6677/01

Edexcel GCE

Mechanics M1
Silver Level S3

Time: 1 hour 30 minutes

Materials required for examination   **Items included with question papers**
Mathematical Formulae (Green)   Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M1), the paper reference (6677), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
68	60	52	44	34	27

1. A firework rocket starts from rest at ground level and moves vertically. In the first 3 s of its motion, the rocket rises 27 m. The rocket is modelled as a particle moving with constant acceleration a m s^{-2}. Find

(a) the value of a,

(2)

(b) the speed of the rocket 3 s after it has left the ground.

(2)

After 3 s, the rocket burns out. The motion of the rocket is now modelled as that of a particle moving freely under gravity.

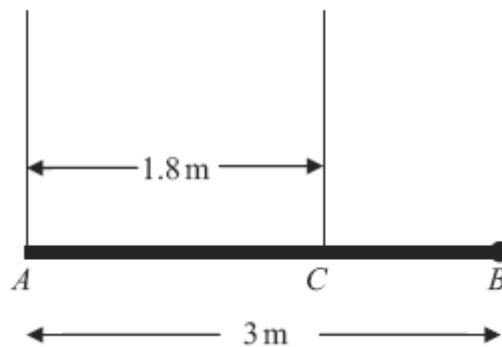(c) Find the height of the rocket above the ground 5 s after it has left the ground.

(4)

2.

Figure 2

A pole AB has length 3 m and weight W newtons. The pole is held in a horizontal position in equilibrium by two vertical ropes attached to the pole at the points A and C where $AC = 1.8$ m, as shown in Figure 2. A load of weight 20 N is attached to the rod at B. The pole is modelled as a uniform rod, the ropes as light inextensible strings and the load as a particle.

(a) Show that the tension in the rope attached to the pole at C is $\left(\dfrac{5}{6}W + \dfrac{100}{3}\right)$ N.

(4)

(b) Find, in terms of W, the tension in the rope attached to the pole at A.

(3)

Given that the tension in the rope attached to the pole at C is eight times the tension in the rope attached to the pole at A,

(c) find the value of W.

(3)

2

3. A stone is projected vertically upwards from a point A with speed u m s^{-1}. After projection the stone moves freely under gravity until it returns to A. The time between the instant that the stone is projected and the instant that it returns to A is $3\frac{4}{7}$ seconds.

Modelling the stone as a particle,

(a) show that $u = 17\frac{1}{2}$,

(3)

(b) find the greatest height above A reached by the stone,

(2)

(c) find the length of time for which the stone is at least $6\frac{3}{5}$ m above A.

(6)

4.

Figure 4
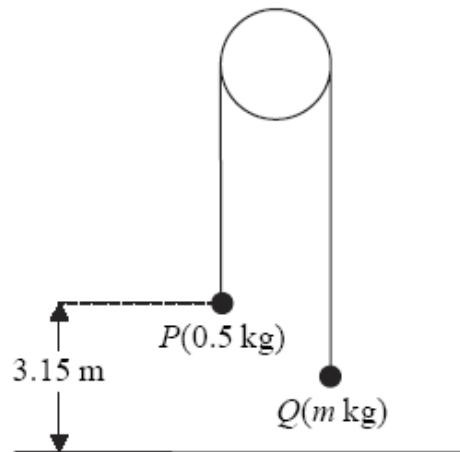
Two particles P and Q have mass 0.5 kg and m kg respectively, where $m < 0.5$. The particles are connected by a light inextensible string which passes over a smooth, fixed pulley. Initially P is 3.15 m above horizontal ground. The particles are released from rest with the string taut and the hanging parts of the string vertical, as shown in Figure 4. After P has been descending for 1.5 s, it strikes the ground. Particle P reaches the ground before Q has reached the pulley.

(*a*) Show that the acceleration of P as it descends is 2.8 m s^{-2}.

(3)

(*b*) Find the tension in the string as P descends.

(3)

(*c*) Show that $m = \frac{5}{18}$.

(4)

(*d*) State how you have used the information that the string is inextensible.

(1)

When P strikes the ground, P does not rebound and the string becomes slack. Particle Q then moves freely under gravity, without reaching the pulley, until the string becomes taut again.

