

**6678/01**

# **Edexcel GCE**

## **Mechanics M2**

### **Gold Level G2**

**Time: 1 hour 30 minutes**

**Materials required for examination**

Mathematical Formulae (Pink)

**Items included with question papers**

Nil

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.**

#### **Instructions to Candidates**

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In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.

Whenever a numerical value of  $g$  is required, take  $g = 9.8 \text{ m s}^{-2}$ .

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

#### **Information for Candidates**

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A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 7 questions in this question paper. The total mark for this paper is 75.

#### **Advice to Candidates**

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You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

#### **Suggested grade boundaries for this paper:**

<b>A*</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>E</b>
<b>59</b>	<b>50</b>	<b>41</b>	<b>34</b>	<b>27</b>	<b>21</b>

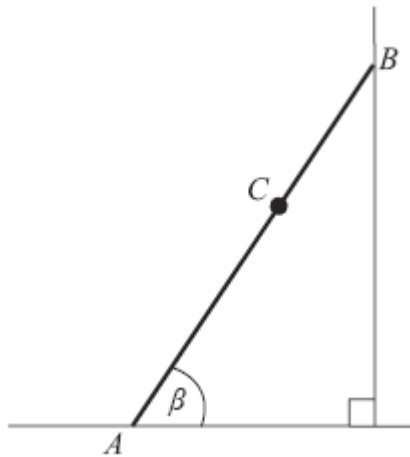
1. A caravan of mass 600 kg is towed by a car of mass 900 kg along a straight horizontal road. The towbar joining the car to the caravan is modelled as a light rod parallel to the road. The total resistance to motion of the car is modelled as having magnitude 300 N. The total resistance to motion of the caravan is modelled as having magnitude 150 N. At a given instant the car and the caravan are moving with speed  $20 \text{ m s}^{-1}$  and acceleration  $0.2 \text{ m s}^{-2}$ .

(a) Find the power being developed by the car's engine at this instant. (5)

(b) Find the tension in the towbar at this instant. (2)

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2.



**Figure 1**

Figure 1 shows a ladder  $AB$ , of mass 25 kg and length 4 m, resting in equilibrium with one end  $A$  on rough horizontal ground and the other end  $B$  against a smooth vertical wall. The ladder is in a vertical plane perpendicular to the wall. The coefficient of friction between the ladder and the ground is  $\frac{11}{25}$ . The ladder makes an angle  $\beta$  with the ground. When Reece, who has mass 75 kg, stands at the point  $C$  on the ladder, where  $AC = 2.8 \text{ m}$ , the ladder is on the point of slipping. The ladder is modelled as a uniform rod and Reece is modelled as a particle.

(a) Find the magnitude of the frictional force of the ground on the ladder. (3)

(b) Find, to the nearest degree, the value of  $\beta$ . (6)

(c) State how you have used the modelling assumption that Reece is a particle. (1)

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3. A cyclist and her cycle have a combined mass of 75 kg. The cyclist is cycling up a straight road inclined at  $5^\circ$  to the horizontal. The resistance to the motion of the cyclist from non-gravitational forces is modelled as a constant force of magnitude 20 N. At the instant when the cyclist has a speed of  $12 \text{ m s}^{-1}$ , she is decelerating at  $0.2 \text{ m s}^{-2}$ .

(a) Find the rate at which the cyclist is working at this instant.

(5)

When the cyclist passes the point *A* her speed is  $8 \text{ m s}^{-1}$ . At *A* she stops working but does not apply the brakes. She comes to rest at the point *B*.

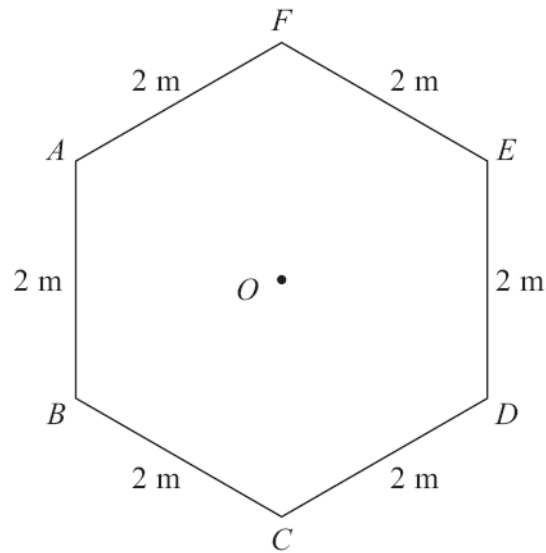
The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 20 N.

