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## 6666/01

## Edexcel GCE

## Core Mathematics C4

 Silver Level S3
## Time: 1 hour 30 minutes

| Materials required for examination | Items included with question |
| :--- | :--- |
| papers Nil <br> Mathematical Formulae (Green) $>l$ |  |

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

## Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

## Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75 .

## Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

| A* $^{*}$ | A | B | C | D | E |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 67 | 59 | 51 | 45 | 38 | 32 |

1. Express in partial fractions

$$
\frac{5 x+3}{(2 x+1)(x+1)^{2}}
$$

2. (a) Use integration to find

$$
\begin{equation*}
\int \frac{1}{x^{3}} \ln x \mathrm{~d} x . \tag{5}
\end{equation*}
$$

(b) Hence calculate

$$
\begin{equation*}
\int_{1}^{2} \frac{1}{x^{3}} \ln x \mathrm{~d} x . \tag{2}
\end{equation*}
$$

January 2013
3.

$$
\mathrm{f}(x)=\frac{27 x^{2}+32 x+16}{(3 x+2)^{2}(1-x)},|x|<\frac{2}{3}
$$

Given that $\mathrm{f}(x)$ can be expressed in the form

$$
\mathrm{f}(x)=\frac{A}{(3 x+2)}+\frac{B}{(3 x+2)^{2}}+\frac{C}{(1-x)},
$$

(a) find the values of $B$ and $C$ and show that $A=0$.
(b) Hence, or otherwise, find the series expansion of $\mathrm{f}(x)$, in ascending powers of $x$, up to and including the term in $x^{2}$. Simplify each term.
(c) Find the percentage error made in using the series expansion in part (b) to estimate the value of $f(0.2)$. Give your answer to 2 significant figures.

January 2009
4. A curve $C$ has parametric equations

$$
x=\sin ^{2} t, y=2 \tan t, 0 \leq t<\frac{\pi}{2} .
$$

(a) Find $\frac{\mathrm{d} y}{\mathrm{~d} x}$ in terms of $t$.

The tangent to $C$ at the point where $t=\frac{\pi}{3}$ cuts the $x$-axis at the point $P$.
(b) Find the $x$-coordinate of $P$.
5. Find the gradient of the curve with equation

$$
\ln y=2 x \ln x, \quad x>0, \quad y>0,
$$

at the point on the curve where $x=2$. Give your answer as an exact value.

June 2011
6. With respect to a fixed origin $O$, the lines $l_{1}$ and $l_{2}$ are given by the equations

$$
l_{1}: \mathbf{r}=\left(\begin{array}{r}
6 \\
-3 \\
-2
\end{array}\right)+\lambda\left(\begin{array}{r}
-1 \\
2 \\
3
\end{array}\right), \quad l_{2}: \mathbf{r}=\left(\begin{array}{r}
-5 \\
15 \\
3
\end{array}\right)+\mu\left(\begin{array}{r}
2 \\
-3 \\
1
\end{array}\right),
$$

where $\mu$ and $\lambda$ are scalar parameters.
(a) Show that $l_{1}$ and $l_{2}$ meet and find the position vector of their point of intersection $A$.
(b) Find, to the nearest $0.1^{\circ}$, the acute angle between $l_{1}$ and $l_{2}$.

The point $B$ has position vector $\left(\begin{array}{r}5 \\ -1 \\ 1\end{array}\right)$.
(c) Show that $B$ lies on $l_{1}$.
(d) Find the shortest distance from $B$ to the line $l_{2}$, giving your answer to 3 significant figures.
7.


Figure 2

Figure 2 shows a sketch of the curve $C$ with parametric equations

$$
x=5 t^{2}-4, \quad y=t\left(9-t^{2}\right)
$$

The curve $C$ cuts the $x$-axis at the points $A$ and $B$.
(a) Find the $x$-coordinate at the point $A$ and the $x$-coordinate at the point $B$.

The region $R$, as shown shaded in Figure 2, is enclosed by the loop of the curve.
(b) Use integration to find the area of $R$.

