

6677/01

Edexcel GCE

Mechanics M1

Gold Level G1

Time: 1 hour 30 minutes

Materials required for examination

Mathematical Formulae (Pink)

Items included with question papers

Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M1), the paper reference (6677), your surname, other name and signature.

Whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$.

When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.

Full marks may be obtained for answers to ALL questions.

There are 6 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.

You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
66	58	49	41	31	24

1. Particle P has mass 3 kg and particle Q has mass 2 kg. The particles are moving in opposite directions on a smooth horizontal plane when they collide directly. Immediately before the collision, P has speed 3 m s^{-1} and Q has speed 2 m s^{-1} . Immediately after the collision, both particles move in the same direction and the difference in their speeds is 1 m s^{-1} .

(a) Find the speed of each particle after the collision.

(5)

(b) Find the magnitude of the impulse exerted on P by Q .

(3)

2. A particle P of mass 2 kg is moving under the action of a constant force \mathbf{F} newtons. The velocity of P is $(2\mathbf{i} - 5\mathbf{j}) \text{ m s}^{-1}$ at time $t = 0$, and $(7\mathbf{i} + 10\mathbf{j}) \text{ m s}^{-1}$ at time $t = 5 \text{ s}$.

Find

(a) the speed of P at $t = 0$,

(2)

(b) the vector \mathbf{F} in the form $a\mathbf{i} + b\mathbf{j}$,

(5)

(c) the value of t when P is moving parallel to \mathbf{i} .

(4)

3. A particle of mass 0.8 kg is held at rest on a rough plane. The plane is inclined at 30° to the horizontal. The particle is released from rest and slides down a line of greatest slope of the plane. The particle moves 2.7 m during the first 3 seconds of its motion. Find

(a) the acceleration of the particle,

(3)

(b) the coefficient of friction between the particle and the plane.

(5)

The particle is now held on the same rough plane by a horizontal force of magnitude X newtons, acting in a plane containing a line of greatest slope of the plane, as shown in Figure 3. The particle is in equilibrium and on the point of moving up the plane.

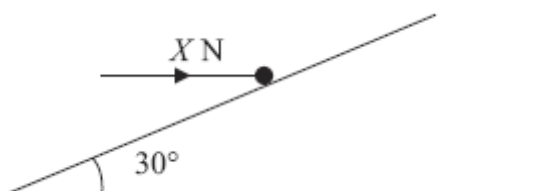


Figure 3

(c) Find the value of X .

(7)

4.

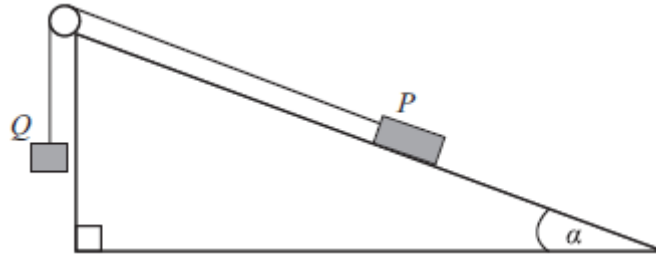


Figure 2

Two particles P and Q have masses 0.3 kg and $m \text{ kg}$ respectively. The particles are attached to the ends of a light inextensible string. The string passes over a small smooth pulley which is fixed at the top of a fixed rough plane. The plane is inclined to the horizontal at an angle α , where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between P and the plane is $\frac{1}{2}$.

The string lies in a vertical plane through a line of greatest slope of the inclined plane. The particle P is held at rest on the inclined plane and the particle Q hangs freely below the pulley with the string taut, as shown in Figure 2.

The system is released from rest and Q accelerates vertically downwards at 1.4 m s^{-2} .

Find

(a) the magnitude of the normal reaction of the inclined plane on P , (2)

(b) the value of m . (8)

When the particles have been moving for 0.5 s , the string breaks. Assuming that P does not reach the pulley,

(c) find the further time that elapses until P comes to instantaneous rest. (6)

5. [In this question, \mathbf{i} and \mathbf{j} are horizontal unit vectors due east and due north respectively and position vectors are given with respect to a fixed origin.]

A ship S is moving along a straight line with constant velocity. At time t hours the position vector of S is \mathbf{s} km. When $t = 0$, $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j}$. When $t = 4$, $\mathbf{s} = 21\mathbf{i} + 10\mathbf{j}$. Find

- (a) the speed of S , (4)
- (b) the direction in which S is moving, giving your answer as a bearing. (2)
- (c) Show that $\mathbf{s} = (3t + 9)\mathbf{i} + (4t - 6)\mathbf{j}$. (2)

A lighthouse L is located at the point with position vector $(18\mathbf{i} + 6\mathbf{j})$ km. When $t = T$, the ship S is 10 km from L .