(b) Use the work-energy principle to find the distance *AB*.

(5)

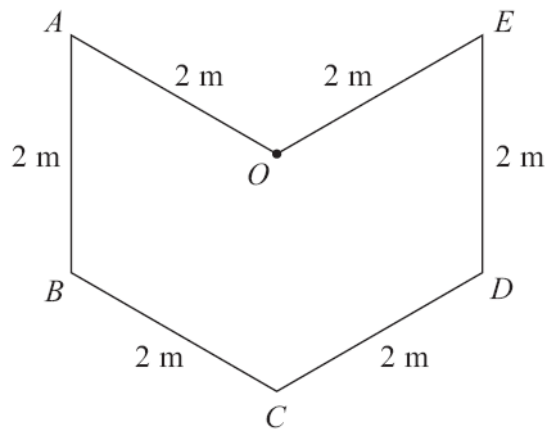
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4.



**Figure 1**

The uniform lamina  $ABCDEF$  is a regular hexagon with centre  $O$  and sides of length 2 m, as shown in Figure 1.



**Figure 2**

The triangles  $OAF$  and  $OEF$  are removed to form the uniform lamina  $OABCDE$ , shown in Figure 2.

(a) Find the distance of the centre of mass of  $OABCDE$  from  $O$ . (5)

The lamina  $OABCDE$  is freely suspended from  $E$  and hangs in equilibrium.

(b) Find the size of the angle between  $EO$  and the downward vertical. (6)

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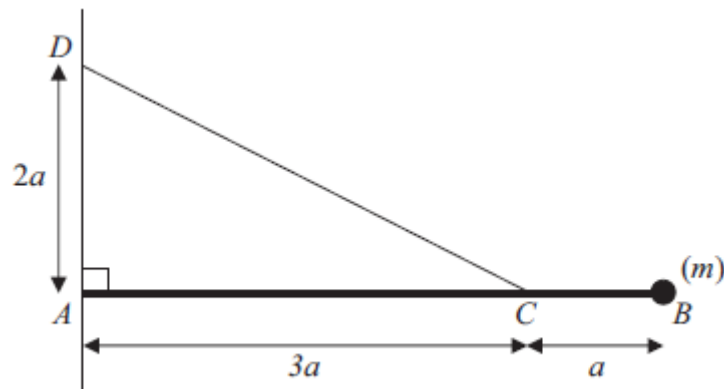
5. Two particles  $P$  and  $Q$ , of masses  $2m$  and  $m$  respectively, are on a smooth horizontal table. Particle  $Q$  is at rest and particle  $P$  collides directly with it when moving with speed  $u$ . After the collision the total kinetic energy of the two particles is  $\frac{3}{4}mu^2$ . Find

(a) the speed of  $Q$  immediately after the collision, (10)

(b) the coefficient of restitution between the particles. (3)

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6.



**Figure 2**

Figure 2 shows a uniform rod  $AB$  of mass  $m$  and length  $4a$ . The end  $A$  of the rod is freely hinged to a point on a vertical wall. A particle of mass  $m$  is attached to the rod at  $B$ . One end of a light inextensible string is attached to the rod at  $C$ , where  $AC = 3a$ . The other end of the string is attached to the wall at  $D$ , where  $AD = 2a$  and  $D$  is vertically above  $A$ . The rod rests horizontally in equilibrium in a vertical plane perpendicular to the wall and the tension in the string is  $T$ .

