Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4
Gold Level (Harder) G3

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C4), the paper reference (6666), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
65	58	47	42	36	28

1. $$f(x) = (3 + 2x)^{-3}, \quad |x| < \tfrac{3}{2}.$$

Find the binomial expansion of f(x), in ascending powers of x, as far as the term in x^3.

Give each coefficient as a simplified fraction.

(5)

June 2007

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2. Using the substitution $u = \cos x + 1$, or otherwise, show that

$$\int_0^{\frac{\pi}{2}} e^{\cos x + 1} \sin x \, dx = e(e - 1).$$

(6)

June 2010

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3. Express $\dfrac{9x^2 + 20x - 10}{(x+2)(3x-1)}$ in partial fractions.

(4)

January 2013

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4. With respect to a fixed origin O the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \qquad l_2 : \mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$$

where λ and μ are parameters and p and q are constants. Given that l_1 and l_2 are perpendicular,

(a) show that $q = -3$.

(2)

Given further that l_1 and l_2 intersect, find

(b) the value of p,

(6)

(c) the coordinates of the point of intersection.

(2)

The point A lies on l_1 and has position vector $\begin{pmatrix} 9 \\ 3 \\ 13 \end{pmatrix}$. The point C lies on l_2.

Given that a circle, with centre C, cuts the line l_1 at the points A and B,

(d) find the position vector of B.

(3)

January 2009

5.

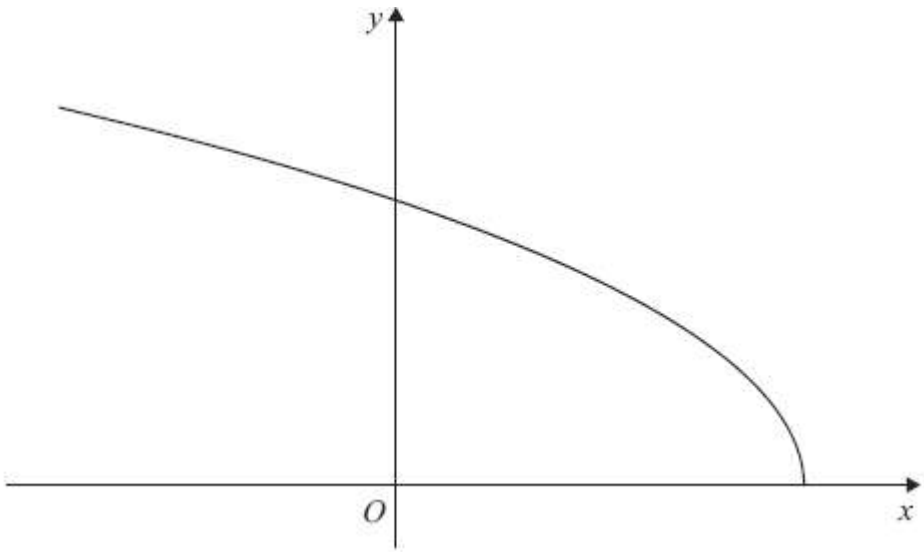

Figure 2

Figure 2 shows a sketch of the curve with parametric equations

$$x = 2\cos 2t, \qquad y = 6\sin t, \qquad 0 \le t \le \frac{\pi}{2}.$$

(*a*) Find the gradient of the curve at the point where $t = \frac{\pi}{3}$.

(4)

(*b*) Find a cartesian equation of the curve in the form

$$y = \mathrm{f}(x), \quad -k \le x \le k,$$

stating the value of the constant k.

(4)

(*c*) Write down the range of $\mathrm{f}(x)$.

(2)

January 2009

6. The points A and B have position vectors $2\mathbf{i} + 6\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} + 4\mathbf{j} + \mathbf{k}$ respectively.

The line l_1 passes through the points A and B.

(*a*) Find the vector $\overrightarrow{AB}$.

(2)

(*b*) Find a vector equation for the line l_1.

(2)

A second line l_2 passes through the origin and is parallel to the vector $\mathbf{i} + \mathbf{k}$. The line l_1 meets the line l_2 at the point C.

