Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Silver Level S4

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
65	58	50	44	37	31

1. (*a*) Find the binomial expansion of

$$\sqrt{(1-8x)}, \qquad |x| < \frac{1}{8},$$

in ascending powers of x up to and including the term in x^3, simplifying each term.

(4)

(*b*) Show that, when $x = \frac{1}{100}$, the exact value of $\sqrt{(1-8x)}$ is $\frac{\sqrt{23}}{5}$.

(2)

(*c*) Substitute $x = \frac{1}{100}$ into the binomial expansion in part (*a*) and hence obtain an approximation to $\sqrt{23}$. Give your answer to 5 decimal places.

(3)

January 2010

2.

$$\mathrm{f}(x) = \frac{1}{\sqrt{(9+4x^2)}}, \qquad |x| < \frac{3}{2}.$$

Find the first three non-zero terms of the binomial expansion of f(x) in ascending powers of x. Give each coefficient as a simplified fraction.

(6)

June 2011

3.

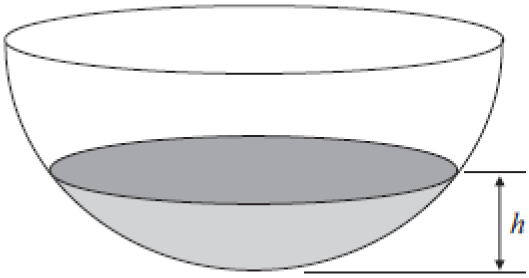

Figure 1

A hollow hemispherical bowl is shown in Figure 1. Water is flowing into the bowl.

When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \le h \le 0.25.$$

(a) Find, in terms of π, $\frac{dV}{dh}$ when $h = 0.1$.

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^3 s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$.

(2)

June 2011

4.

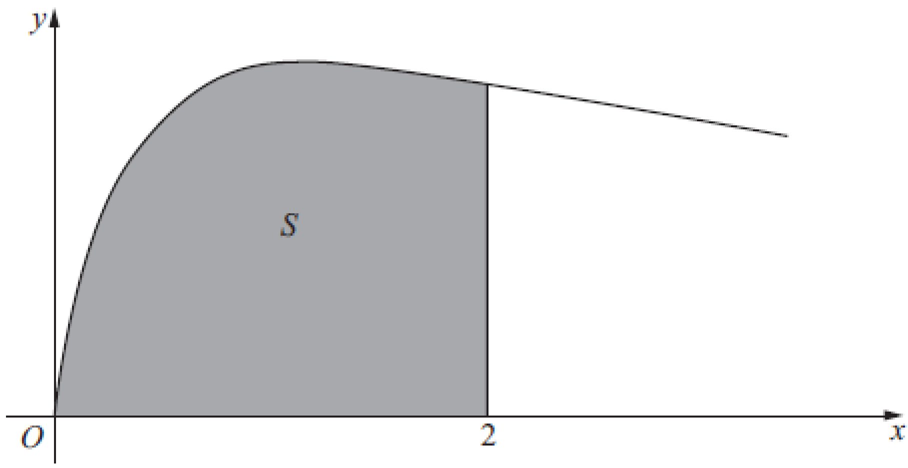

Figure 1

Figure 1 shows the curve with equation

$$y = \sqrt{\left(\frac{2x}{3x^2 + 4}\right)}, \quad x \geq 0.$$

The finite region S, shown shaded in Figure 1, is bounded by the curve, the x-axis and the line $x = 2$.

The region S is rotated 360° about the x-axis.

Use integration to find the exact value of the volume of the solid generated, giving your answer in the form $k \ln a$, where k and a are constants.

(5)

January 2012

5.