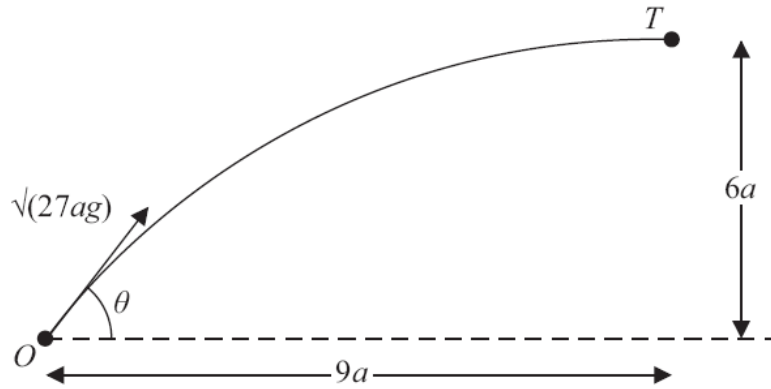
(a) Show that  $T = mg\sqrt{13}$ . (5)

The particle of mass  $m$  at  $B$  is removed from the rod and replaced by a particle of mass  $M$  which is attached to the rod at  $B$ . The string breaks if the tension exceeds  $2mg\sqrt{13}$ . Given that the string does not break,

(b) show that  $M \leq \frac{5}{2}m$ . (3)

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7.



**Figure 4**

A small ball is projected from a fixed point  $O$  so as to hit a target  $T$  which is at a horizontal distance  $9a$  from  $O$  and at a height  $6a$  above the level of  $O$ . The ball is projected with speed  $\sqrt{27ag}$  at an angle  $\theta$  to the horizontal, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

(a) Show that  $\tan^2 \theta - 6 \tan \theta + 5 = 0$ . (7)

The two possible angles of projection are  $\theta_1$  and  $\theta_2$ , where  $\theta_1 > \theta_2$ .

(b) Find  $\tan \theta_1$  and  $\tan \theta_2$ . (3)

The particle is projected at the larger angle  $\theta_1$ .

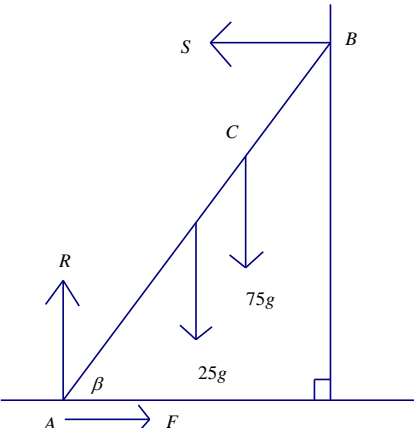
(c) Show that the time of flight from  $O$  to  $T$  is  $\sqrt{\left(\frac{78a}{g}\right)}$ . (3)

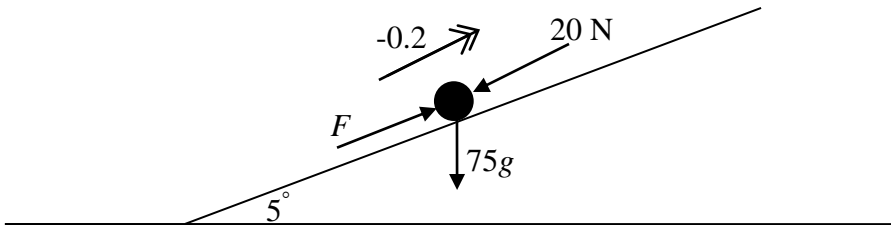
(d) Find the speed of the particle immediately before it hits  $T$ . (3)