(*c*) Find the acute angle between l_1 and l_2.

(3)

(*d*) Find the position vector of the point C.

(4)

January 2008

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7. The line l_1 has equation $\mathbf{r} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, where λ is a scalar parameter.

The line l_2 has equation $\mathbf{r} = \begin{pmatrix} 0 \\ 9 \\ -3 \end{pmatrix} + \mu \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$, where μ is a scalar parameter.

Given that l_1 and l_2 meet at the point C, find

(*a*) the coordinates of C.

(3)

The point A is the point on l_1 where $\lambda = 0$ and the point B is the point on l_2 where $\mu = -1$.

(*b*) Find the size of the angle ACB. Give your answer in degrees to 2 decimal places.

(4)

(*c*) Hence, or otherwise, find the area of the triangle ABC.

(5)

June 2010

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8. A population growth is modelled by the differential equation

$$\frac{dP}{dt} = kP,$$

where P is the population, t is the time measured in days and k is a positive constant.

Given that the initial population is P_0,

(a) solve the differential equation, giving P in terms of P_0, k and t.

(4)

Given also that $k = 2.5$,

(b) find the time taken, to the nearest minute, for the population to reach $2P_0$.

(3)

In an improved model the differential equation is given as

$$\frac{dP}{dt} = \lambda P \cos \lambda t,$$

where P is the population, t is the time measured in days and λ is a positive constant.

Given, again, that the initial population is P_0 and that time is measured in days,

(c) solve the second differential equation, giving P in terms of P_0, λ and t.

(4)

Given also that $\lambda = 2.5$,

(d) find the time taken, to the nearest minute, for the population to reach $2P_0$ for the first time, using the improved model.

(3)

June 2007

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	** represents a constant	
	$f(x) = (3+2x)^{-3} = \underline{(3)^{-3}}\left(1+\frac{2x}{3}\right)^{-3} = \underline{\frac{1}{27}}\left(1+\frac{2x}{3}\right)^{-3}$ Takes 3 outside the bracket to give any of $(3)^{-3}$ or $\frac{1}{27}$. **See note below.**	B1
	$= \frac{1}{27}\left\{\underline{1+(-3)(**x);+\frac{(-3)(-4)}{2!}(**x)^2+\frac{(-3)(-4)(-5)}{3!}(**x)^3+\ldots}\right\}$ Expands $(1+**x)^{-3}$ to give a simplified or an un-simplified $1+(-3)(**x)$;	M1;
	with $** \neq 1$ A correct simplified or an un-simplified $\{\underline{\ldots\ldots\ldots}\}$ expansion with candidate's followed thro' $(**x)$	A1 $\surd$
	$= \frac{1}{27}\left\{\underline{1+(-3)(\frac{2x}{3})+\frac{(-3)(-4)}{2!}(\frac{2x}{3})^2+\frac{(-3)(-4)(-5)}{3!}(\frac{2x}{3})^3+\ldots}\right\}$	
	$= \frac{1}{27}\left\{1-2x+\frac{8x^2}{3}-\frac{80}{27}x^3+\ldots\right\}$	
	$= \frac{1}{27}-\frac{2x}{27};+\frac{8x^2}{81}-\frac{80x^3}{729}+\ldots$ Anything that cancels to $\frac{1}{27}-\frac{2x}{27}$; Simplified $\frac{8x^2}{81}-\frac{80x^3}{729}$	A1; A1
		[5]
		5 marks

2.	$\frac{du}{dx} = -\sin x$	B1
	$\int \sin x\, e^{\cos x+1} dx = -\int e^u du$	M1 A1
	$= -e^u$ ft sign error	A1ft
	$= -e^{\cos x+1}$	
	$\left[-e^{\cos x+1}\right]_0^{\frac{\pi}{2}} = -e^1 - \left(-e^2\right)$ or equivalent with u	M1
	$= e\left(e-1\right)$ ∗ cso	A1 **(6)**
		[6]