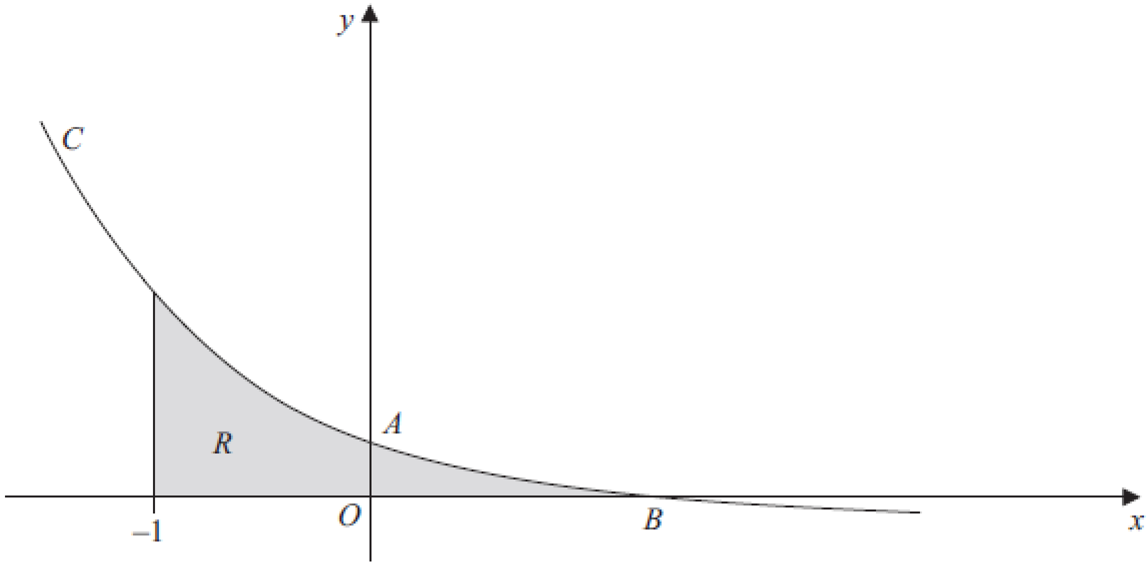

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \qquad y = 2^t - 1.$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(*a*) Show that A has coordinates (0, 3).

(2)

(*b*) Find the x-coordinate of the point B.

(2)

(*c*) Find an equation of the normal to C at the point A.

(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = –1$ and the x-axis.

(*d*) Use integration to find the exact area of R.

(6)

January 2013

6. A curve has parametric equations

$$x = \tan^2 t, \qquad y = \sin t, \qquad 0 < t < \frac{\pi}{2}.$$

(*a*) Find an expression for $\frac{dy}{dx}$ in terms of *t*. You need not simplify your answer.

(3)

(*b*) Find an equation of the tangent to the curve at the point where $t = \frac{\pi}{4}$.

Give your answer in the form $y = ax + b$, where *a* and *b* are constants to be determined.

(5)

(*c*) Find a cartesian equation of the curve in the form $y^2 = f(x)$.

(4)

June 2007

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7. A curve is described by the equation

$$x^2 + 4xy + y^2 + 27 = 0$$

(*a*) Find $\frac{dy}{dx}$ in terms of *x* and *y*.

(5)

A point *Q* lies on the curve.

The tangent to the curve at *Q* is parallel to the *y*-axis.

Given that the *x*-coordinate of *Q* is negative,

(*b*) use your answer to part (*a*) to find the coordinates of *Q*.

(7)

June 2013

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8. (*a*) Using the identity $\cos 2\theta = 1 - 2\sin^2\theta$, find $\int \sin^2\theta \; d\theta$.

(2)

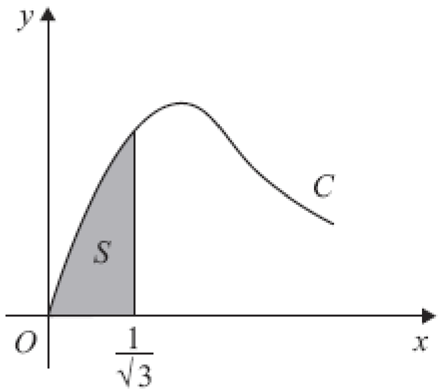

Figure 4

Figure 4 shows part of the curve C with parametric equations

$$x = \tan\theta, \quad y = 2\sin 2\theta, \quad 0 \leq \theta < \frac{\pi}{2}.$$

The finite shaded region S shown in Figure 4 is bounded by C, the line $x = \frac{1}{\sqrt{3}}$ and the x-axis. This shaded region is rotated through 2π radians about the x-axis to form a solid of revolution.