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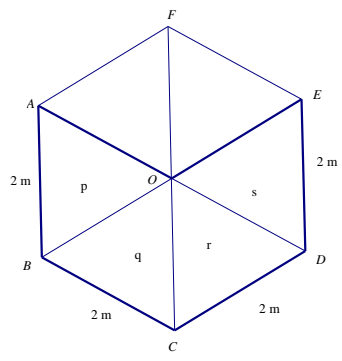
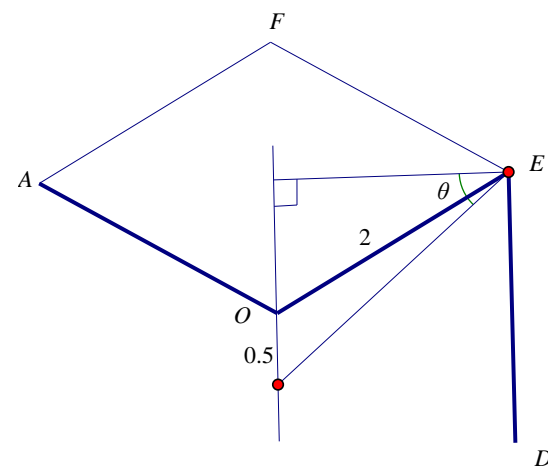
**TOTAL FOR PAPER: 75 MARKS**

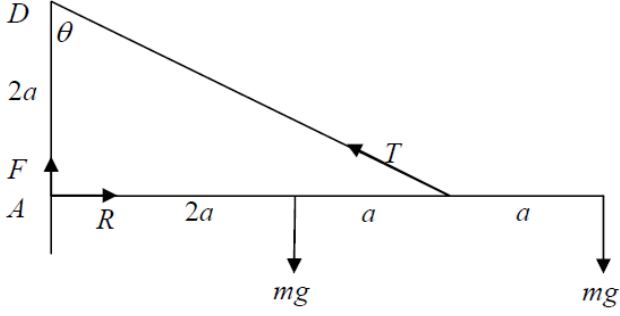
**END**

Question Number	Scheme	Marks
<p>1.</p> <p>(a)</p>	$F - 150 - 300 = 1500 \times 0.2$ $F = 750$ $P = 750 \times 20 = 15000 \text{ watts}$	<p>M1</p> <p>A1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>(5)</p>
<p>(b)</p>	<p>Use their mass as a guide to which of these two alternatives is being used.</p> <p>For caravan: <math>T - 150 = 600 \times 0.2</math></p> $T = 270 \text{ N}$	<p>M1</p> <p>A1</p> <p>(2)</p> <p>[7]</p>
<p>2.</p> <p>(a)</p>	 $R(\uparrow) : R = 25g + 75g (= 100g)$ $F = \mu R \Rightarrow F = \frac{11}{25} \times 100g$ $= 44g (= 431)$	<p>B1</p> <p>M1</p> <p>A1</p> <p>(3)</p>
<p>(b)</p>	<p>M(A):</p> $25g \times 2 \cos \beta + 75g \times 2.8 \cos \beta$ $= S \times 4 \sin \beta$ <p>R(<math>\leftrightarrow</math>): <math>F = S</math></p> $176g \sin \beta = 260g \cos \beta$ $\beta = 56(^{\circ})$	<p>M1</p> <p>A2,1,0</p> <p>M1A1</p> <p>A1</p> <p>(6)</p>
<p>(c)</p>	<p>So that Reece's weight acts directly at the point C.</p>	<p>B1</p> <p>[10]</p>

Question Number	Scheme	Marks
<p>3.</p> <p>(a)</p> <p>(b)</p>	 <p>Driving force = F</p> <p>Resolving parallel to the plane: <math>F - 20 - 75g \sin 5 = -75 \times 0.2 = -15</math></p> $F = 5 + 75g \sin 5^\circ$ <p><math>P = Fv \quad \therefore</math> working at <math>12 \times (5 + 75g \sin 5^\circ) = 828.7 \dots</math></p> $\approx 830 \text{ W}$ <p>Loss in KE = gain in GPE + work done against resistance</p> $\frac{1}{2} \times 75 \times 64 = 75 \times 9.8 \times \sin 5^\circ d + 20d = d \times 84.059 \dots$ $d = 28.6 \text{ m}$	<p><b>M1</b></p> <p><b>A2 – lee</b></p> <p><b>DM1</b></p> <p><b>A1</b></p> <p>(5)</p> <p><b>M1</b></p> <p><b>A2 – lee</b></p> <p><b>DM1 A1</b></p> <p>(5)</p> <p><b>(10 marks)</b></p>