Question Number	Scheme	Marks
3.	$\frac{9x^2+20x-10}{(x+2)(3x-1)} \equiv A + \frac{B}{(x+2)} + \frac{C}{(3x-1)}$	
	$A = 3$ their constant term $= 3$	B1
	$9x^2 + 20x - 10 \equiv A(x+2)(3x-1) + B(3x-1) + C(x+2)$ Forming a correct identity.	B1
	Either x^2: $9 = 3A$, x: $20 = 5A + 3B + C$; constant: $-10 = -2A - B + 2C$ Attempts to find the value of either one of their B or their C from their identity.	M1
	or $x = -2 \Rightarrow 36 - 40 - 10 = -7B \Rightarrow -14 = -7B \Rightarrow B = 2$	
	$x = \frac{1}{3} \Rightarrow 1 + \frac{20}{3} - 10 = \frac{7}{3}C \Rightarrow -\frac{7}{3} = \frac{7}{3}C \Rightarrow C = -1$ Correct values for their B and their C, which are found using a correct identity.	A1
		[4]

Question Number	Scheme		Marks
4 (a)	$\mathbf{d}_1 = -2\mathbf{i} + \mathbf{j} - 4\mathbf{k}$, $\mathbf{d}_2 = q\mathbf{i} + 2\mathbf{j} + 2\mathbf{k}$		
	As $\left\{ \mathbf{d}_1 \bullet \mathbf{d}_2 = \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} \bullet \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix} \right\} = \underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	Apply dot product calculation between two direction vectors, ie. $\underline{(-2\times q) + (1\times 2) + (-4\times 2)}$	M1
	$\mathbf{d}_1 \bullet \mathbf{d}_2 = 0 \Rightarrow -2q + 2 - 8 = 0$ $-2q = 6 \Rightarrow \underline{q = -3}$ AG	Sets $\mathbf{d}_1 \bullet \mathbf{d}_2 = 0$ and solves to find $\underline{q = -3}$	A1 cso
			(2)
(b)	Lines meet where:		
	$\begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix} = \begin{pmatrix} -5 \\ 11 \\ p \end{pmatrix} + \mu \begin{pmatrix} q \\ 2 \\ 2 \end{pmatrix}$		
	First two of $\mathbf{i}$: $11 - 2\lambda = -5 + q\mu$ (1) $\mathbf{j}$: $2 + \lambda = 11 + 2\mu$ (2) $\mathbf{k}$: $17 - 4\lambda = p + 2\mu$ (3)	Need to see equations (1) and (2). Condone one slip. (Note that $q = -3$.)	M1
	(1) + 2**(2)** gives: $15 = 17 + \mu \Rightarrow \mu = -2$	Attempts to solve (1) and (2) to find one of either λ or μ	dM1
	(2) gives: $2 + \lambda = 11 - 4 \Rightarrow \lambda = 5$	Any one of $\underline{\lambda = 5}$ or $\underline{\mu = -2}$ Both $\underline{\lambda = 5}$ and $\underline{\mu = -2}$	A1 A1
	(3) $\Rightarrow 17 - 4(5) = p + 2(-2)$	Attempt to substitute their λ and μ into their **k** component to give an equation in p alone.	ddM1
	$\Rightarrow p = 17 - 20 + 4 \Rightarrow \underline{p = 1}$	$\underline{p = 1}$	A1 cso
			(6)
(c)	$\mathbf{r} = \begin{pmatrix} 11 \\ 2 \\ 17 \end{pmatrix} + 5\begin{pmatrix} -2 \\ 1 \\ -4 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} -5 \\ 11 \\ 1 \end{pmatrix} - 2\begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix}$	Substitutes their value of λ or μ into the correct line l_1 or l_2.	M1
	Intersect at $\mathbf{r} = \underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	$\underline{\begin{pmatrix} 1 \\ 7 \\ -3 \end{pmatrix}}$ or $\underline{(1, 7, -3)}$	A1
			(2)

