Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1
Gold Level G1

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
67	58	49	41	33	25

1. The complex numbers z and w are given by

$$z = 8 + 3\text{i}, \qquad w = -2\text{i}$$

Express in the form $a + b\text{i}$, where a and b are real constants,

(*a*) $z - w$,

(1)

(*b*) zw.

(2)

June 2013 (R)

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2. $$\text{f}(x) = \cos(x^2) - x + 3, \qquad 0 < x < \pi$$

(*a*) Show that the equation $\text{f}(x) = 0$ has a root α in the interval [2.5, 3].

(2)

(*b*) Use linear interpolation once on the interval [2.5, 3] to find an approximation for α, giving your answer to 2 decimal places.

(3)

June 2013

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3. $$\text{f}(x) = \frac{1}{2}x^4 - x^3 + x - 3$$

(*a*) Show that the equation $\text{f}(x) = 0$ has a root α between $x = 2$ and $x = 2.5$.

(2)

(*b*) Starting with the interval [2, 2.5] use interval bisection twice to find an interval of width 0.125 which contains α.

(3)

The equation $\text{f}(x) = 0$ has a root β in the interval [–2, –1].

(*c*) Taking –1.5 as a first approximation to β, apply the Newton-Raphson process once to $\text{f}(x)$ to obtain a second approximation to β.
Give your answer to 2 decimal places.

(5)

June 2013 (R)

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4. The rectangular hyperbola H has Cartesian equation $xy = 4$.

The point $P\left(2t, \frac{2}{t}\right)$ lies on H, where $t \neq 0$.

(*a*) Show that an equation of the normal to H at the point P is

$$ty - t^3x = 2 - 2t^4$$

(5)

The normal to H at the point where $t = -\frac{1}{2}$ meets H again at the point Q.

(*b*) Find the coordinates of the point Q.

(4)

June 2013

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5. (*a*) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2+9n+26)$$

for all positive integers n.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n}(r+2)(r+3) = \frac{2}{3}n(an^2+bn+c)$$

where a, b and c are integers to be found.

(4)

June 2013

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6. A parabola C has equation $y^2 = 4ax$, $\quad a > 0$

The points $P(ap^2, 2ap)$ and $Q(aq^2, 2aq)$ lie on C, where $p \neq 0$, $q \neq 0$, $p \neq q$.

(*a*) Show that an equation of the tangent to the parabola at P is

$$py - x = ap^2$$

(4)

(*b*) Write down the equation of the tangent at Q.

(1)

The tangent at P meets the tangent at Q at the point R.

(*c*) Find, in terms of p and q, the coordinates of R, giving your answers in their simplest form.

(4)

Given that R lies on the directrix of C,

(*d*) find the value of pq.

(2)

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7. $$z_1 = 2 + 3\text{i}, \qquad z_2 = 3 + 2\text{i}, \qquad z_3 = a + b\text{i}, \qquad a, b \in \mathbb{R}$$

(*a*) Find the exact value of $|z_1 + z_2|$.

(2)

Given that $w = \dfrac{z_1 z_3}{z_2}$,

(*b*) find w in terms of a and b, giving your answer in the form $x + \text{i}y$, $\quad x, y \in \mathbb{R}$.

(4)

Given also that $w = \dfrac{17}{13} - \dfrac{7}{13}\text{i}$,

(*c*) find the value of a and the value of b,

(3)

(*d*) find arg w, giving your answer in radians to 3 decimal places.

(2)

June 2013

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8. (*a*) Prove by induction, that for $n \in \mathbb{Z}^+$,

$$\sum_{r=1}^{n} r(2r-1) = \frac{1}{6}n(n+1)(4n-1)$$

(6)

(*b*) Hence, show that

$$\sum_{r=n+1}^{3n} r(2r-1) = \frac{1}{3}n(an^2+bn+c)$$

where *a*, *b* and *c* are integers to be found.