Question Number	Scheme	Marks									
<p>4</p> <p>(a)</p>	 <table border="1" data-bbox="343 649 909 772"> <thead> <tr> <th>AOCB</th> <th>OCDE</th> <th>whole</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>1</td> <td>2</td> </tr> <tr> <td>1/2</td> <td>1/2</td> <td><math>\bar{y}</math></td> </tr> </tbody> </table> $2\bar{y} = 1 \times \frac{1}{2} + 1 \times \frac{1}{2}$ $\bar{y} = 0.5 \text{ (m)}$	AOCB	OCDE	whole	1	1	2	1/2	1/2	$\bar{y}$	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(5)</p>
AOCB	OCDE	whole									
1	1	2									
1/2	1/2	$\bar{y}$									
<p>(b)</p>	 $2 \cos 30 (= \sqrt{3}), \quad "0.5" + 2 \sin 30 (= 1.5)$ $\tan \theta = \frac{\text{their } 1.5}{\text{their } \sqrt{3}}$ $\text{Required angle} = \theta - 30 = \tan^{-1} \frac{1.5}{\sqrt{3}} - 30 = 40.89... - 30 = 10.9^\circ$	<p>M1A1ft</p> <p>DM1</p> <p>A1ft</p> <p>DM1</p> <p>A1</p> <p>(6)</p> <p>[11]</p>									

Question Number	Scheme	Marks
<p>5.</p> <p>(a)</p>	$2mu = 2mv_p + mv_Q$ $\frac{3}{4}mu^2 = \frac{1}{2}2mv_p^2 + \frac{1}{2}mv_Q^2$ $3v_Q^2 - 4uv_Q + u^2 = 0 \text{ or } 12v_p^2 - 16uv_p + 5u^2 = 0$ $v_Q = \frac{u}{3}, v_p = \frac{5u}{6} \text{ or } v_Q = u, v_p = \frac{u}{2}$ $v_Q = u$ <p>..... since <math>v_Q &gt; v_p</math></p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p>DM1</p> <p>A1</p> <p>(10)</p>
<p>(b)</p>	$e = \frac{u - \frac{u}{2}}{u} \quad \left( \frac{v_Q - v_p}{u} \right)$ $= \frac{1}{2}$	<p>M1</p> <p>A1 ft</p> <p>A1</p> <p>(3)</p> <p>[13]</p>
<p>6.</p>	 <p>(a)</p> $M(A) \quad 3a \times T \cos\theta = 2amg + 4amg$ $\cos\theta = \left( \frac{2}{\sqrt{9+4}} \right) = \frac{2}{\sqrt{13}}$ $\frac{6}{\sqrt{13}}T = 6mg$ $T = mg\sqrt{13}$ <p>(b)</p> $3a \times T \times \cos\theta = 2amg + 4aMg$ $T = \left( \frac{2mg + 4Mg}{6} \right) \sqrt{13} \leq 2mg\sqrt{13}$ $mg + 2Mg < 6mg$ $M \leq \frac{5}{2}m$	<p>M1</p> <p>A1 A1</p> <p>B1</p> <p>A1</p> <p>(5)</p> <p>M1</p> <p>A1</p> <p>A1</p> <p>(3)</p> <p>[8]</p>