- (d) Find the possible values of T . (6)
-

6. [In this question \mathbf{i} and \mathbf{j} are unit vectors due east and due north respectively. Position vectors are given relative to a fixed origin O .]

Two ships P and Q are moving with constant velocities. Ship P moves with velocity $(2\mathbf{i} - 3\mathbf{j})$ km h⁻¹ and ship Q moves with velocity $(3\mathbf{i} + 4\mathbf{j})$ km h⁻¹.

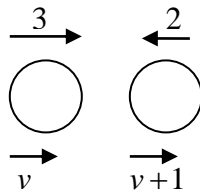
- (a) Find, to the nearest degree, the bearing on which Q is moving. (2)

At 2 p.m., ship P is at the point with position vector $(\mathbf{i} + \mathbf{j})$ km and ship Q is at the point with position vector $(-2\mathbf{j})$ km.

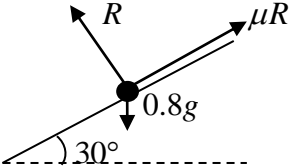
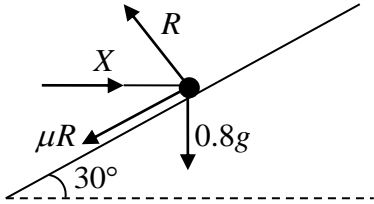
At time t hours after 2 p.m., the position vector of P is \mathbf{p} km and the position vector of Q is \mathbf{q} km.

- (b) Write down expressions, in terms of t , for
- (i) \mathbf{p} ,
- (ii) \mathbf{q} ,
- (iii) \overrightarrow{PQ} . (5)
- (c) Find the time when
- (i) Q is due north of P ,
- (ii) Q is north-west of P . (4)

TOTAL FOR PAPER: 75 MARKS

Question Number	Scheme	Marks
1. (a)	 <p>CLM: $3 \times 3 - 2 \times 2 = 3v + 2(v+1)$</p> <p>$v_P = 0.6 \text{ m s}^{-1}; v_Q = 1.6 \text{ m s}^{-1}$</p>	M1 A1 M1A1 (A1 ft) (5)

Question Number	Scheme	Marks
2. (a)	$\text{speed} = \sqrt{2^2 + (-5)^2}$ $= \sqrt{29} = 5.4 \text{ or better}$	M1 A1 (2)
(b)	$((7\mathbf{i} + 10\mathbf{j}) - (2\mathbf{i} - 5\mathbf{j}))/5$ $= (5\mathbf{i} + 15\mathbf{j})/5 = \mathbf{i} + 3\mathbf{j}$ $\mathbf{F} = m\mathbf{a} = 2(\mathbf{i} + 3\mathbf{j}) = 2\mathbf{i} + 6\mathbf{j}$	M1 A1 A1 DM1 A1ft (5)
(c)	$\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 5\mathbf{j}) + (\mathbf{i} + 3\mathbf{j})t$ $(-5 + 3t)\mathbf{j}$ <p>Parallel to $\mathbf{i} \Rightarrow -5 + 3t = 0$</p> $t = 5/3$	M1 A1 M1 A1 (4) [11]

Question Number	Scheme	Marks
Q3.	<p>(a) $s = ut + \frac{1}{2}at^2 \Rightarrow 2.7 = \frac{1}{2}a \times 9$ $a = 0.6 \text{ (m s}^{-2}\text{)}$</p> <p>(b) </p> <p>$R = 0.8g \cos 30^\circ (\approx 6.79)$ Use of $F = \mu R$ $0.8g \sin 30^\circ - \mu R = 0.8 \times a$ $(0.8g \sin 30^\circ - \mu 0.8g \cos 30^\circ = 0.8 \times 0.6)$ $\mu \approx 0.51$ accept 0.507</p> <p>(c) </p> <p>$R \cos 30^\circ = \mu R \cos 60^\circ + 0.8g$ $(R \approx 12.8)$ $\rightarrow X = R \sin 30^\circ + \mu R \sin 60^\circ$ Solving for X, $X \approx 12$ accept 12.0</p> <p>Alternative to (c)</p> <p>$R = X \sin 30^\circ + 0.8 \times 9.8 \sin 60^\circ$ $\mu R + 0.8g \cos 60^\circ = X \cos 30^\circ$</p> <p>$X = \frac{\mu 0.8g \sin 60^\circ + 0.8g \cos 60^\circ}{\cos 30^\circ - \mu \sin 30^\circ}$ Solving for X, $X \approx 12$ accept 12.0</p>	<p>M1 A1 A1 (3)</p> <p>B1 B1 M1 A1 A1 (5)</p> <p>M1 A2 (1,0) M1 A1 DM1 A1 (7) [15]</p> <p>M1 A2 (1,0) M1 A1</p> <p>DM1 A1 (7)</p>