(*e*) Find the time between the instant when P strikes the ground and the instant when the string becomes taut again.
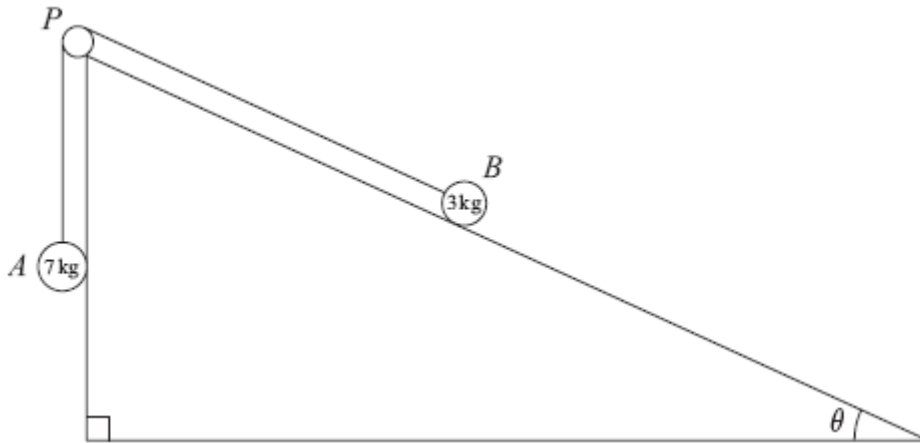
(6)

5.

Figure 4

Two particles A and B, of mass 7 kg and 3 kg respectively, are attached to the ends of a light inextensible string. Initially B is held at rest on a rough fixed plane inclined at angle θ to the horizontal, where $\tan \theta = \frac{5}{12}$. The part of the string from B to P is parallel to a line of greatest slope of the plane. The string passes over a small smooth pulley, P, fixed at the top of the plane. The particle A hangs freely below P, as shown in Figure 4. The coefficient of friction between B and the plane is $\frac{2}{3}$. The particles are released from rest with the string taut and B moves up the plane.

(*a*) Find the magnitude of the acceleration of B immediately after release.

(10)

(*b*) Find the speed of B when it has moved 1 m up the plane.

(2)

When B has moved 1 m up the plane the string breaks. Given that in the subsequent motion B does not reach P,

(*c*) find the time between the instants when the string breaks and when B comes to instantaneous rest.

(4)

6. [*In this question* **i** *and* **j** *are horizontal unit vectors due east and due north respectively.*]

A hiker *H* is walking with constant velocity $(1.2\mathbf{i} - 0.9\mathbf{j})$ m s^{-1}.
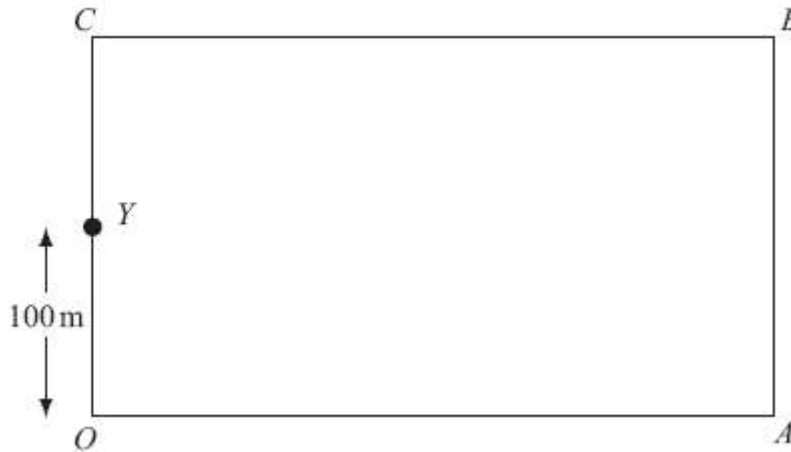
(*a*) Find the speed of *H*.

(2)

Figure 3

A horizontal field *OABC* is rectangular with *OA* due east and *OC* due north, as shown in Figure 3. At twelve noon hiker *H* is at the point *Y* with position vector 100**j** m, relative to the fixed origin *O*.