January 2010
8. (a) Using the substitution $x=2 \cos u$, or otherwise, find the exact value of

$$
\int_{1}^{\sqrt{2}} \frac{1}{x^{2} \sqrt{ }\left(4-x^{2}\right)} \mathrm{d} x
$$



Figure 3
Figure 3 shows a sketch of part of the curve with equation $y=\frac{4}{x\left(4-x^{2}\right)^{\frac{1}{4}}}, 0<x<2$.
The shaded region $S$, shown in Figure 3, is bounded by the curve, the $x$-axis and the lines with equations $x=1$ and $x=\sqrt{ } 2$. The shaded region $S$ is rotated through $2 \pi$ radians about the $x$-axis to form a solid of revolution.
(b) Using your answer to part (a), find the exact volume of the solid of revolution formed.

January 2010


\begin{tabular}{|c|c|c|}
\hline \multirow[t]{3}{*}{2. (a)} \& \[
\int \frac{1}{x^{3}} \ln x \mathrm{~d} x, \quad\left\{\begin{array}{ll}
u=\ln x \& \Rightarrow \frac{\mathrm{~d} u}{\mathrm{~d} x}=\frac{1}{x} \\
\frac{\mathrm{~d} v}{\mathrm{~d} x}=x^{-3} \& \Rightarrow v=\frac{x^{-2}}{-2}=\frac{-1}{2 x^{2}}
\end{array}\right\}
\] \& \\
\hline \& \begin{tabular}{l}
In the form \(\frac{ \pm \lambda}{x^{2}} \ln x \pm \int \mu \frac{1}{x^{2}} \cdot \frac{1}{x}\)
\[
=\underline{\frac{-1}{2 x^{2}} \ln x} \underline{\underline{\int \frac{-1}{2 x^{2}} \cdot \frac{1}{x}} \mathrm{~d} x}
\] \\
\(\frac{-1}{2 x^{2}} \ln x\) simplified or un-simplified. \(\underline{\underline{-\int \frac{-1}{2 x^{2}} \cdot \frac{1}{x}}}\) simplified or un-simplified.
\[
\left\{=\frac{-1}{2 x^{2}} \ln x+\frac{1}{2} \int \frac{1}{x^{3}} \mathrm{~d} x\right\}
\]
\end{tabular} \& M1
\(\underline{\text { A1 }}\)
\(\underline{\underline{A 1}}\) \\
\hline \& \begin{tabular}{l}
\[
=-\frac{1}{2 x^{2}} \ln x+\frac{1}{2}\left(-\frac{1}{2 x^{2}}\right)\{+c\}
\]
\[
\pm \int \mu \frac{1}{x^{2}} \cdot \frac{1}{x} \rightarrow \pm \beta x^{-2}
\] \\
Correct answer, with/without \(+c\)
\end{tabular} \& \(\begin{array}{ll}\text { dM1 } \\ \text { A1 } \\ \& \\ \& \end{array}\) \\
\hline \multirow[t]{2}{*}{(b)} \& Applies limits of \(\left\{\left[-\frac{1}{2 x^{2}} \ln x-\frac{1}{4 x^{2}}\right]_{1}^{2}\right\}=\left(-\frac{1}{2(2)^{2}} \ln 2-\frac{1}{4(2)^{2}}\right)-\left(-\frac{1}{2(1)^{2}} \ln 1-\frac{1}{4(1)^{2}}\right)\) 2 and 1 to their part (a) answer and subtracts the correct way round. \(=\frac{3}{16}-\frac{1}{8} \ln 2 \quad\) or \(\quad \frac{3}{16}-\ln 2^{\frac{1}{8}}\) or \(\frac{1}{16}(3-2 \ln 2)\), etc, or awrt 0.1 or equivalent. \& M1