(4)

June 2013 (R)

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9. Prove by induction that, for $n \in \mathbb{Z}^+$,

$$f(n) = 2^{2n-1} + 3^{2n-1}$$

is divisible by 5.

(6)

June 2012

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$z = 8 + 3\text{i}, \quad w = -2\text{i}$	
	$z - w \left\{= (8 + 3\text{i}) - (-2\text{i})\right\} = 8 + 5\text{i}$	B1
		(1)
(b)	$zw \left\{= (8 + 3\text{i})(-2\text{i})\right\} = 6 - 16\text{i}$	M1 A1
		(2)
		[3]
2. (a)	$\text{f}(x) = \cos\left(x^2\right) - x + 3$	
	f(2.5) = 1.499..... f(3) = -0.9111.....	M1
	Sign change (positive, negative) (and $\text{f}(x)$ is continuous) therefore root or equivalent.	A1
	Use of degrees gives f(2.5) = 1.494 and f(3) = 0.988 which is awarded M1A0	
		(2)
(b)	$\dfrac{3 - \alpha}{\text{"}0.91113026188\text{"}} = \dfrac{\alpha - 2.5}{\text{"}1.4994494182\text{"}}$	M1 A1ft
	$\alpha = \dfrac{3 \times 1.499... + 2.5 \times 0.9111....}{1.499... + 0.9111....}$	
	$\alpha = 2.81\,(2\text{d.p.})$	A1 cao
		(3)
		[5]

Question Number	Scheme	Marks
3. (a)	$\mathrm{f}(x) = \frac{1}{2}x^4 - x^3 + x - 3$	
	$\mathrm{f}(2) = -1$ $\mathrm{f}(2.5) = 3.40625$	M1
	Sign change (and $\mathrm{f}(x)$ is continuous) therefore a root α exists between $x = 2$ and $x = 2.5$	A1
		(2)
(b)	$\mathrm{f}(2.25) = 0.673828125 \left\{= \frac{345}{512}\right\}$ $\left\{\Rightarrow 2 \leqslant \alpha \leqslant 2.25\right\}$	B1 M1
	$\mathrm{f}(2.125) = -0.2752685547...$ $\Rightarrow 2.125 \leqslant \alpha \leqslant 2.25$	A1
		(3)
(c)	$\mathrm{f}'(x) = 2x^3 - 3x^2 + 1 \{+0\}$	M1 A1
	$\mathrm{f}(-1.5) = 1.40625 \left(= 1\frac{13}{32}\right)$	B1
	$\{\mathrm{f}'(-1.5) = -12.5\}$	
	$\beta_2 = -1.5 - \left(\frac{\text{"}1.40625\text{"}}{\text{"}-12.5\text{"}}\right)$	M1
	$= -1.3875 \quad \left(= -1\frac{31}{80}\right)$	
	$= -1.39$ (2dp)	A1 **cao**
		(5)
		[10]

Question Number	Scheme	Marks
4. (a)	$y = \frac{4}{x} = 4x^{-1} \Rightarrow \frac{dy}{dx} = -4x^{-2} = -\frac{4}{x^2}$	M1
	$xy = 4 \Rightarrow x\frac{dy}{dx} + y = 0$	
	$\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{2}{t^2}.\frac{1}{2}$	
	$\frac{dy}{dx} = -4x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ or $\frac{dy}{dx} = -\frac{2}{t^2}.\frac{1}{2}$ or equivalent expressions	A1
	So, $m_N = t^2$	M1
	$y - \frac{2}{t} = t^2(x - 2t)$	M1
	$ty - t^3x = 2 - 2t^4$ *	A1* cso (5)
(b)	$t = -\frac{1}{2} \Rightarrow -\frac{1}{2}y - \left(-\frac{1}{2}\right)^3 x = 2 - 2\left(-\frac{1}{2}\right)^4$	M1
	$4y - x + 15 = 0$	
	$y = \frac{4}{x} \Rightarrow x^2 - 15x - 16 = 0$ or $\left(2t, \frac{2}{t}\right) \to \frac{8}{t} - 2t + 15 = 0 \Rightarrow 2t^2 - 15t - 8 = 0$ or $x = \frac{4}{y} \Rightarrow 4y^2 + 15y - 4 = 0$.	M1
	$(x+1)(x-16) = 0 \Rightarrow x =$ or $(2t+1)(t-8) = 0 \Rightarrow t =$ or $(4y-1)(y+4) = 0 \Rightarrow y =$	M1
	$(P: x = -1, y = -4)(Q:) x = 16, y = \frac{1}{4}$	A1
		(4) **[9]**