(*b*) Write down the position vector of *H* at time *t* seconds after noon.

(2)

At noon, another hiker *K* is at the point with position vector $(9\mathbf{i} + 46\mathbf{j})$ m. Hiker *K* is moving with constant velocity $(0.75\mathbf{i} + 1.8\mathbf{j})$ m s^{-1}.

(*c*) Show that, at time *t* seconds after noon,

$$\overrightarrow{HK} = [(9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j}] \text{ metres.}$$

(4)

Hence,

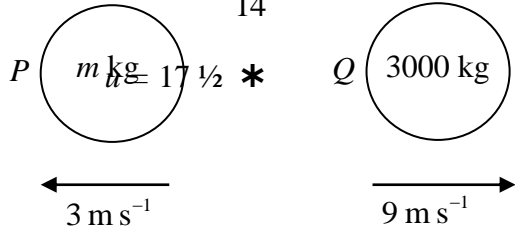(*d*) show that the two hikers meet and find the position vector of the point where they meet.
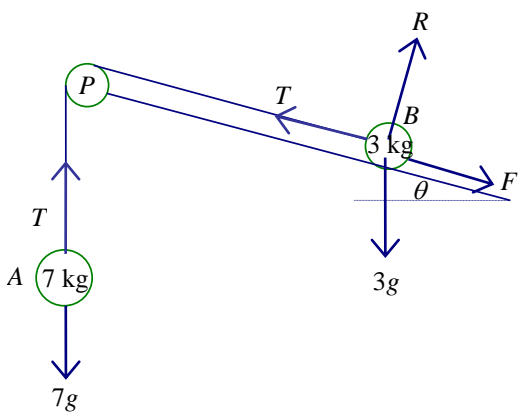
(5)

TOTAL FOR PAPER: 75 MARKS

END

1	(a)	$27 = 0 + \frac{1}{2}.a.3^2 \quad \Rightarrow a = \underline{6}$	M1 A1 (2)
	(b)	$v = 6 \times 3 = \underline{18 \text{ m s}^{-1}}$	M1 A1 f.t. (2)
	(c)	From $t = 3$ to $t = 5$, $s = 18 \times 2 - \frac{1}{2} \times 9.8 \times 2^2$	M1 A1 f.t.
		Total ht. $= s + 27 = \underline{43.4 \text{ m, } 43 \text{ m}}$	M1 A1 (4)
			8

Q2.

(a)

$$\text{M}(A) \qquad W \times 1.5 + 20 \times 3 = Y \times 1.8 \qquad\qquad \text{M1 A2 (1, 0)}$$

$$Y = \frac{5}{6}W + \frac{100}{3} \quad \ast \qquad\qquad \text{cso} \qquad \text{A1} \quad (4)$$

(b)   ↑ $X + Y = W + 20$ or equivalent   M1 A1

$$X = \frac{1}{6}W - \frac{40}{3} \qquad\qquad\qquad \text{A1} \quad (3)$$

(c)

$$\frac{5}{6}W + \frac{100}{3} = 8\left(\frac{1}{6}W - \frac{40}{3}\right) \qquad\qquad \text{M1 A1 ft}$$

$$W = 280 \qquad\qquad\qquad\qquad \text{A1} \quad (3)$$
                                                                      [10]

Alternative to (b)

M(C) $X \times 1.8 + 20 \times 1.2 = W \times 0.3$ M1 A1

$$X = \frac{1}{6}W - \frac{40}{3} \qquad\qquad\qquad \text{A1}$$

Question Number	Scheme	Marks
3. (a)	$v = u + at(\rightarrow) \Rightarrow 0 = u - g(\frac{25}{14})$ $u = 17\frac{1}{2}$	M1 M(A)1 A1 (3)
(b)	$v^2 = u^2 + 2as(-) \Rightarrow 0^2 = 17.5^2 - 2gs$ $s = 15.6 \ (\text{m}) \ \text{or} \ 16 \ (\text{m})$	M1 A1 (2)
(c)	$s = ut + \frac{1}{2}at^2 (-) \Rightarrow 6.6 = 17.5t - \frac{1}{2}gt^2$ $4.9t^2 - 17.5t + 6.6 = 0$ $t = \dfrac{17.5 \pm \sqrt{(17.5^2 - 129.36)}}{9.8} = \dfrac{17.5 \pm 13.3}{9.8}$ $t = 3.142.. \ (22/7) \ \text{or} \ 0.428\ldots(3/7)$ $T = t_2 - t_1 = 2.71 \ (2.7)$	M1 A1 DM1 A1 DM1 A1 (6) **(11 marks)**