5.	(a)	$\frac{dx}{dt} = -4\sin 2t$, $\frac{dy}{dt} = 6\cos t$	B1, B1	
		$\frac{dy}{dx} = -\frac{6\cos t}{4\sin 2t}$ $\left(= -\frac{3}{4\sin t}\right)$	M1	
		At $t = \frac{\pi}{3}$, $m = -\frac{3}{4\times\frac{\sqrt{3}}{2}} = -\frac{\sqrt{3}}{2}$ accept equivalents, awrt -0.87	A1	(4)
	(b)	Use of $\cos 2t = 1 - 2\sin^2 t$	M1	
		$\cos 2t = \frac{x}{2}$, $\sin t = \frac{y}{6}$		
		$\frac{x}{2} = 1 - 2\left(\frac{y}{6}\right)^2$	M1	
		Leading to $y = \sqrt{(18-9x)}$ $\left(= 3\sqrt{(2-x)}\right)$ cao	A1	
		$-2 \le x \le 2$ $k = 2$	B1	(4)
	(c)	$0 \le f(x) \le 6$ either $0 \le f(x)$ or $f(x) \le 6$	B1	
		Fully correct. Accept $0 \le y \le 6$, $[0, 6]$	B1	(2)
				(10 marks)

Question Number	Scheme		Marks
6. (a)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix}$ & $\overrightarrow{OB} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} = \underline{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}}$	Finding the difference between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. Correct answer.	M1 ± A1 **[2]**
(b)	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}$	An expression of the form $(\text{vector}) \pm \lambda(\text{vector})$ $\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{AB})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{AB})$ or	M1
	$l_1 : \mathbf{r} = \begin{pmatrix} 2 \\ 6 \\ -1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$ or $\mathbf{r} = \begin{pmatrix} 3 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ -2 \end{pmatrix}$	$\mathbf{r} = \overrightarrow{OA} \pm \lambda(\text{their } \overrightarrow{BA})$ or $\mathbf{r} = \overrightarrow{OB} \pm \lambda(\text{their } \overrightarrow{BA})$ ($\mathbf{r}$ is needed.)	A1 $\sqrt{}$ aef **[2]**
(c)	$l_2 : \mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \Rightarrow \mathbf{r} = \mu \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$		
	$\overrightarrow{AB} = \mathbf{d}_1 = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\mathbf{d}_2 = \mathbf{i} + 0\mathbf{j} + \mathbf{k}$ & θ is angle		
	$\cos\theta = \dfrac{\overrightarrow{AB} \bullet \mathbf{d}_2}{\left(\lvert\overrightarrow{AB}\rvert . \lvert\mathbf{d}_2\rvert\right)} = \dfrac{\begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix} \bullet \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}{\left(\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}\right)}$	Considers dot product between $\mathbf{d}_2$ and their $\overrightarrow{AB}$.	M1 $\sqrt{}$
	$\cos\theta = \dfrac{1 + 0 + 2}{\sqrt{(1)^2 + (-2)^2 + (2)^2} . \sqrt{(1)^2 + (0)^2 + (1)^2}}$	Correct followed through expression or **equation**.	A1 $\sqrt{}$
	$\cos\theta = \dfrac{3}{3.\sqrt{2}} \Rightarrow \underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79.}$	$\underline{\theta = 45^\circ \text{ or } \tfrac{\pi}{4} \text{ or awrt } 0.79}$	A1 **cao** **[3]**

This means that $\cos\theta$ does not necessarily have to be the subject of the equation. It could be of the form $3\sqrt{2}\cos\theta = 3$.