(*b*) Show that the volume of the solid of revolution formed is given by the integral

$$k\int_0^{\frac{\pi}{6}} \sin^2\theta \; d\theta,$$

where k is a constant.

(5)

(*c*) Hence find the exact value for this volume, giving your answer in the form $p\pi^2 + q\pi\sqrt{3}$, where p and q are constants.

(3)

June 2009

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
Q1	(a) $(1-8x)^{\frac{1}{2}} = 1+\left(\tfrac{1}{2}\right)(-8x)+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-8x)^2+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{3!}(-8x)^3+\dots$	M1 A1
	$= 1-4x-8x^2; -32x^3 - \dots$	A1; A1 (4)
	(b) $\sqrt{(1-8x)} = \sqrt{\left(1-\dfrac{8}{100}\right)}$	M1
	$= \sqrt{\dfrac{92}{100}} = \sqrt{\dfrac{23}{25}} = \dfrac{\sqrt{23}}{5}$ *	A1 (2)
	cso	
	(c) $1-4x-8x^2-32x^3 = 1-4(0.01)-8(0.01)^2-32(0.01)^3$	
	$= 1-0.04-0.0008-0.000\,032 = 0.959\,168$	M1
	$\sqrt{23} = 5\times 0.959\,168$	M1
	$= 4.795\,84$	A1 (3)
	cao	
		[9]

2.	$f(x) = (\ \dots + \dots)^{-\frac{1}{2}}$	M1
	$= 9^{-\frac{1}{2}}(\ \dots + \dots)^{\dots}$ 3^{-1}, $\dfrac{1}{3}$ or $\dfrac{1}{9^{\frac{1}{2}}}$	B1
	$(1+kx^2)^n = 1+nkx^2+\dots$ n not a natural number, $k \neq 1$	M1
	$(1+kx^2)^{-\frac{1}{2}} = \dots + \dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx^2)^2$ ft their $k \neq 1$	A1 ft
	$\left(1+\dfrac{4}{9}x^2\right)^{-\frac{1}{2}} = 1-\dfrac{2}{9}x^2+\dfrac{2}{27}x^4$	A1
	$f(x) = \dfrac{1}{3}-\dfrac{2}{27}x^2+\dfrac{2}{81}x^4$	A1 **(6)**
		[6]

3.	(a) $\dfrac{dV}{dh}=\dfrac{1}{2}\pi h-\pi h^2$	or equivalent	M1 A1
	At $h=0.1$, $\dfrac{dV}{dh}=\dfrac{1}{2}\pi(0.1)-\pi(0.1)^2=0.04\pi$	$\dfrac{\pi}{25}$	M1 A1 **(4)**
	(b) $\dfrac{dh}{dt}=\dfrac{dV}{dt}\div\dfrac{dV}{dh}=\dfrac{\pi}{800}\times\dfrac{1}{\frac{1}{2}\pi h-\pi h^2}$	or $\dfrac{\pi}{800}\div$ their (a)	M1
	At $h=0.1$, $\dfrac{dh}{dt}=\dfrac{\pi}{800}\times\dfrac{25}{\pi}=\dfrac{1}{32}$	awrt 0.031	A1 **(2)**
			[6]

4.	Volume $=\underline{\pi\displaystyle\int_0^2\left(\sqrt{\left(\frac{2x}{3x^2+4}\right)}\right)^2\,dx}$	Use of $\underline{V=\pi\int y^2}\,dx$.	<u>B1</u>
	$=(\pi)\left[\dfrac{1}{3}\ln(3x^2+4)\right]_0^2$	$\pm k\ln(3x^2+4)$	M1
		$\dfrac{1}{3}\ln(3x^2+4)$	A1
	$=(\pi)\left[\left(\dfrac{1}{3}\ln 16\right)-\left(\dfrac{1}{3}\ln 4\right)\right]$	Substitutes limits of 2 and 0 and subtracts the correct way round.	dM1
	So Volume $=\underline{\dfrac{1}{3}\pi\ln 4}$	$\underline{\dfrac{1}{3}\pi\ln 4}$ or $\underline{\dfrac{2}{3}\pi\ln 2}$	A1 oe isw
			[5] **(5 marks)**