Question Number	Scheme	Marks
4. (a)	$R = 0.3g \cos \alpha$ $= 0.24g = 2.35 \text{ (3sf)} = 2.4 \text{ (2sf)}$	M1 A1 (2)
(b)	$mg - T = 1.4m$ $T - 0.3g \sin \alpha - F = 0.3 \times 1.4$ $F = 0.5R$ Eliminating R and T $m = 0.4$	M1 A1 M1 A2 M1 DM1 A1 (8)
(c)	$v = 1.4 \times 0.5$ $-0.3g \sin \alpha - F = 0.3a$ $a = -9.8$ $0 = 0.7 - 9.8t$ $t = 0.071 \text{ s or } 0.0714 \text{ s (1/14 A0)}$	B1 M1 A1 A1 M1 A1 (6) 16

Question Number	Scheme	Marks
Q5.	<p>(a)</p> $\mathbf{v} = \frac{21\mathbf{i} + 10\mathbf{j} - (9\mathbf{i} - 6\mathbf{j})}{4} = 3\mathbf{i} + 4\mathbf{j}$ <p>speed is $\sqrt{(3^2 + 4^2)} = 5 \text{ (km h}^{-1}\text{)}$</p> <p>(b)</p> $\tan \theta = \frac{3}{4} \quad (\Rightarrow \theta \approx 36.9^\circ)$ <p>bearing is 37, 36.9, 36.87, ...</p> <p>(c)</p> $\mathbf{s} = 9\mathbf{i} - 6\mathbf{j} + t(3\mathbf{i} + 4\mathbf{j})$ $= (3t + 9)\mathbf{i} + (4t - 6)\mathbf{j} \quad *$ <p style="text-align: right;">cso</p> <p>(d) Position vector of S relative to L is</p> $(3T + 9)\mathbf{i} + (4T - 6)\mathbf{j} - (18\mathbf{i} + 6\mathbf{j}) = (3T - 9)\mathbf{i} + (4T - 12)\mathbf{j}$ $(3T - 9)^2 + (4T - 12)^2 = 100$ $25T^2 - 150T + 125 = 0 \quad \text{or equivalent}$ $(T^2 - 6T + 5 = 0)$ $T = 1, 5$	M1 A1 M1 A1 (4) M1 A1 (2) M1 A1 (2) M1 A1 M1 DM1 A1 A1 (6) [14]

6. (a)	$\tan\theta = \frac{3}{4}$; bearing is 37° (nearest degree)	M1; A1 (2)
(b) (i) (ii) (iii) (c) (i) (ii)	$\mathbf{p} = (\mathbf{i} + \mathbf{j}) + t(2\mathbf{i} - 3\mathbf{j})$ $\mathbf{q} = (-2\mathbf{j}) + t(3\mathbf{i} + 4\mathbf{j})$ $\mathbf{PQ} = \mathbf{q} - \mathbf{p} = (-\mathbf{i} - 3\mathbf{j}) + t(\mathbf{i} + 7\mathbf{j})$ $-1 + t = 0$ $t = 1$ or 3pm $-1 + t = -(-3 + 7t)$ $t = \frac{1}{2}$ or 2.30 pm	M1 A1 A1 M1 A1 (5) M1 A1 M1 A1 (4) 11

Examiner reports

Question 1

In the first part, many candidates showed the velocities in the correct direction on a diagram but surprisingly many failed to realise that Q would be the faster particle. Most candidates could use the conservation of momentum principle to produce an equation of the form $3v_P + 2v_Q = (3 \times 3) \pm (2 \times 2)$, but fewer than half realised how to deal with the difference between the final speeds. Some had P travelling faster (1.4 and 0.4), others used v for *both* final speeds, found $v = 1$ then decided either that both were travelling at 1 m s^{-1} or that the speeds were 1 m s^{-1} and 2 m s^{-1} , either way round.

Part (b) was rather better done with most of those obtaining wrong answers in part (a) managing to score M1 A1ft A0. Only a few of those who had done part (a) correctly lost the final mark in part (b) for an answer of -7.2 .