A1 <br>
\hline \& \& [2]
7 <br>
\hline
\end{tabular}



| Question Number | Scheme |  | Marks |
| :---: | :---: | :---: | :---: |
| 3 (c) | $\begin{aligned} \text { Actual }=\mathrm{f}(0.2) & =\frac{1.08+6.4+16}{(6.76)(0.8)} \\ & =\frac{23.48}{5.408}=4.341715976 \ldots=\frac{2935}{676} \end{aligned}$ | Attempt to find the actual value of $f(0.2)$ or seeing awrt 4.3 and believing it is candidate's actual $\mathrm{f}(0.2)$. | M1 |
|  | Or $\begin{aligned} \text { Actual }=\mathrm{f}(0.2) & =\frac{4}{(3(0.2)+2)^{2}}+\frac{3}{(1-0.2)} \\ & =\frac{4}{6.76}+3.75=4.341715976 \ldots=\frac{2935}{676} \end{aligned}$ | Candidates can also attempt to find the actual value by $\frac{A}{(3 x+2)}+\frac{B}{(3 x+2)^{2}}+\frac{C}{(1-x)}$ <br> with their $A, B$ and $C$. |  |
|  | $\begin{aligned} & \begin{aligned} \text { Estimate }=\mathrm{f}(0.2) & =4+\frac{39}{4}(0.2)^{2} \\ & =4+0.39=4.39 \end{aligned} \\ & \text { \%age error }=\frac{\|4.39-4.341715976 \ldots\|}{4.341715976 \ldots} \times 100 \end{aligned}$ | Attempt to find an estimate for $f(0.2)$ using their answer to (b) | M1 $\sqrt{ }$ |
|  |  | $\left\|\frac{\text { their estimate - actual }}{\text { actual }}\right\| \times 100$ | M1 |
|  | $=1.112095408 \ldots=1.1 \%(2 \mathrm{sf})$ | 1.1\% | A1 cao (4) |
|  |  |  | [14] |

\begin{tabular}{|c|c|c|c|c|c|}
\hline \begin{tabular}{l}
Question \\
Number
\end{tabular} \& \multicolumn{3}{|c|}{Scheme} \& \multicolumn{2}{|c|}{Marks} \\
\hline \multirow[t]{6}{*}{4.} \& \multirow[t]{2}{*}{(a)

(b)} \& $$
\begin{aligned}
& \frac{\mathrm{d} x}{\mathrm{~d} t}=2 \sin t \cos t, \frac{\mathrm{~d} y}{\mathrm{~d} t}=2 \sec ^{2} t \\
& \frac{\mathrm{~d} y}{\mathrm{~d} x}=\frac{\sec ^{2} t}{\sin t \cos t}\left(=\frac{1}{\sin t \cos ^{3} t}\right)
\end{aligned}
$$ \& \multirow[t]{6}{*}{or equivalent} \& B1 B1

M1 A1 \& (4) <br>
\hline \& \& At $t=\frac{\pi}{3}, \quad x=\frac{3}{4}, \quad y=2 \sqrt{ } 3$ \& \& B1 \& <br>

\hline \& \multirow{4}{*}{(b)} \& $$
\frac{\mathrm{d} y}{\mathrm{~d} x}=\frac{\sec ^{2} \frac{\pi}{3}}{\sin \frac{\pi}{3} \cos \frac{\pi}{3}}=\frac{16}{\sqrt{3}}
$$ \& \& M1 A1 \& <br>

\hline \& \& $$
y-2 \sqrt{ } 3=\frac{16}{\sqrt{ } 3}\left(x-\frac{3}{4}\right)
$$ \& \& M1 \& <br>

\hline \& \& $$
y=0 \Rightarrow x=\frac{3}{8}
$$ \& \& M1 A1 \& (6) <br>

\hline \& \& \& \& \& [10] <br>
\hline
\end{tabular}






## Question 1

This question was generally well answered with about $74 \%$ of candidates gaining all 4 marks. The majority of candidates were able to split up $\frac{5 x+3}{(2 x+1)(x+1)^{2}}$ in the correct form of $\frac{A}{(2 x+1)}+\frac{B}{(x+1)} \frac{C}{(x+1)^{2}}$ although some missed the $(x+1)$ factor to give the incorrect form of $\frac{A}{(2 x+1)}+\frac{B}{(x+1)^{2}}$. Many candidates were successful in either substituting values and/or equating coefficients in order to find their constants. A minority who unnecessarily formed three simultaneous equations by equating coefficients were less successful in finding all three constants correctly.

## Question 2

Only about 45\% of the candidates were able to gain all 7 marks in this question as it involved a challenging integration by parts, on account of the term $\frac{1}{x^{3}}$. This meant that candidates had to be especially careful when dealing with negative powers of $x$.

In Q2(a), the majority of candidates applied the integration by parts formula correctly in the right direction to gain 3 out of the 5 marks available. Many of them then proceeded to integrate an expression of the form $\frac{\mu}{x^{3}}$ to give an expression of the form $\frac{\beta}{x^{2}}$ although a minority gave an expression of the form $\frac{\beta}{x^{4}}$. A significant number of candidates failed to gain the final accuracy mark due to sign errors or errors with the constants $\alpha$ and $\beta$ in $\frac{\alpha}{x^{2}} \ln x+\frac{\beta}{x^{2}}+c$. A minority of candidates applied the by parts formula in the 'wrong direction' and incorrectly stated that $\frac{\mathrm{d} v}{\mathrm{~d} x}=\ln x$ implied $v=\frac{1}{x}$.