Question Number	Scheme		Marks
6. (d)	If l_1 and l_2 intersect then: $\begin{pmatrix}2\\6\\-1\end{pmatrix}+\lambda\begin{pmatrix}1\\-2\\2\end{pmatrix}=\mu\begin{pmatrix}1\\0\\1\end{pmatrix}$		
	$\mathbf{i}:\ 2+\lambda=\mu$ (1) $\mathbf{j}:\ 6-2\lambda=0$ (2) $\mathbf{k}:\ -1+2\lambda=\mu$ (3)	**Either** seeing equation (2) written down correctly with or without any other equation **or** seeing equations (1) and (3) written down correctly.	M1 $\sqrt{}$
	(2) yields $\lambda=3$ Any two yields $\lambda=3,\quad \mu=5$	Attempt to solve either equation (2) or simultaneously solve any two of the three equations to find …	dM1
		either one of λ or μ correct.	A1
	$l_1:\ \mathbf{r}=\begin{pmatrix}2\\6\\-1\end{pmatrix}+3\begin{pmatrix}1\\-2\\2\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ *or* $\mathbf{r}=5\begin{pmatrix}1\\0\\1\end{pmatrix}=\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$	$\underline{\begin{pmatrix}5\\0\\5\end{pmatrix}}$ or $5\mathbf{i}+5\mathbf{k}$ Fully correct solution & no incorrect values of λ or μ seen earlier.	A1 **cso**
			[4]

7.	(a) **j** components $3+2\lambda=9 \quad \Rightarrow \lambda=3$ $\quad (\mu=1)$	M1 A1	
	Leading to $C:(5,9,-1)$ accept vector forms	A1	**(3)**
	(b) Choosing correct directions or finding $\overrightarrow{AC}$ and $\overrightarrow{BC}$	M1	
	$\begin{pmatrix}1\\2\\1\end{pmatrix}.\begin{pmatrix}5\\0\\2\end{pmatrix}=5+2=\sqrt{6}\sqrt{29}\cos\angle ACB$ use of scalar product	M1 A1	
	$\angle ACB=57.95°$ awrt $57.95°$	A1	**(4)**
	(c) $A:(2,3,-4) \quad B:(-5,9,-5)$		
	$\overrightarrow{AC}=\begin{pmatrix}3\\6\\3\end{pmatrix}, \quad \overrightarrow{BC}=\begin{pmatrix}10\\0\\4\end{pmatrix}$		
	$AC^2=3^2+6^2+3^2 \quad \Rightarrow \quad AC=3\sqrt{6}$	M1 A1	
	$BC^2=10^2+4^2 \quad \Rightarrow \quad BC=2\sqrt{29}$	A1	
	! $ABC=\frac{1}{2}AC\times BC\sin\angle ACB$		
	$=\frac{1}{2}3\sqrt{6}\times 2\sqrt{29}\sin\angle ACB\approx 33.5$ $\quad 15\sqrt{5}$, awrt 34	M1 A1	**(5)**
			[12]

Question Number	Scheme		Marks
8. (a)	$\frac{dP}{dt} = kP$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int k\ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int k\,dt$ on either side with integral signs not necessary.	M1
	$\ln P = kt;(+c)$	Must see $\ln P$ and kt; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \Rightarrow \ln P_0 = c$ $\left(\text{or } P = Ae^{kt} \Rightarrow P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = kt + \ln P_0 \Rightarrow e^{\ln P} = e^{kt + \ln P_0} = e^{kt}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{kt}}$	$\underline{P = P_0 e^{kt}}$	A1
			[4]
(b)	$P = 2P_0$ & $k = 2.5 \Rightarrow \underline{2P_0 = P_0 e^{2.5t}}$	Substitutes $P = 2P_0$ into an expression involving P	M1
	$e^{2.5t} = 2 \Rightarrow \underline{\ln e^{2.5t} = \ln 2}$ or $\underline{2.5t = \ln 2}$ …or $e^{kt} = 2 \Rightarrow \underline{\ln e^{kt} = \ln 2}$ or $\underline{kt = \ln 2}$	Eliminates P_0 and takes ln of both sides	M1
	$\Rightarrow t = \frac{1}{2.5}\ln 2 = 0.277258872...$ days		
	$t = 0.277258872... \times 24 \times 60 = 399.252776...$ minutes		
	$t = \underline{399\,\text{min}}$ or $t = \underline{6\text{ hr }39\text{ mins}}$ (to nearest minute)	awrt $t = \underline{399}$ or $\underline{6\text{ hr }39\text{ mins}}$	A1
			[3]

