the basis of substituting one or more possible values for *e*, and did not consider the full set of possible values.

Statistics for M2 Practice Paper Silver 3

				Mean average scored by candidates achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	6		82.5	4.95	5.57	5.39	4.74	3.68	3.48	1.94	1.51
2	9		74.8	6.73		8.12	6.66	5.55	4.68	3.55	2.45
3	9		73.4	6.61	7.63	6.79	6.24	6.11	6.00	5.97	4.80
4	11		64.4	7.08	8.78	7.63	6.22	5.37	2.96	2.94	0.93
5	12		69.3	8.31		9.23	6.67	4.65	4.22	3.02	1.25
6	12		64.8	7.77	9.41	8.50	6.01	4.65	3.30	2.57	1.92
7	16	11	61.0	9.72	11.53	10.74	8.82	7.61	6.55	5.06	3.06
	75		**68.2**	**51.17**		**56.40**	**45.36**	**37.62**	**31.19**	**25.05**	**15.92**