In Q2(b), most candidates gained the method mark for substitution of $x=2$ and $x=1$ into their answer in Q2(a) and subtracting the correct way round. The final mark was largely dependent upon their having obtained the correct answer in Q2(a).

## Question 3

Part (a) was tackled well by many candidates. The majority of candidates were able to write down the correct identity. The most popular strategy at this stage (and the best!) was for candidates to substitute $x=1$ and $x=-\frac{2}{3}$ into their identity to find the values of the constants $B$ and $C$. The substitution of $x=-\frac{2}{3}$ caused problems for a few candidates which led them to find an incorrect value for $B$. Many candidates demonstrated that constant $A$ was zero by use of a further value of $x$ or by comparing coefficients in their identity. A significant minority of candidates manipulated their original identity and then compared coefficients to produce three equations in order to solve them simultaneously.

In part (b), most candidates were able to rewrite their partial fractions with negative powers and apply the two binomial expansions correctly, usually leading to the correct answer. A
significant minority of candidates found the process of manipulating $4(3 x+2)^{-2}$ to $\left(1+\frac{3}{2} x\right)^{-2}$ challenging.

A significant number of candidates were unsure of what to do in part (c). Some candidates found the actual value only. Other candidates found the estimated value only. Of those who progressed further, the most common error was to find the difference between these values and then divide by their estimate rather than the actual value. Some candidates did not follow the instruction to give their final answer correct to 2 significant figures and thus lost the final accuracy mark.

## Question 4

The majority of candidates knew how to tackle this question and solutions gaining all the method marks were common. However there were many errors of detail and only about $32 \%$ of the candidates gained full marks. In part (a), many candidates had difficult in differentiating $\sin ^{2} t$ and $2 \tan t$. $2 \tan t$ was more often differentiated correctly, possibly because the differential of $\tan t$ is given in the formula book, although $2 \ln \sec t$ or $\ln \sec ^{2} t$ were often seen. Many could not differentiate $\sin ^{2} t$ correctly. $\cos ^{2} t, 2 \cos t$ and $2 \sin t$ were all common. Nearly all candidates knew they had to divide $\frac{\mathrm{d} y}{\mathrm{~d} t}$ by $\frac{\mathrm{d} x}{\mathrm{~d} t}$, although there was some confusion in notation, with candidates mixing up their $x \mathrm{~s}$ and $t \mathrm{~s}$. The majority knew how to approach part (b), finding the linear equation of the tangent to the curve at $\left(\frac{3}{4}, 2 \sqrt{ } 3\right)$, putting $y=0$ and solving for $x$. Some candidates used $y=0$ prematurely and found the tangent to the curve at $\left(\frac{3}{4}, 0\right)$ rather than at $\left(\frac{3}{4}, 2 \sqrt{ } 3\right)$.

## Question 5

The majority of those who used implicit differentiation were successful on this question. The commonest error was to differentiate $2 x \ln x$ incorrectly and, occasionally, integration by parts was seen. Those who differentiated correctly were usually able to find that $y=16$ when $x=2$ and complete the question. The commonest error at this stage was $\ln y=4 \ln 2$ leading to $y=8$.

Those who started by making $y$ the subject of the formula rarely made progress beyond the first step of writing $y=\mathrm{e}^{2 x \ln x}$. This can be differentiated using the chain rule but the majority made some attempt to transform this expression before differentiating and this was often done incorrectly, $\mathrm{e}^{2 x \ln x}=\mathrm{e}^{2 x} \mathrm{e}^{\ln x}$ being a common error. Those who transformed, correctly, to $x^{2 x}$ often differentiated this to $(2 x) x^{2 x-1}$.

## Question 6

In part (a) the majority of candidates were able to set up equations in $\lambda$ and $\mu$ and, with a few exceptions were able to solve them correctly. Substitution into one of the given line equations to obtain the coordinates of $A$ usually followed correctly, although a substantial number of candidates were unable, or forgot, to show that the lines did indeed intersect. In
part (b), the great majority of candidates realised that a scalar product was involved although substantial numbers of candidates selected the position vectors of the fixed points on the line rather than the direction vectors. Among those who did select the correct vectors, the commonest error was to give an obtuse angle rather than the acute angle which the question asked for. Part (c) was well done although some did not show the consistency of $\lambda$ for all three components.