Question Number	Scheme		Marks
8. (c)	$\frac{dP}{dt} = \lambda P \cos \lambda t$ and $t = 0,\ P = P_0$ (1)		
	$\int \frac{dP}{P} = \int \lambda \cos \lambda t \ dt$	Separates the variables with $\int \frac{dP}{P}$ and $\int \lambda \cos \lambda t \, dt$ on either side with integral signs not necessary.	M1
	$\ln P = \sin \lambda t ; (+ c)$	Must see $\ln P$ and $\sin \lambda t$; Correct equation with/without + c.	A1
	When $t = 0,\ P = P_0 \ \Rightarrow \ \ln P_0 = c$ $\left(\text{or } P = Ae^{\sin \lambda t} \ \Rightarrow \ P_0 = A\right)$	Use of boundary condition (1) to attempt to find the constant of integration.	M1
	$\ln P = \sin \lambda t + \ln P_0 \quad \Rightarrow e^{\ln P} = e^{\sin \lambda t + \ln P_0} = e^{\sin \lambda t}.e^{\ln P_0}$		
	Hence, $\underline{P = P_0 e^{\sin \lambda t}}$	$\underline{P = P_0 e^{\sin \lambda t}}$	A1
			[4]
(d)	$P = 2P_0$ & $\lambda = 2.5 \quad \Rightarrow \ 2P_0 = P_0 e^{\sin 2.5t}$		
	$e^{\sin 2.5t} = 2 \ \Rightarrow \ \underline{\sin 2.5t = \ln 2}$ …or … $e^{\lambda t} = 2 \ \Rightarrow \ \underline{\sin \lambda t = \ln 2}$	Eliminates P_0 and makes $\sin \lambda t$ or $\sin 2.5t$ the subject by taking ln's	M1
	$\underline{t = \frac{1}{2.5} \sin^{-1}(\ln 2)}$ $t = 0.306338477...$	Then rearranges to make t the subject. (must use $\sin^{-1}$)	dM1
	$t = 0.306338477... \times 24 \times 60 = 441.1274082...$ minutes		
	$t = \underline{441\text{min}}$ or $t = \underline{7 \text{ hr } 21 \text{ mins}}$ (to nearest minute)	awrt $t = \underline{441}$ or $\underline{7 \text{ hr } 21 \text{ mins}}$	A1
			[3]
			14 marks

Question 1

The majority of candidates produced correct solutions to this question, but a substantial minority of candidates were unable to carry out the first step of writing $(3+2x)^{-3}$ as $\frac{1}{27}\left(1+\frac{2x}{3}\right)^{-2}$. Those who were able to do this could usually complete the remainder of the question but some sign errors and manipulation errors were seen. Another common error was for candidates to apply $\frac{n(n-1)}{2!}$ and/or $\frac{n(n-1)(n-2)}{3!}$ in the third and fourth terms of their expansion.

Question 2

This question was generally well done and, helped by the printed answer, many produced fully correct answers. The commonest error was to omit the negative sign when differentiating $\cos x+1$. The order of the limits gave some difficulty. Instead of the correct $-\int_2^1 e^u\,du$, an incorrect version $-\int_1^2 e^u\,du$ was produced and the resulting expressions manipulated to the printed result and working like $-\left(e^2-e^1\right)=-e^2+e^1=e\left(e-1\right)$ was not uncommon.

Some candidates got into serious difficulties when, through incorrect algebraic manipulation, they obtained $-\int e^u \sin^2 x\,du$ instead of $-\int e^u\,du$. This led to expressions such as $\int e^u\left(u^2-2u\right)du$ and the efforts to integrate this, either by parts twice or a further substitution, often ran to several supplementary sheets. The time lost here inevitably led to difficulties in finishing the paper. Candidates need to have some idea of the amount of work and time appropriate to a 6 mark question and, if they find themselves exceeding this, realise that they have probably made a mistake and that they would be well advised to go on to another question.

Question 3

This was correctly answered by about 40% of the candidates.

A majority incorrectly expressed $\frac{9x^2+20x-10}{(x+2)(3x-1)}$ as $\frac{2}{(x+2)}-\frac{1}{(3x-1)}$, having failed to realise that the algebraic fraction given in the question is improper, thereby losing 3 of the 4 marks available.