Question Number	Scheme		Marks
5.	***Working parametrically:***		
	$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$ or $y = e^{t\ln 2} - 1$		
(a)	$\{x = 0 \Rightarrow\}\ 0 = 1 - \frac{1}{2}t \Rightarrow t = 2$	Applies $x = 0$ to obtain a value for t.	M1
	When $t = 2$, $y = 2^2 - 1 = 3$	Correct value for y.	A1 **[2]**
(b)	$\{y = 0 \Rightarrow\}\ 0 = 2^t - 1 \Rightarrow t = 0$	Applies $y = 0$ to obtain a value for t. (Must be seen in part (b)).	M1
	When $t = 0$, $x = 1 - \frac{1}{2}(0) = 1$	$x = 1$	A1 **[2]**
(c)	$\frac{dx}{dt} = -\frac{1}{2}$ and either $\frac{dy}{dt} = 2^t \ln 2$ or $\frac{dy}{dt} = e^{t\ln 2}\ln 2$		B1
	$\frac{dy}{dx} = \frac{2^t \ln 2}{-\frac{1}{2}}$	Attempts their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$.	M1
	At A, $t = "2"$, so $m(\mathbf{T}) = -8\ln 2 \Rightarrow m(\mathbf{N}) = \frac{1}{8\ln 2}$	Applies $t = "2"$ and $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1
	$y - 3 = \frac{1}{8\ln 2}(x - 0)$ or $y = 3 + \frac{1}{8\ln 2}x$ or equivalent.		M1 A1 oe cso **[5]**
(d)	$\text{Area}(R) = \int (2^t - 1).\left(-\frac{1}{2}\right)dt$	Complete substitution for both y and dx	M1
	$x = -1 \to t = 4$ and $x = 1 \to t = 0$		B1
	$= \left\{-\frac{1}{2}\right\}\left(\frac{2^t}{\ln 2} - t\right)$	Either $2^t \to \frac{2^t}{\ln 2}$ or $(2^t - 1) \to \frac{(2^t)}{\pm\alpha(\ln 2)} - t$ or $(2^t - 1) \to \pm\alpha(\ln 2)(2^t) - t$	M1*
		$(2^t - 1) \to \frac{2^t}{\ln 2} - t$	A1
	$\left\{-\frac{1}{2}\left[\frac{2^t}{\ln 2} - t\right]_4^0\right\} = -\frac{1}{2}\left(\left(\frac{1}{\ln 2}\right) - \left(\frac{16}{\ln 2} - 4\right)\right)$	**Depends on the previous method mark.** Substitutes their changed limits in t and subtracts either way round.	dM1*
	$= \frac{15}{2\ln 2} - 2$	$\frac{15}{2\ln 2} - 2$ or equivalent.	A1
			[6] **15**