In the diagram for part (a): P (m kg), Q (3000 kg), with 15 m s^{-1}, 3 m s^{-1} to the left, 9 m s^{-1} to the right.

Question Number	Scheme	Marks
4.	(a) $s = ut + \frac{1}{2}at^2 \implies 3.15 = \frac{1}{2}a \times \frac{9}{4}$ $\qquad\qquad a = 2.8 \ \left(\mathrm{ms^{-2}}\right)$ ✳ cso	M1 A1 A1 **(3)**
	(b) N2L for P: $0.5g - T = 0.5 \times 2.8$ $\qquad\qquad\qquad T = 3.5 \ \left(\mathrm{N}\right)$	M1 A1 A1 **(3)**
	(c) N2L for Q: $T - mg = 2.8m$ $\qquad\qquad m = \dfrac{3.5}{12.6} = \dfrac{5}{18}$ ✳ cso	M1 A1 DM1 A1 **(4)**
	(d) The acceleration of P is equal to the acceleration of Q.	B1 **(1)**
	(e) $\qquad\qquad\qquad v = u + at \implies v = 2.8 \times 1.5$ $\qquad$ (or $\quad v^2 = u^2 + 2as \implies v^2 = 2 \times 2.8 \times 3.15$) $\qquad\qquad\qquad \left(v^2 = 17.64, v = 4.2\right)$ $\qquad v = u + at \implies 4.2 = -4.2 + 9.8t$ $\qquad\qquad\qquad t = \dfrac{6}{7}, \ 0.86, \ 0.857 \ \ (\mathrm{s})$	M1 A1 DM1 A1 DM1 A1 **(6)** **[17]**

Question Number	Scheme	Marks
5. **(a)**	$\tan\theta = \dfrac{5}{12}$ $\sin\theta = \dfrac{5}{13}$ $\cos\theta = \dfrac{12}{13}$	

For A: $7g - T = 7a$ — M1 A1

For B: parallel to plane $T - F - 3g\sin\theta = 3a$ — M1 A1

perpendicular to plane $R = 3g\cos\theta$ — M1 A1

$F = \mu R = \tfrac{2}{3}\,3g\cos\theta = 2g\cos\theta$ — M1

Eliminating T, $7g - F - 3g\sin\theta = 10a$ — DM1

Equation in g and a: $7g - 2g \times \dfrac{12}{13} - 3g\dfrac{5}{13} = 7g - \dfrac{39}{13}g = 4g = 10a$ — DM1

$a = \dfrac{2g}{5}$ oe or 3.9 or 3.92 — A1

(10)

(b) After 1 m,

$v^2 = u^2 + 2as$, $\quad v^2 = 0 + 2\times\dfrac{2g}{5}\times 1$ — M1

$v = 2.8$ — A1

(2)

(c) $-(F + 3g\,\sin\theta) = 3a$ — M1

$\dfrac{2}{3}\times 3g \times \dfrac{12}{13} + 3g\times\dfrac{5}{13} = 3g = -3a,\ a = -g$ — A1

$v = u + at$, $0 = 2.8 - 9.8t$, — DM1

$t = \dfrac{2}{7}$ oe, 0.29. 0.286 — A1

(4)

[16]