Question Number	Scheme	Marks
5. (a)	$(r+2)(r+3)=r^2+5r+6$	B1
	$\sum(r^2+5r+6)=\frac{1}{6}n(n+1)(2n+1)+5\times\frac{1}{2}n(n+1),+6n$	M1,B1ft
	$=\frac{1}{3}n\left[\frac{1}{2}(n+1)(2n+1)+\frac{15}{2}(n+1)+18\right]$	M1 A1
	$\left(=\frac{1}{3}n\left[n^2+\frac{3}{2}n+\frac{1}{2}+\frac{15}{2}n+\frac{15}{2}+18\right]\right)$	
	$=\frac{1}{3}n\left[n^2+9n+26\right]$ *	A1*cso
		(6)
(b)	$\sum_{r=n+1}^{3n} =\frac{1}{3}3n\left((3n)^2+9(3n)+26\right)-\frac{1}{3}n\left(n^2+9n+26\right)$	M1A1
	3f(***n***) – f(n or n+1) is M0	
	$(=n\left(9n^2+27n+26\right)-\frac{1}{3}n\left(n^2+9n+26\right))$	
	$=\frac{2}{3}n\left(\frac{27}{2}n^2+\frac{81}{2}n+39-\frac{1}{2}n^2-\frac{9}{2}n-13\right)$	dM1
	$=\frac{2}{3}n\left(13n^2+36n+26\right)$	A1
	$(a=13, b=36, c=26)$	
		(4) **[10]**

Question Number	Scheme	Marks
6. (a)	$y = 2a^{\frac{1}{2}}x^{\frac{1}{2}} \Rightarrow \frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$	
	$y^2 = 4ax \Rightarrow 2y\frac{dy}{dx} = 4a$	M1
	or $\frac{dy}{dx} = \frac{dy}{dt}.\frac{dt}{dx} = 2a.\frac{1}{2ap}$	
	$\frac{dy}{dx} = a^{\frac{1}{2}}x^{-\frac{1}{2}}$ or $2y\frac{dy}{dx} = 4a$ or $\frac{dy}{dx} = 2a.\frac{1}{2ap}$	A1
	$y - 2ap = \frac{1}{p}(x - ap^2)$	M1
	$py - x = ap^2$ *	A1 cso (4)
(b)	$qy - x = aq^2$	B1 (1)
(c)	$qy - aq^2 = py - ap^2$	M1
	$y(q-p) = aq^2 - ap^2$	
	$y = \frac{aq^2 - ap^2}{q-p}$	M1
	$y = a(p+q)$ *or* $ap + aq$ $x = apq$	A1,A1
	$(R(apq, ap+aq))$	(4)
(d)	$'apq' = -a$	M1
	$pq = -1$	A1 (2) **[11]**