For those achieving the correct partial fractions, a process of long division was typically used to find the value of the constant term, and the resulting remainder, usually $5x-4$, became the LHS of the subsequent identity. A minority of them, however, applied $9x^2+20x-10\equiv A(x+2)(3x-1)+B(3x-1)+C(x+2)$ in order to obtain the correct partial fractions.

Question 4

The majority of candidates identified the need for some form of dot product calculation in part (a). Taking the dot product $l_1.l_2$, was common among candidates who did not correctly proceed, while others did not make any attempt at a calculation, being unable to identify the vectors required. A number of candidates attempted to equate l_1 and l_2 at this stage. The majority of candidates, however, were able to show that $q = -3$.

In part (b), the majority of candidates correctly equated the **i**, **j** and **k** components of l_1 and l_2, and although some candidates made algebraic errors in solving the resulting simultaneous equations, most correctly found λ and μ. In almost all such cases the value of *p* and the point of intersection in part (c) was then correctly determined.

There was a failure by many candidates to see the link between part (d) and the other three parts of this question with the majority of them leaving this part blank. Those candidates who decided to draw a diagram usually increased their chance of success. Most candidates who were successful at this part applied a vector approach as detailed in the mark scheme. The easiest vector approach, adopted by a few candidates, is to realise that $\lambda = 1$ at *A*, $\lambda = 5$ at the point of intersection and so $\lambda = 9$ at *B*. So substitution of $\lambda = 9$ into l_1 yields the correct position vector $-7\mathbf{i} + 11\mathbf{j} - 19\mathbf{k}$. A few candidates, by deducing that the intersection point is the midpoint of *A* and *B* were able to write down $\frac{9+x}{2} = 1$, $\frac{3+y}{2} = 7$ and $\frac{13+z}{2} = -3$, in order to find the position vector of *B*.

Question 5

Nearly all candidates knew the method for solving part (a), although there were many errors in differentiating trig functions. In particular $\frac{\mathrm{d}}{\mathrm{d}t}(2\cos 2t)$ was often incorrect. It was clear from both this question and question 2 that, for many, the calculus of trig functions was an area of weakness. Nearly all candidates were able to obtain an exact answer in surd form. In part (b), the majority of candidates were able to eliminate *t* but, in manipulating trigonometric identities, many errors, particularly with signs, were seen. The answer was given in a variety of forms and all exact equivalent answers to that printed in the mark scheme were accepted. The value of *k* was often omitted and it is possible that some simply overlooked this. Domain and range remains an unpopular topic and many did not attempt part (c). In this case, inspection of the printed figure gives the lower limit and was intended to give candidates a lead to identifying the upper limit.

Question 6

In part (a), a majority of candidates were able to subtract the given position vectors correctly in order to find $\overrightarrow{AB}$. Common errors in this part included some candidates subtracting the position vector the wrong way round and a few candidates who could not deal with the double negative when finding the **k** component of $\overrightarrow{AB}$.

In part (b), a significant majority of candidates were able to state a vector equation of l_1. A significant number of these candidates, however, wrote 'Line = 'and omitted the '**r**' on the left hand side of the vector equation, thereby losing one mark.

Many candidates were able to apply the dot product correctly in part (c) to find the correct angle. Common errors here included applying a dot product formula between $\overrightarrow{OA}$ and $\overrightarrow{OB}$; or applying the dot product between either $\overrightarrow{OA}$ or $\overrightarrow{OB}$ and the direction vector of l_1. Interestingly, a surprising number of candidates either simplified $\sqrt{(1)^2+(-2)^2+(2)^2}$ to $\sqrt{5}$ or when finding the dot product multiplied -2 by 0 to give -2.

Part (d) proved more discriminating. The majority of candidates realised that they needed to put the line l_1 equal to line l_2. A significant number of these candidates, however, were unable to write l_2 as $\mu(\mathbf{i}+\mathbf{k})$ or used the same parameter (usually λ) as they had used for l_1. Such candidates then found difficulty in making further progress with this part.