Question Number	Scheme		Marks
6. (a)	$x = \tan^2 t$, $y = \sin t$		
	$\frac{dx}{dt} = 2(\tan t)\sec^2 t$, $\frac{dy}{dt} = \cos t$	Correct $\frac{dx}{dt}$ and $\frac{dy}{dt}$	B1
	$\therefore \frac{dy}{dx} = \frac{\cos t}{2\tan t \sec^2 t}$ $\left(= \frac{\cos^4 t}{2\sin t}\right)$	$\frac{\pm\cos t}{\text{their } \frac{dx}{dt}}$	M1
		$\frac{+\cos t}{\text{their } \frac{dx}{dt}}$	A1 $\sqrt{}$
			[3]
(b)	When $t = \frac{\pi}{4}$, $x = 1$, $y = \frac{1}{\sqrt{2}}$ (need values)	The point $\left(1, \frac{1}{\sqrt{2}}\right)$ or $(1, \text{awrt } 0.71)$ **These coordinates can be implied.** ($y = \sin\left(\frac{\pi}{4}\right)$ is not sufficient for B1)	B1, B1
	When $t = \frac{\pi}{4}$, m(**T**) $= \frac{dy}{dx} = \frac{\cos\frac{\pi}{4}}{2\tan\frac{\pi}{4}\sec^2\frac{\pi}{4}}$		
	$= \frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{\sqrt{2}}}\right)^2} = \frac{\frac{1}{\sqrt{2}}}{2.(1)\left(\frac{1}{\frac{1}{2}}\right)} = \frac{\frac{1}{\sqrt{2}}}{2.(1)(2)} = \frac{1}{4\sqrt{2}} = \frac{\sqrt{2}}{8}$	any of the five underlined expressions or awrt 0.18	B1 aef
	T: $y - \frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(x - 1)$	Finding an equation of a tangent with ***their point*** and ***their tangent gradient*** or finds c by using $y = (\text{their gradient})x + "c"$.	M1 $\sqrt{}$ aef
	T: $y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}$ or $y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$	Correct simplified EXACT equation of tangent	A1 aef **cso**
	or $\frac{1}{\sqrt{2}} = \frac{1}{4\sqrt{2}}(1) + c \Rightarrow c = \frac{1}{\sqrt{2}} - \frac{1}{4\sqrt{2}} = \frac{3}{4\sqrt{2}}$		
	Hence **T:** $y = \frac{1}{4\sqrt{2}}x + \frac{3}{4\sqrt{2}}$ or $y = \frac{\sqrt{2}}{8}x + \frac{3\sqrt{2}}{8}$		
			[5]

Question Number	Scheme	Marks
7.	$x^2 + 4xy + y^2 + 27 = 0$	
(a)	$\left\{ \frac{\cancel{dy}}{\cancel{dx}} = \right\}$ $\underline{2x} + \left(\underline{\underline{4y + 4x\frac{dy}{dx}}}\right) + \underline{2y\frac{dy}{dx}} = \underline{0}$	M1 A1 B1
	$2x + 4y + (4x + 2y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{-2x - 4y}{4x + 2y}$ $\left\{ = \frac{-x - 2y}{2x + y} \right\}$	A1 **cso oe**
		(5)
(b)	$4x + 2y = 0$	M1
	$y = -2x$ | $x = -\frac{1}{2}y$	A1
	$x^2 + 4x(-2x) + (-2x)^2 + 27 = 0$ | $\left(-\frac{1}{2}y\right)^2 + 4\left(-\frac{1}{2}y\right)y + y^2 + 27 = 0$	M1*
	$-3x^2 + 27 = 0$ | $-\frac{3}{4}y^2 + 27 = 0$	
	$x^2 = 9$ | $y^2 = 36$	dM1*
	$x = -3$ | $y = 6$	A1
	When $x = -3$, $y = -2(-3)$ | When $y = 6$, $x = -\frac{1}{2}(6)$	ddM1*
	$y = 6$ | $x = -3$	A1 **cso**
		(7)
		[12]

8.	(a)	$\int \sin^2\theta \, d\theta = \frac{1}{2}\int (1-\cos 2\theta)\, d\theta = \frac{1}{2}\theta - \frac{1}{4}\sin 2\theta \quad (+C)$	M1 A1 (2)
		$x = \tan\theta \Rightarrow \frac{dx}{d\theta} = \sec^2\theta$	
	(b)	$\pi\int y^2\, dx = \pi\int y^2 \frac{dx}{d\theta}\, d\theta = \pi\int (2\sin 2\theta)^2 \sec^2\theta \, d\theta$	M1 A1
		$= \pi\int \frac{(2\times 2\sin\theta\cos\theta)^2}{\cos^2\theta}\, d\theta$	M1
		$= 16\pi\int \sin^2\theta \, d\theta \qquad k = 16\pi$	A1
		$x = 0 \Rightarrow \tan\theta = 0 \Rightarrow \theta = 0, \quad x = \frac{1}{\sqrt{3}} \Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$	B1 (5)
	(c)	$\left(V = 16\pi\int_0^{\frac{\pi}{6}} \sin^2\theta \, d\theta\right)$	
		$V = 16\pi\left[\frac{1}{2}\theta - \frac{\sin 2\theta}{4}\right]_0^{\frac{\pi}{6}}$	M1
		$= 16\pi\left[\left(\frac{\pi}{12} - \frac{1}{4}\sin\frac{\pi}{3}\right) - (0-0)\right]$ Use of correct limits	M1
		$= 16\pi\left(\frac{\pi}{12} - \frac{\sqrt{3}}{8}\right) = \frac{4}{3}\pi^2 - 2\pi\sqrt{3} \qquad p = \frac{4}{3},\ q = -2$	A1 (3)
			(10 marks)