Question Number	Scheme	Marks
7. (a)	$z_1 = 2+3\text{i}, \quad z_2 = 3+2\text{i}$	
	$z_1 + z_2 = 5+5\text{i} \Rightarrow \lvert z_1 + z_2 \rvert = \sqrt{5^2+5^2}$	M1
	$\sqrt{50}\ (= 5\sqrt{2})$	A1 cao
		(2)
(b)	$\dfrac{z_1 z_3}{z_2} = \dfrac{(2+3\text{i})(a+b\text{i})}{3+2\text{i}}$	
	$= \dfrac{(2+3\text{i})(a+b\text{i})(3-2i)}{(3+2\text{i})(3-2\text{i})}$	M1
	$(3+2\text{i})(3-2\text{i}) = 13$	B1
	$\dfrac{z_1 z_3}{z_2} = \dfrac{(12a-5b)+(5a+12b)\text{i}}{13}$	dM1A1
		(4)
(c)	$12a - 5b = 17$ $5a + 12b = -7$	M1
	$\begin{matrix} 60a - 25b = 85 \\ 60a + 144b = -84 \end{matrix} \quad \Rightarrow \quad b = -1$	dM1
	a = 1, b = -1	A1
		(3)
(d)	$\arg(w) = -\tan^{-1}\left(\dfrac{7}{17}\right)$	M1
	=awrt -0.391 or awrt 5.89	A1
		(2) **[11]**
8. (a)	$\sum_{r=1}^{n} r(2r-1) = \dfrac{1}{6}n(n+1)(4n-1)$	
	$n=1;\quad \text{LHS} = \sum_{r=1}^{1} r(2r-1) = 1$	
	$\text{RHS} = \frac{1}{6}(1)(2)(3) = 1$	B1
	As LHS = RHS, the summation formula is true for $n = 1$.	
	Assume that the summation formula is true for $n = k$.	
	ie. $\sum_{r=1}^{k} r(2r-1) = \dfrac{1}{6}k(k+1)(4k-1).$	
	With $n = k+1$ terms the summation formula becomes:	

Question Number	Scheme	Marks
	$\sum_{r=1}^{k+1} r(2r-1) = \underline{\frac{1}{6}k(k+1)(4k-1) + (k+1)(2(k+1)-1)}$	M1
	$= \frac{1}{6}k(k+1)(4k-1) + (k+1)(2k+1)$	
	$= \frac{1}{6}(k+1)\left(k(4k-1) + 6(2k+1)\right)$	dM1
	$= \frac{1}{6}(k+1)\left(4k^2 + 11k + 6\right)$	A1
	$= \frac{1}{6}(k+1)(k+2)(4k+3)$	
	$= \frac{1}{6}(k+1)(k+1+1)(4(k+1)-1)$	dM1
	If the summation formula is <u>true for</u> $\underline{n=k}$, then it is shown to be <u>true for</u> $\underline{n=k+1}$. As the result is <u>true for</u> $\underline{n=1}$, it is now also <u>true for all</u> $\underline{n}$ and $n \in \mathbb{Z}^+$ by mathematical induction.	A1 cso
		(6)
(b)	$\sum_{r=n+1}^{3n} r(2r-1) = S_{3n} - S_n$	
	$= \frac{1}{6}.3n(3n+1)(12n-1) - \frac{1}{6}n(n+1)(4n-1)$	M1 A1
	$= \frac{1}{6}n\{3(3n+1)(12n-1) - (n+1)(4n-1)\}$	
	$= \frac{1}{6}n\{3(36n^2 + 9n - 1) - (4n^2 + 3n - 1)\}$	dM1
	$= \frac{1}{6}n\{108n^2 + 27n - 3 - 4n^2 - 3n + 1\}$	
	$= \frac{1}{6}n\{104n^2 + 24n - 2\}$	
	$= \frac{1}{3}n\left(52n^2 + 12n - 1\right)$	A1
	$\{a = 52, b = 12, c = -1\}$	(4) **[10]**