Question 7

Part (a) was fully correct in the great majority of cases but the solutions were often unnecessarily long and nearly two pages of working were not unusual. The simplest method is to equate the **j** components. This gives one equation in λ, leading to $\lambda = 3$, which can be substituted into the equation of l_1 to give the coordinates of *C*. In practice, the majority of candidates found both λ and μ and many proved that the lines were coincident at *C*. However the question gave the information that the lines meet at *C* and candidates had not been asked to prove this. This appeared to be another case where candidates answered the question that they had expected to be set, rather than the one that actually had been.

The great majority of candidates demonstrated, in part (b), that they knew how to find the angle between two vectors using a scalar product. However the use of the position vectors of *A* and *B*, instead of vectors in the directions of the lines was common. Candidates could have used either the vectors $\begin{pmatrix}1\\2\\1\end{pmatrix}$ and $\begin{pmatrix}5\\0\\2\end{pmatrix}$, given in the question, or $\overrightarrow{AC}$ and $\overrightarrow{BC}$. The latter was much the commoner choice but many made errors in signs. Comparatively few chose to use the cosine rule. In part (c), many continued with the position vectors they had used incorrectly in part (b) and so found the area of the triangle *OAB* rather than triangle *ABC*. The easiest method of completing part (c) was usually to use the formula Area = $\frac{1}{2}ab\sin C$ and most chose this. Attempts to use Area = $\frac{1}{2}$base $\times$ height were usually fallacious and often assumed that the triangle was isosceles. A few complicated attempts were seen which used vectors to find the coordinates of the foot of a perpendicular from a vertex to the opposite side. In principle, this is possible but, in this case, the calculations proved too difficult to carry out correctly under examination conditions.

Question 8

Many candidates, who answered part (a), were able to separate the variables correctly and integrate both sides of their equation to obtain $\ln P = kt$. At this point a significant number of candidates either omitted the constant of integration or were unable to deal with the boundary conditions given in the question. Some candidates, for example, wrote down $P = e^{kt} + c$; and stated that $c = P_0$ to give the common incorrect solution of $P = P_0 + e^{kt}$. Other candidates used P_0 instead of P in their attempts, and then struggled to find the constant of integration. Some candidates, who correctly evaluated the constant of integration, did not make P the subject of the equation but left their answer as $\ln P = kt + \ln P_0$.

Those candidates who had successfully answered part (a) were able to gain most of the marks available in part (b). A few of these candidates, however, struggled to convert the correct time in hours to the correct time in minutes. Those who did not progress well in part (a) may have gained only a method mark in part (b) by replacing P in their part (a) equation with $2P_0$.

Those candidates who were successful in the first two parts of this question usually succeeded to score most of the marks available in parts (c) and (d). In part (c) some candidates incorrectly integrated $\lambda \cos \lambda t$. In part (d), a significant number of candidates found difficultly in solving the equation $\sin(2.5t) = \ln 2$. It was not uncommon for some of these candidates to write $t = \frac{\ln 2}{\sin(2.5)}$. Also, in part (d), some candidates did not work in radians when evaluating $t = \arcsin(\ln 2)$.

Statistics for C4 Practice Paper G3

					Mean score for students achieving grade:						
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	5		78	3.88		4.59	4.06	3.64	3.00	2.30	1.38
2	6		64	3.81	5.84	5.13	4.00	2.69	1.71	0.94	0.36
3	4	1	57	2.26	3.53	2.49	2.09	1.73	1.58	1.50	1.15
4	13		61	7.94		10.15	7.19	4.59	3.25	1.74	0.58
5	10		54	5.38		7.41	5.34	3.97	2.72	1.64	0.63
6	11		57	6.30		8.66	5.80	4.15	3.11	1.68	1.27
7	12		54	6.42	10.86	8.23	6.15	4.39	3.01	2.02	1.02
8	14		36	5.09		8.99	3.90	1.81	0.80	0.35	0.09
	75		**55**	**41.08**		**55.65**	**38.53**	**26.97**	**19.18**	**12.17**	**6.48**