Question 1

This proved a suitable starting question and there were many completely correct solutions. The majority of candidates could complete part (a) successfully. In part (b), those who realised that working in common (vulgar) fractions was needed usually gained the method mark but, as noted in the introduction, the working needed to establish the printed result was frequently incomplete. It is insufficient to write down $\sqrt{1-\frac{8}{100}}=\frac{\sqrt{23}}{5}$. The examiners accepted, for example, $\sqrt{1-\frac{8}{100}}=\sqrt{\frac{92}{100}}=\sqrt{\frac{23}{25}}=\frac{\sqrt{23}}{5}$. In part (c), most candidates realised that they had to evaluate their answer to part (a) with $x=0.01$. However many failed to recognise the implication of part (b), that this evaluation needed to be multiplied by 5. It was not uncommon for candidates to confuse parts (b) and (c) with the expansion and decimal calculation appearing in (b) and fraction work leading to $\sqrt{23}$ appearing in (c).

Question 2

Many candidates got off to a very bad start to this question by writing $\sqrt{\left(9+4x^2\right)}=3+2x$ or $\left(9+4x^2\right)^{-\frac{1}{2}}=\left(3+2x\right)^{-1}$. Such errors in algebra are heavily penalised as the resulting binomial expansions are significantly simplified and, in this case, gave answers in incorrect powers of *x*. Those who obtained $\frac{1}{3}\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}}$ showed that they understood the binomial theorem but there were many errors in signs, often due to the failure to use brackets correctly. Some candidates seemed to lose the thread of the question and, having expanded $\left(1+\frac{4}{9}x^2\right)^{-\frac{1}{2}}$ correctly, failed to multiply by $\frac{1}{3}$. It was not unusual to see an, often correct, term in x^6 provided. The examiners ignore this but such additional work does lose time.

Question 3

This question was well done and full marks were common. Candidates were roughly equally divided between those who expanded and differentiated and those who differentiated using the product rule. The latter method was the more complicated and more subject to error but many correct solutions were seen using both methods. If the differentiation was correct, nearly all completed part (a) correctly. Rather oddly, a number of cases were seen where $\frac{1}{12}\pi h^2\left(3-4h\right)$ was misread as $\frac{1}{2}\pi h^2\left(3-4h\right)$. Part (b) was generally well done although there were a minority of students who made no attempt at it at all. The large majority correctly interpreted $\frac{\pi}{800}$ as $\frac{\mathrm{d}V}{\mathrm{d}t}$ and realised they had to divide $\frac{\pi}{800}$ by $\frac{\mathrm{d}V}{\mathrm{d}h}$. Inverting $\frac{\mathrm{d}V}{\mathrm{d}h}$ did cause difficulty for some candidates. For example, $\frac{1}{0.5\pi h-\pi h^2}=\frac{1}{0.5\pi h}-\frac{1}{\pi h^2}$ was seen from time to time and $\frac{1}{\frac{\pi}{25}} = 25\pi$ leading to the answer $\frac{\pi^2}{32}$, instead of the correct $\frac{1}{32}$, was relatively common.

Question 4

At least 90% of the candidature was able to apply the volume of revolution formula correctly. Only a few candidates did not include π in their volume formula or did not square the expression for y.