Question Number	Scheme	Marks
9.	$\mathrm{f}(n) = 2^{2n-1} + 3^{2n-1}$ is divisible by 5.	
	$\mathrm{f}(1) = 2^1 + 3^1 = 5,$	B1
	Assume that for $n = k$,	
	$\mathrm{f}(k) = 2^{2k-1} + 3^{2k-1}$ is divisible by 5 for $k \in \mathbb{Z}^+$.	
	$\mathrm{f}(k+1) - \mathrm{f}(k) = 2^{2(k+1)-1} + 3^{2(k+1)-1} - \left(2^{2k-1} + 3^{2k-1}\right)$	M1A1
	$= 2^{2k+1} + 3^{2k+1} - 2^{2k-1} - 3^{2k-1}$	
	$= 2^{2k-1+2} + 3^{2k-1+2} - 2^{2k-1} - 3^{2k-1}$	
	$= 4\left(2^{2k-1}\right) + 9\left(3^{2k-1}\right) - 2^{2k-1} - 3^{2k-1}$	M1
	$= 3\left(2^{2k-1}\right) + 8\left(3^{2k-1}\right)$	
	$= 3\left(2^{2k-1}\right) + 3\left(3^{2k-1}\right) + 5\left(3^{2k-1}\right)$	
	$= 3\mathrm{f}(k) + 5\left(3^{2k-1}\right)$	
	$\therefore \mathrm{f}(k+1) = 4\mathrm{f}(k) + 5\left(3^{2k-1}\right)$ or $4(2^{2k-1} + 3^{2k-1}) + 5\left(3^{2k-1}\right)$	A1
	If the result is true for $n = k$, then it is now true for $n = k+1$. As the result has shown to be true for $n = 1$, then the result is true for all n.	A1 cso
		[6]

Examiner reports

Question 1

This was a good opening question for the candidates and solutions were almost always correct. They demonstrated that they knew how to deal with real and imaginary parts when subtracting and multiplying very well.

Question 2

Most candidates realised that a conclusion was required to part (a) and gained full marks here. Some had used degrees instead of radians and were confused by the lack of sign change. These candidates rarely continued to attempt part (b) although they could have gained the method and follow through mark had they done so.

In part (b) there were often problems with the sign of 0.911 and the fraction was often inverted and both of these errors resulted in no marks being awarded for this part of the question. Some candidates attempted to use interval bisection instead of linear interpolation. Occasionally, candidates went back to first principles and found the equation of the line, then putting $y = 0$ to find the intercept on the x-axis. A long method, but often successful.

Question 3

In part (a) nearly all responses seen demonstrated the sign change in $f(x)$, but some candidates failed to make a satisfactory conclusion and lost the accuracy mark. Part (b) was a problem for some and a few tried linear interpolation rather than interval bisection. There were a few candidates that used interval bisection correctly, but unfortunately gave the wrong interval as their final answer. The Newton-Raphson method in part (c) was correctly applied in many solutions. The differentiation was usually done well, but there was the occasional slip. A number of responses did not contain much evidence of the calculations involved and some candidates used 1.5 instead of -1.5. This was not a problem where answers were correct, but a lack of working did cost some candidates marks where the final answer was incorrect.

Question 4

There were many good answers to part (a). Candidates were able to find the gradient of the tangent using calculus and realised that they had to take the negative reciprocal to find the gradient of the normal. Very few candidates left these gradients in terms of x and most realised that a substitution was needed. As a piece of bookwork, this had been well understood and there were many fully correct solutions.

Part (b) proved to be more challenging. A few candidates substituted t as $\frac{1}{2}$ instead of $-\frac{1}{2}$ into their equation of the normal and many did not realise that they also needed to use the equation of the hyperbola. Those that did use the equation of the hyperbola were generally successful in obtaining a quadratic, which they solved to get the correct coordinates. Those who drew a sketch generally showed a better understanding of what was required in this part of the question.

Question 5

There were some good answers to part (a). The correct formulae were used and the term 6n was achieved by the majority of candidates. Factorising went ahead correctly, possibly because there was a given answer to achieve. A few candidates tried to use mathematical induction to prove the result and they gained no marks.

In part (b) most realised that they needed to find the difference of two sums. Marks were lost here when 3f(n) was used in place of f(3n). Also $3n^2$ instead of $(3n)^2$ was a common error. Overall this question was very well answered this year.

Question 6

The mathematics required here had been learnt well and many candidates achieved the required result successfully. Only a few candidates just quoted the gradient of the tangent and again, few left the gradient as a function of x, which was encouraging.