10

6.	(a)	$\|\mathbf{v}\| = \sqrt{1.2^2 + (-0.9)^2} = 1.5 \text{ m s}^{-1}$	M1 A1 (2)
	(b)	$(\mathbf{r}_H =)100\mathbf{j} + t(1.2\mathbf{i} - 0.9\mathbf{j}) \text{ m}$	M1 A1 (2)
			M1 A1
	(c)	$(\mathbf{r}_K =)9\mathbf{i} + 46\mathbf{j} + t(0.75\mathbf{i} + 1.8\mathbf{j}) \text{ m}$	
		$\overrightarrow{HK} = \mathbf{r}_K - \mathbf{r}_H = (9 - 0.45t)\mathbf{i} + (2.7t - 54)\mathbf{j} \text{ m}$ **Printed Answer**	M1 A1 (4)
	(d)	Meet when $\overrightarrow{HK} = \mathbf{0}$	
		$(9 - 0.45t) = 0$ and $(2.7t - 54) = 0$	M1 A1
		$t = 20$ from both equations	A1
		$\mathbf{r}_K = \mathbf{r}_H = (24\mathbf{i} + 82\mathbf{j}) \text{ m}$	M1 A1 **cso** (5)
			(13 marks)

Examiner reports

Question 1

There were various approaches that could be applied successfully to answer this question. Those who fully understood the implications of projecting from above ground level could achieve full marks by the most direct method although sign errors were not uncommon. Another popular approach was to split the motion into two stages (to and from the highest point) in both part (a) to find the initial velocity, and in part (b) to find the whole time. Although this required more working, there tended to be fewer sign errors. Premature approximation occasionally led to inaccuracy in the final answer. The weakest candidates sometimes only considered motion to or from the highest point. It should be noted that the rubric requires g = 9.8 to be used and not 9.81, which was penalised.

Question 2

This question was well done by the majority of candidates and was the next best answered question after 1 and 2. Most made valid attempts at taking moments, in part (a) about A and often also about C in part (b).The printed answer was an additional help to the less able students who were able to score the marks in part (b) by using it in a vertical resolution. There was some confusion in the last part over the interpretation and use of the information given. Correct statements of simply $Y = 8X$ or else $8X + X = W + 20$ were seen but also $X = 8Y$ was common as were the more surprising $X + 8Y = W + 20$ and $8X + Y = W + 20$, both of which scored nothing.

Question 3

In the first part, most candidates derived the value of u in a valid way, either by considering motion to the highest point and using half the given time, or using a displacement of zero with the full $3\frac{4}{7}$ seconds; confusion between the two methods was usually avoided since the answer was given in the question, although occasionally an answer of '35', obtained from using inconsistent values for s and t, was divided by two with no explanation. Finding the greatest height in part (b) was generally well done with the main source of error being in giving an answer to more than 3 significant figures (not justified with $g = 9.8$).

In part (c) there were many alternative valid approaches to finding the time for which the particle was above the given height. Perhaps the most common of these was to set up a quadratic equation in t. This was generally solved successfully, with occasional sign errors, but the significance of the two solutions was not always recognised; it was necessary to find their difference to reach the final answer. Another common approach was to find the velocity 'v =13.3' at 's = 6.6' and then, either find the time to the highest point (and double), or find the time to return to that level. Alternatively, the time taken to reach 's = 6.6' was calculated, doubled, and subtracted from $3\frac{4}{7}$. Although a wide variety of correct working was seen, some candidates did lose their way and calculated inappropriate values for t. Again, the final answer was required to 2 or 3 significant figures, although over-accuracy is only penalised once per question. The use of $g = 9.81$ was seen occasionally and led to the loss of one mark for the whole question.

Question 4

Part (a)
Many candidates seemed to expect that the first part of the question would require equations of motion for each particle. Once into relevant calculations, however, most candidates were very successful in obtaining 2.8 m s $^{-2}$. The majority of successful candidates attempted this part directly using $s = ut + \frac{1}{2}at^2$. Others used a two step approach using $v = u + at$ to give $v = 4.2$, followed by use of another *suvat* formula to get 2.8. A very few tried a verification method which did gain them maximum marks at this stage.

Part (b) Candidates generally formed an $F = ma$ equation with the majority obtaining the correct equation and getting $T = 3.5$N. However there was still a sizeable number who mistakenly wrote T- 0.5g = 0.5a =1.4. It is noticeable that despite regular comment from Edexcel some candidates still use g = 9.81 which leads to marks being lost in a variety of places where accuracy matters. *Part (c)* Many candidates formed a relevant equation, using the correct forces, reaching the stage of $T = 3.5 = (2.8 + g) m$ and then went straight to $m = 5/18$, resulting in the loss of a mark. For many candidates there is still a lack of dexterity with the manipulation of fractions. Moreover, there is still a sizeable number of candidates who try to use one equation of motion for the whole system, despite advice to the contrary in several recent examiners reports.