Question 2

In part (a) almost all candidates found the magnitude of the velocity to give the speed correctly. Most derived the acceleration, in the second part, by subtracting the velocities and dividing by the time appropriately (often by setting up a $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ equation first); however, a significant minority continue to lack confidence in dealing with vectors and vector notation. Most realised they then had to multiply by the mass to find the force, but sometimes the answer given was the same as the acceleration. The final part proved to be a good discriminator. Some failed to set up an equation in terms of t for the velocity, using their acceleration and the initial velocity; various combinations of terms were often seen. Many realised the \mathbf{j} -component had to be zero, but some equated the \mathbf{i} -component to zero whilst others equated the two components. Nevertheless, there were a significant number of candidates who correctly deduced the answer, often with very little working.

Question 3

Part (a) had a very high success rate and all three marks were regularly scored but the second part was found to be more challenging. Most were able to resolve perpendicular to the plane to find the reaction and use it to find the limiting friction. However, all too often there were omissions from the equation of motion parallel to the plane, either the mass \times acceleration term and/or the weight component or else g was missing. Part (c) was a good discriminator and candidates needed to realise that this was a new system and that there was no acceleration. Those who failed to appreciate this and used their friction force from part (b) scored no marks. The majority of successful candidates resolved parallel and perpendicular to the plane (although a sizeable minority resolved vertically and horizontally) but even then a correct final answer was rarely seen due to premature approximation or else it was given to too many figures.

Question 4

This question proved to be very discriminating, particularly part (c). There were an impressive number of correct solutions to the first two parts. A mark was often lost in part (a) due to over accuracy whilst a few got their sines and cosines mixed up and others left out g . In the second part it was rare to see an attempt at the whole system solution and too often the friction or the weight component was left out of the appropriate equation. The vast majority considered the two particles separately and most candidates who obtained correct equations were able to obtain the correct value of m without further algebraic errors. The final part proved to be

considerably more difficult. Many found the speed of P when the string broke but then failed to appreciate that they had to then find the new deceleration before they could move on, with many just assuming it was g , without justification. A number of candidates had the masses the wrong way round but could still pick up the majority of the marks.

Question 5

There was some confusion in parts (a), (b) and (c) over which vectors were velocities and which were displacements, with some even using acceleration. In the first part, many did not appreciate the distinction between velocity and speed and in part (b) many were unable to convert an appropriate angle into a bearing. The third part tended to be well-answered but a few used 'verification' at $t = 0$ and $t = 4$ and scored nothing. Part (d) was a good discriminator and the less able were often unable to make much progress. The majority of candidates who used Pythagoras to find the magnitude of the relative position vector and equated it to 10 scored at least 3/6 but many often lost the accuracy marks due to poor algebra. There were a number of other methods seen which used the fact that the lighthouse was on the path of the ship and that the speed of the ship was 5 km/h and these received full credit.

Question 6

The majority were able to find the correct bearing in part (a), but it was surprising that so many candidates firstly, seemed not to realise that bearings need to be measured from North and secondly, threw away a mark by not rounding to the nearest degree – perhaps they were rushing at the end of the paper. Others tried more exotic and totally unnecessary methods such as the cosine rule.

Except for the few who seemed to have no idea of how to work with vectors (and those who thought PQ meant magnitude of PQ) part (b) was done well. There were many who wrote down the vector $\mathbf{q} - \mathbf{p}$ correctly and were able to score a mark– often for a correct unsimplified expression but then made errors in simplifying which sometimes then led to marks being lost in the final part.

Part (c) (i) was done successfully by most good candidates, but part (c)(ii) had a very low success rate, with most candidates simply equating the \mathbf{i} and \mathbf{j} components of PQ . This proved to be a good discriminator at the end of the paper.

Statistics for M1 Practice Paper Gold Level G1

Original paper	Qu	Mean score	Max score	Mean %	Mean score for students achieving grade:						
					ALL	A	B	C	D	E	U
1106	1	4.92	8	62	4.92	6.31	5.29	4.67	3.90	3.20	1.81
1101	2	7.30	11	66	7.30	8.95	7.15	5.72	4.58	3.67	2.16
1001	3	9.33	15	62	9.33	11.81	9.11	7.37	5.51	4.36	2.51
1106	4	8.27	16	52	8.27	12.18	9.10	6.64	4.61	2.97	1.21
1001	5	7.77	14	56	7.77	10.85	7.13	5.36	4.28	3.19	1.39
1106	6	5.51	11	50	5.51	7.90	6.06	4.61	3.46	2.42	1.05
		43.10	75	57	43.10	58.00	43.84	34.37	26.34	19.81	10.13