Question Number	Scheme	Marks
7. (a)	$(\rightarrow)\sqrt{27ag} \cos \theta. t = 9a$ $(\uparrow)\sqrt{27ag} \sin \theta. t - \frac{1}{2}gt^2 = 6a$ $(\uparrow)\sqrt{27ag} \sin \theta. \frac{9a}{\sqrt{27ag} \cos \theta} - \frac{1}{2}g \left( \frac{9a}{\sqrt{27ag} \cos \theta} \right)^2 = 6a$ $9a \tan \theta - \frac{1}{2}g \cdot 81a^2 \frac{(1 + \tan^2 \theta)}{27ag} = 6a$ $\tan^2 \theta - 6 \tan \theta + 5 = 0$	<b>M1</b> <b>A1</b> <b>M1</b> <b>A1</b> <b>DM1</b> <b>DM1</b> <b>A1 (7)</b>
(b)	$\tan^2 \theta - 6 \tan \theta + 5 = 0$ $(\tan \theta - 1)(\tan \theta - 5) = 0$ $\tan \theta_2 = 1 \quad \text{or} \quad \tan \theta_1 = 5$	<b>M1</b> <b>A1 A1 (3)</b>
(c)	$t = \frac{9a}{\sqrt{27ag} \cos \theta} = \frac{9a}{\sqrt{27ag}} \times \frac{\sqrt{26}}{1}$	<b>M1</b> <b>A1ft</b>
(d)	$= \sqrt{\frac{81a^2 \cdot 26}{27a}} = \sqrt{\frac{78a}{g}} \text{ * Answer given*}$	<b>A1 (3)</b>
(d)	$\frac{1}{2}m(27ag - v^2) = mg6a$ $v = \sqrt{15ag}$	<b>M1</b> <b>A1</b> <b>A1 (3)</b>
		<b>[16]</b>

## Examiner reports

### Question 1

(a) Those candidates who realised that they needed to start with an equation for the motion of the car and caravan combined had little difficulty with this question. Most errors were due to the omission of forces, or equations which included all the forces but only the mass of one vehicle.

(b) Many candidates completed this part successfully; by considering the equation of motion for the caravan, the solution is independent of the work in (a).

### Question 2

This question was answered well, with few instances this time of the reaction at the ground or at the wall being in the wrong direction.

In (a) candidates had little problem finding the frictional force acting but then the majority gave the answer to 4 significant figures, losing the final mark.

Most candidates then went on to take moments about  $A$  or  $B$ . Errors at this stage were usually due to terms being dimensionally incorrect – often leaving out the distance in one or more terms of the moments equation. Virtually all candidates went on to find a value for  $\beta$ , but this was not always expressed to the required degree of accuracy.

Most candidates demonstrated some understanding of what it meant to model Reece as a particle, but few were sufficiently precise in their responses. Many mentioned mass acting at a point rather than weight and few were specific about where the weight was assumed to act.

### Question 3

Part (a): The majority of the candidates found the correct equation of motion and almost all went on to use ' $P = Fv$ ' correctly. Too many candidates gave a final answer of 828.7 watts, which is an inappropriate level of accuracy following the use of  $g = 9.8$ . Some candidates mistakenly assumed that there was an acceleration of  $0.2 \text{ m s}^{-2}$ , and some ignored the deceleration completely and used  $a = 0$ .

Part (b): This question clearly states that candidates are required to use the work-energy principle. Candidates who approached the question by using  $F = ma$  scored no marks. It was disappointing to find several candidates using this alternative method. Some candidates chose to use both methods, answering the question twice, as a means of checking their work-energy solution, this is of course perfectly acceptable.

The most common errors were to miss out a term in the work-energy equation or to include an extra GPE term. Some missed out either the work done against resistance or the GPE term, but a substantial number of candidates considered both the increase in GPE and the work done against the weight, apparently not realising that these terms are the same thing. Candidates need to be warned that incomplete work-energy equations and equations with duplicated terms are given no marks. Less common, but equally serious in terms of marks, was the inclusion of  $20$  rather than  $20d$  as the work done against resistance - an inconsistent equation mixing energy and forces is considered a method error.

With the correct number of terms in the energy equation, there were still sign errors made. There were also errors in the weight term, such as using mass instead of weight and confusion of sine and cosine.

#### Question 4

There was a wide variety of approaches to part (a), which resulted in a wide range of responses, from those which made the question look trivial to those which struggled to deal with the chosen dissection of the lamina. The most surprising feature of many responses was that not all candidates understood the geometrical properties of a hexagon. In particular, candidates could not find the ratios of masses of the parts without attempting to find the areas, apparently not realising that the regular hexagon comprises six identical equilateral triangles. The most elegant solutions split the lamina into two identical rhombuses, but popular alternative approaches were based on four triangles or on a hexagon with a rhombus removed. Some candidates did not appreciate the line of symmetry  $OC$  and that they only needed to consider vertical distances from  $O$ . A few thought the shape was just made up of rods and did not seem to understand what a lamina is.