The integration was well attempted and the majority of candidates recognised that the integral could be manipulated into the form $\int \frac{f'(x)}{f(x)}\,dx$ and integrated to give their result in the form $k \ln(3x^2 + 4)$ usually with $k = \frac{1}{3}$. A variety of incorrect values of k were seen with the most common being either 3 or 1. A significant number of candidates integrated incorrectly to give answers such as $x^2 \ln(3x^2 + 4)$ or $2x \ln(3x^2 + 4)$.
Those candidates who applied the substitution $u = 3x^2 + 4$ proceeded to achieve $\frac{1}{3}\ln u$, and changed their x-limits of 0 and 2 to give correct u-limits of 4 and 16. Other substitutions of $u = 3x^2$ or $u = x^2$, were also used, usually successfully.

Unproductive attempts were seen by a minority of candidates, such as integration by parts or simplifying $\frac{2x}{3x^2+4}$ to give $\frac{2x}{3x^2} + \frac{2x}{4}$, or integrating $2x$ and $3x^2 + 4$ separately and then multiplying or dividing the two results together.

The majority of candidates were able to apply the limits correctly and examiners observed the correct answer in a variety of different forms.

Question 5

This question, and in particular the final Q5(d), proved challenging for a large number of candidates, with about 18% of the candidature scoring at least 12 of the 15 marks available and only about 7% scoring all 15 marks.

Q5(a) and Q5(b) were almost invariably completed correctly, the main source of error in Q5(b) being that a very small number of candidates did not realise that $t = 0$ follows from $2^t = 1$.

Many correct solutions to Q5(c) were seen. The principal reason for loss of marks came from candidates being unable to find the derivative of $2^t - 1$. Dividing $\frac{dy}{dt}$ by $\frac{dx}{dt}$ $\left(\text{i.e. dividing by } -\frac{1}{2}\right)$ proved challenging for a number of candidates. Some candidates, having correctly established $\frac{dy}{dx}$ as being $(-2)2^t \ln 2$, then proceeded incorrectly to equate this to $-4^t \ln 2$. Most knew how to obtain the gradient of the normal, and could write down the equation of a straight line.

Q5(d) was answered well by small number of candidates, and, although a significant number could write the area as $\int_4^0 (2^t - 1).\left(-\frac{1}{2}\right)\mathrm{d}t$, many were unable to perform the integration of 2^t with respect to *t*. Some wrote 2^t as $2t$, thus simplifying the problem, whilst attempts such as $\frac{2^{t+1}}{t+1}$ were not uncommon. Candidates who were unable to make an attempt at the integration of 2^t were unable to access the final 4 marks in this part. Approaches that facilitated integration included re-writing 2^t as $e^{t\ln 2}$ or substituting $u = 2^t$, leading to $\frac{\mathrm{d}u}{\mathrm{d}t} = 2^t \ln 2 = u \ln 2$, and thereby circumventing a direct integration of 2^t.

Other candidates used a cartesian approach, giving the area as $\int_{-1}^{1} \left(2^{2-2x} - 1\right)\mathrm{d}x$ (or equivalent), but again a number were unable to carry out the integration.

Question 6

In part (a), a significant number of candidates struggled with applying the chain rule in order to differentiate $\tan^2 t$ with respect to *t*. Some candidates replaced $\tan^2 t$ with $\frac{\sin^2 t}{\cos^2 t}$ and proceeded to differentiate this expression using both the chain rule and the quotient rule. Very few candidates incorrectly differentiated $\sin t$ to give $-\cos t$. A majority of candidates were then able to find $\frac{\mathrm{d}y}{\mathrm{d}x}$ by dividing their $\frac{\mathrm{d}y}{\mathrm{d}t}$ by their $\frac{\mathrm{d}x}{\mathrm{d}t}$.

In part (b), the majority of candidates were able to write down the point $\left(1, \frac{1}{\sqrt{2}}\right)$ and find the equation of the tangent using this point and their tangent gradient. Some candidates found the incorrect value of the tangent gradient using $t = \frac{\pi}{4}$ even though they had correctly found $\frac{\mathrm{d}y}{\mathrm{d}x}$ in part (a). There were a significant number of candidates, who having correctly written down the equation of the tangent as $y - \frac{1}{\sqrt{2}} = \frac{3\sqrt{2}}{8}(x-1)$ were unable to correctly rearrange this equation into the form $y = ax + b$.