The equation in part (b) was usually quoted accurately although there were a few candidates who tried to start from scratch for 1 mark.

In part (c) most attempted to eliminate either x or y to find the coordinates of the point of intersection but simplifying the result proved to be more of a challenge. Often poor algebraic skills meant the loss of the last three marks for this question. The equation of the directrix was not generally known and only the more able candidates achieved the final two marks for this question. Use of $x = –4a$ and $x = a$ as the directrix were common errors.

Question 7

Part a proved a challenge to some candidates and common errors seen were finding the modulus of z_1 and z_2, then adding. Unfortunately some candidates just added z_1 and z_2 and made no attempt to find the modulus and a few left i in the square root. Most candidates knew what they were supposed to do to achieve the answer but the amount of simplification required defeated them. Many ended up, after many lines of working, with only one variable in either the real or complex part of their answer. Few failed to spot that multiplying the two numerical terms, (2 + 3i) and (3 – 2i) before multiplying by the algebraic term simplified the working.

In part (c) the algebra involved in achieving an answer, following an earlier error, proved to be a challenge.

The mark for the use of tan was usually achieved in part (d) and common errors in the final answer were omitting the negative sign or inverting the fraction.

Question 8

A large number of candidates knew induction well and picked up most of the marks in this question, though there was a reluctance to take out the factor (k + 1) early. A common error was to show it was true for n = 1, but then just substitute k + 1 into the formula instead of adding the (k + 1)th term to S_k. A significant number of candidates failed to identify the (k + 1) in each bracket following them finding the correct factors. A minority scored no marks by trying to prove by use of the standard summation formulae from the formula book.

In part (b) those who used their answer to part (a) generally did so correctly, but a minority did not correctly follow their answer to part (a). An occasional error seen was $3S_n – S_n$ instead of $S_{3n} – S_n$.

Question 9

This question proved to be a good discriminator. Many candidates could make a start and proved the result was true for $n = 1$. There were then varying approaches at the induction with $f(k) - f(k + 1)$ being the most popular, but there were also other valid methods that met with varying degrees of success such as $f(k) + f(k + 1)$ or attempts to deal with $f(k + 1)$ directly. Candidates who made it this far then often made some attempt to obtain an expression in terms of 2^{2k-1} and 3^{2k-1} but were then less successful in reaching an expression that was divisible by 5. The penultimate mark for all methods required completion to an expression for $f(k + 1)$ that was clearly shown to be divisible by 5. For the final mark the candidate needed to make a sensible conclusion, bringing the various parts of the proof together. An example of a minimum acceptable comment here, following completely correct work, would be 'if the result is true for $n = k$ then it has been shown to be true for $n = k + 1$ and as it was shown true for $n = 1$ then the result is true for all positive integers'.

Statistics for FP1 Practice Paper Gold Level G1

				Mean score for students achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	3		97	2.90	2.98	2.96	2.94	2.93	2.94	2.82	2.71
2	5	5	77	3.86	4.80	4.48	3.93	3.65	3.11	2.67	1.84
3	10		90	8.98	9.66	9.59	9.31	8.99	8.68	7.83	6.70
4	9	9	81	7.30	8.84	8.43	7.72	6.95	5.84	5.17	2.84
5	10	10	78	7.79	9.83	9.25	8.25	7.24	5.93	5.13	2.99
6	11	11	74	8.12	10.82	9.93	8.64	7.33	5.78	4.49	2.08
7	11	11	74	8.17	10.56	9.58	8.45	7.59	6.46	5.67	3.71
8	10		68	6.75	9.21	8.77	7.55	5.62	4.81	3.32	2.43
10	6		61	3.67	5.09	4.35	3.50	3.14	2.86	2.55	2.10
	75		**77**	**57.54**	**71.79**	**67.34**	**60.29**	**53.44**	**46.41**	**39.65**	**27.40**