Part (d) proved very demanding. Those who were able to draw a simple diagram to represent the situation and who remembered that each part of a question frequently relies on previous parts were able to find the length of $A B$, using the results of parts (a) and (c), and the angle found in part (b) to complete the question using elementary trigonometry, although some used the tangent of the angle rather than the sine. These were however a small minority of candidates and the majority either just left the question blank or thought that $A B$ was the length they were looking for. Some tried more complicated methods such as finding a general expression for $\overrightarrow{B X}$ and taking the scalar product of this vector with the direction of $l_{2}$ and equating to zero. The working for this method gets very complicated $\left(\mu=\frac{33}{7}\right)$ but a few correct solutions of this type were seen.

## Question 7

Part (a) was well done. The majority of candidates correctly found the $x$-coordinates of $A$ and $B$, by putting $y=0$, solving for $t$ and then substituting in $x=5 t^{2}-4$. Full marks were common. Part (b) proved difficult. A substantial minority of candidates failed to substitute for the $\mathrm{d} x$ when substituting into $\int y \mathrm{~d} x$ or used $\frac{\mathrm{d} t}{\mathrm{~d} x}$ rather than $\frac{\mathrm{d} x}{\mathrm{~d} t}$. A surprising feature of the solutions seen was the number of candidates who, having obtained the correct $\int t\left(9-t^{2}\right) 10 t \mathrm{~d} t$, were unable to remove the brackets correctly to obtain $\int\left(90 t^{2}-10 t^{4}\right) \mathrm{d} t$. Weaknesses in elementary algebra flawed many otherwise correct solutions. Another source of error was using the $x$-coordinates for the limits when the variable in the integral was $t$. At the end of the question, many failed to realise that $\int_{0}^{3}\left(90 t^{2}-10 t^{4}\right) \mathrm{d} t$ gives only half of the required area.

Some candidates made either the whole of the question, or just part (b), more difficult by eliminating parameters and using the cartesian equation. This is a possible method but the indices involved are very complicated and there were very few successful solutions using this method.

## Question 8

Answers to part (a) were mixed, although most candidates gained some method marks. A surprisingly large number of candidates failed to deal with $\sqrt{4-4 \cos ^{2} u}$ correctly and many did not recognise that $\int \frac{1}{\cos ^{2} x} \mathrm{~d} x=\int \sec ^{2} x \mathrm{~d} x=\tan x(+C)$ in this context. Nearly all converted the limits correctly. Answers to part (b) were also mixed. Some could not get beyond stating the formula for the volume of revolution while others gained the first mark, by substituting the equation given in part (b) into this formula, but could not see the connection with part (a). Candidates could recover here and gain full follow through marks in part (b) after an incorrect attempt at part (a).

Statistics for C4 Practice Paper Silver Level S3

| Qu | Max score | Modal score | Mean \% | Mean score for students achieving grade: |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  | ALL | A* | A | B | C | D | E | U |
| 1 | 4 |  | 82 | 3.29 | 3.87 | 3.51 | 3.24 | 2.89 | 2.23 | 1.97 | 0.81 |
| 2 | 7 | 7 | 71 | 4.98 | 6.86 | 5.97 | 4.94 | 3.90 | 2.81 | 2.09 | 0.94 |
| 3 | 14 |  | 72 | 10.04 |  | 12.02 | 9.57 | 8.16 | 6.42 | 5.42 | 2.69 |
| 4 | 10 |  | 65 | 6.50 | 9.62 | 8.38 | 6.73 | 4.96 | 3.47 | 2.03 | 0.93 |
| 5 | 7 |  | 64 | 4.47 | 6.79 | 5.76 | 4.56 | 3.36 | 2.24 | 1.42 | 0.70 |
| 6 | 14 |  | 62 | 8.70 | 12.92 | 10.84 | 8.84 | 6.78 | 4.88 | 3.35 | 1.75 |
| 7 | 9 |  | 60 | 5.36 |  | 6.92 | 4.68 | 3.52 | 2.62 | 1.58 | 1.25 |
| 8 | 10 |  | 41 | 4.07 |  | 5.71 | 2.63 | 1.67 | 0.85 | 0.41 | 0.20 |
|  | 75 |  | 63 | 47.41 |  | 59.11 | 45.19 | 35.24 | 25.52 | 18.27 | 9.27 |