In part (c), there were many ways that a candidate could tackle this question and there were many good solutions seen. Errors usually arose when candidates wrote down and used incorrect trigonometric identities. It was disappointing to see a number of candidates who used trigonometric identities correctly and reached $y^2 = x\left(1 - y^2\right)$ but were then unable to rearrange this or, worse still, thought that this was the answer to the question.

Question 7

This question discriminated well between candidates of all abilities, with about 80% of candidates gaining at least 5 marks of the 12 marks available and about 45% gaining at least 8 marks. Only about 15% of candidates gained all 12 marks. Part (a) was answered well with full marks commonly awarded. Part (b) was far more challenging with only a small minority presenting a complete and correct solution.

In part (a), many candidates were able to differentiate correctly, factorise out $\frac{dy}{dx}$, and rearrange their equation to arrive at a correct expression for the gradient function. A minority did not apply the product rule correctly when differentiating $4xy$, whilst a small number left the constant term of 27 in their differentiated equation.

In part (b), those small proportion of candidates who realised they needed to set the denominator of their $\frac{dy}{dx}$ expression equal to zero usually went on to answer this part correctly. Some, however, did not attempt this part, while a majority attempted to solve $\frac{dy}{dx} = 0$, and many proceeded to obtain coordinates of $(-6, 3)$ for the point Q, despite a number of them initially sketching a curve with a vertical tangent. A smaller proportion solved $\frac{dy}{dx} = 1$, presumably because the digit 1 is written as a vertical line; whilst others either substituted $y = 0$ or $x = 0$ into their $\frac{dy}{dx}$ expression. Manipulation and bracketing errors sometimes led to candidates writing equations such as $y^2 = A$ or $x^2 = A$, where A was negative. Examiners were surprised that a fair number of candidates, having obtained their value of x (or y), then proceeded to substitute this into $x^2 + 4xy + y^2 + 27 = 0$, rather than using the much simpler $y = kx$ or $x = ky$.

Question 8

The responses to this question were very variable and many lost marks through errors in manipulation or notation, possibly through mental tiredness. For examples, many made errors in manipulation and could not proceed correctly from the printed $\cos 2\theta = 1 - 2\sin^2\theta$ to $\sin^2\theta = \frac{1}{2} - \frac{1}{2}\cos 2\theta$ and the answer $\frac{x}{2} - \frac{1}{4}\sin 2\theta$ was often seen, instead of $\frac{\theta}{2} - \frac{1}{4}\sin 2\theta$. In part (b), many never found $\frac{dx}{d\theta}$ or realised that the appropriate form for the volume was

$$\pi \int y^2 \frac{dx}{d\theta} d\theta.$$

However the majority did find a correct integral in terms of θ although some were unable to use the identity $\sin 2\theta = 2\sin\theta\cos\theta$ to simplify their integral. The incorrect value $k = 8\pi$ was very common, resulting from a failure to square the factor 2 in $\sin 2\theta = 2\sin\theta\cos\theta$. Candidates were expected to demonstrate the correct change of limits. Minimally a reference to the result $\tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$, or an equivalent, was required. Those who had complete solutions usually gained the two method marks in part (c) but earlier errors often led to incorrect answers.

Statistics for C4 Practice Paper Silver Level S4

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
1	9		78	7.03		7.96	7.01	6.33	5.78	5.20	3.68
2	6		69	4.14	5.54	4.55	4.15	3.73	3.23	2.59	1.66
3	6		66	3.94	5.78	5.11	4.12	2.93	1.90	1.17	0.58
4	5		61	3.07	4.76	3.76	2.72	1.93	1.15	0.73	0.22
5	15	11	60	8.98	13.00	10.01	8.50	7.11	5.89	4.77	2.86
6	12		57	6.87		9.42	6.87	5.16	3.56	2.15	0.98
7	12	8	54	6.51	10.57	7.99	6.48	5.2	4.08	2.93	1.47
8	10		39	3.93		6.21	3.36	1.99	1.06	0.54	0.19
	75		**59**	**44.47**		**55.01**	**43.21**	**34.38**	**26.65**	**20.08**	**11.64**