Part (d) In this part, modelling was being tested and candidates needed to show that they really knew what was happening. A large number of candidates gave the correct answer that "both particles move with same acceleration" , gaining the single mark available. However candidates who tried to play safe and included another irrelevant reason, such as same tension, had not shown full understanding of the model and therefore were penalised. Other wrong answers included saying that acceleration was constant.

Part (e) Here, candidates first needed to find the speed of the system when the particle hit the ground. This required the calculation of $v = 4.2$ which some candidates merely quoted. This is the part of the question where candidates began to lose marks and common errors at this stage included using an incorrect value for acceleration. The question then continued with testing vertical motion under gravity. Successful candidates used a variety of equivalent methods. Some worked out the time to the top , followed by a calculation of distance followed by a calculation for time to fall back to launch point, followed by the addition of the two times, giving the answer to the correct degree of accuracy. Some took a more direct approach and used $s = ut + \frac{1}{2}at^2$ or $v = u + at$, for the whole of the remaining motion i.e. up and down. Many only found the time to the top and lost the final two marks. Common errors involved use of incorrect accelerations, displacements and times. Again candidates seem to want to work in decimals rather than in fractions. Candidates should be encouraged to make greater use of diagrams.

Question 5

Despite the lack of structure in part (a) most candidates knew the methods required. Many gained the marks for resolving perpendicular to the slope and for using $F = \frac{2}{3} R$. The equation of motion for the 7 kg mass was also often correct, but a common error was to replace T by $7g$ when resolving parallel to the plane. Sometimes a term, either friction or the weight component, was omitted from this equation. Although some candidates failed to complete successfully all the substitution and rearranging required to find the acceleration, there were a number of entirely correct solutions. An appropriate constant acceleration formula was generally used for the velocity in part (b), although an incorrect answer from part (a) led to loss of the accuracy mark. In the final part, many did not manage to find the new acceleration by a valid method; some used the value from part (a) or quoted '9.8' without any justification,

whilst others realised that a new value was required but omitted a term from the equation of motion. Amongst those who resolved the two forces, a number had friction acting in the wrong direction. There were a small number of entirely correct solutions seen.

Question 6

Most candidates were able to gain the first six marks and most seemed to know that, in part (c), they needed to perform a subtraction on $\mathbf{r}_H$ and $\mathbf{r}_K$ although some were unsure which way round to do it. Another common error was to equate the position vectors and then fudge the answer. This received no credit.

In part (d) many candidates assumed that the hikers would meet and equated just one pair of components to produce $t = 20$. If they then used just one hiker to find $24\mathbf{i} + 82\mathbf{j}$ they scored only 2 out of 5, if they used **both** hikers, they scored full marks. There were a number of other ways of obtaining $t = 20$, some spurious, but provided that the candidate verified that both hikers were at the point with position vector $24\mathbf{i} + 82\mathbf{j}$ at $t = 20,$ they could score all of the marks.

Statistics for M1 Practice Paper Silver Level S3

Original paper	Qu	Mean score	Max score	Mean %	Mean score for students achieving grade:						
					ALL	A	B	C	D	E	U
0801	1	5.90	8	74	5.90	6.82	5.80	5.20	4.68	3.86	3.05
1001	2	7.38	10	74	7.38	9.21	7.89	6.48	4.45	2.88	1.24
1201	3	7.28	11	66	7.28	8.99	7.42	6.17	4.93	4.10	2.91
0706	4	10.37	17	61	10.37	14.52	11.87	9.51	7.20	5.12	2.20
1101	5	9.10	16	57	9.10	12.27	8.75	6.10	4.17	2.87	0.99
0906	6	7.96	13	61	7.96	11.38	8.99	6.99	5.21	4.01	1.82
		47.99	75	64	47.99	63.19	50.72	40.45	30.64	22.84	12.21

14