In part (b) many candidates correctly identified a triangle with the required angle, but their labelling was often incomplete or inaccurate. A common error was to treat the angle  $EOC$  as a right angle. The two approaches suggested in the mark scheme were equally popular with candidates who reached the correct conclusion. Most solutions involved two stages of working but a few candidates were successful in using the sine rule with angles  $\theta$  and  $60 - \theta$ .

#### Question 5

Although it has been unusual to start an impact question with an equation about kinetic energy, most candidates started part (a) by writing down equations for conservation of momentum and for the kinetic energy after the collision, and using these to form a quadratic equation to find the speed of  $Q$ . A few candidates assumed that both candidates had the same mass in their energy equation. Having found two possible solutions for the velocity of  $Q$ , some candidates continued with both, not realising that it was not possible for the velocity of  $P$  to be greater than the velocity of  $Q$ . Knowing that they wanted a positive value for  $e$ , candidates working with the wrong pair of velocities usually contrived to reach a positive answer, when the negative answer arising from their values if they worked through correctly could have warned them of their error.

Many candidates are so accustomed to starting impact equations by forming equations for conservation of momentum and use of the impact law that it was common to see approaches to this question which started by finding the speeds of  $P$  and  $Q$  after the collision in terms of  $e$  and  $u$ , and using these to form a quadratic equation in  $e$  before going back to determine the speed of  $Q$ .

Candidates often made slips in their algebra, resulting in equations with incorrect and very untidy roots.

## Question 6

There were many correct responses to this question, some considerably more concise than others.

In part (a) many candidates took the direct route of the mark scheme, and most dealt confidently with the exact trig ratio. There were several who had initially made a false start, resolving vertically and horizontally and ignoring all or part of the reaction at the hinge, but they often went on to score full marks by later taking moments about  $A$  correctly. Some did not seem to understand the significance of requiring an exact answer and obtained the given answer in surd form from a decimal value of  $\sin 33.69^\circ$ .

In part (b) most candidates learned from their experience in (a) and started by taking moments about  $A$ . A minority tried several alternative options before deciding to take moments about  $A$ . Some candidates did not deal appropriately with the inequality, either by including it when taking moments or by simply inserting it in the final line.

## Question 7

(a) Almost all candidates started correctly, with equations for the horizontal and vertical displacement of the ball, and went on to use these to form an equation in  $\theta$ . Several candidates were not able to complete the final step to reach an equation in  $\tan \theta$

(b) The solutions of the given quadratic equation were usually correct.

(c) Those candidates who knew how to find the value of  $\cos \theta$  found this straight forward, but several candidates resorted to approximate values in their attempt to derive an exact answer.

(d) Candidates were free to choose their own approach here, but those who elected to consider energy were usually more successful than those who took the *suvat* approach. There were many slips in finding the horizontal and vertical components of speed and combining these using Pythagoras' theorem.

## Statistics for M2 Practice Paper Gold 2

Qu	Max Score	Modal score	Mean %	Mean average scored by candidates achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	7		81.4	5.70	6.65	6.02	5.50	5.18	5.06	3.82	2.84
2	10		65.2	6.52		7.37	5.84	5.06	3.48	3.13	1.13
3	10		62.4	6.24	8.31	6.71	4.67	3.53	2.50	1.69	0.39
4	11	11	49.3	5.42	8.23	6.79	4.93	3.73	2.78	2.21	1.24
5	13		55.6	7.23	10.43	7.81	6.87	5.59	5.21	4.38	2.10
6	8		59.5	4.76	6.75	5.78	4.49	3.37	2.39	1.46	0.85
7	16		60.1	9.62	12.91	10.78	8.32	7.99	6.48	6.29	3.68
	<b>75</b>		<b>60.7</b>	<b>45.49</b>		<b>51.26</b>	<b>40.62</b>	<b>34.45</b>	<b>27.90</b>	<b>22.98</b>	<b>12.23</b>