Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3
Gold Level (Hard) G1

Time: 1 hour 30 minutes

Materials required for examination	Items included with question papers
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
68	59	50	43	37	31

1. The point P lies on the curve with equation

$$y = 4e^{2x + 1}.$$

The y-coordinate of P is 8.

(a) Find, in terms of ln 2, the x-coordinate of P.

(2)

(b) Find the equation of the tangent to the curve at the point P in the form $y = ax + b$, where a and b are exact constants to be found.

(4)

June 2008

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2. A curve C has equation

$$y = \frac{3}{(5-3x)^2}, \qquad x \neq \frac{5}{3}.$$

The point P on C has x-coordinate 2.

Find an equation of the normal to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(7)

June 2010

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3. A curve C has equation $y = x^2e^x$.

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C.

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve C.

(2)

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4. (i) Given that $y = \dfrac{\ln(x^2+1)}{x}$, find $\dfrac{dy}{dx}$.

(4)

(ii) Given that $x = \tan y$, show that $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$.

(5)

January 2010

5.

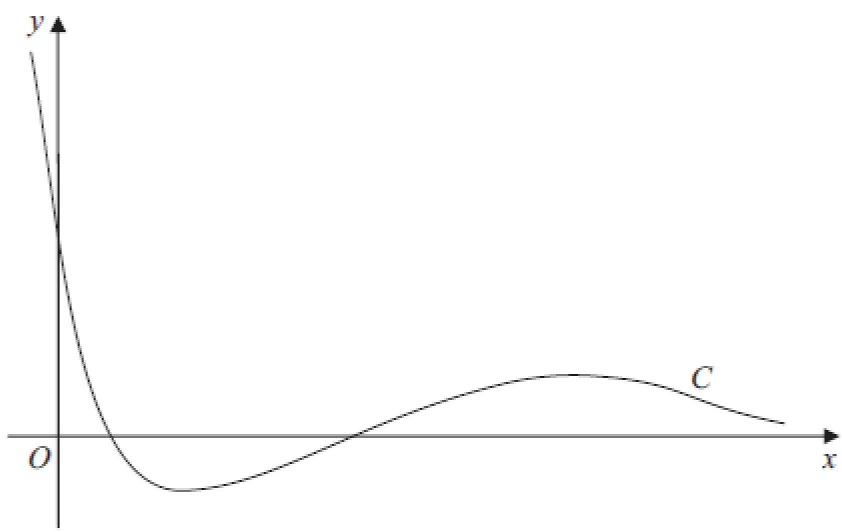

Figure 1

Figure 1 shows a sketch of the curve C with the equation $y = (2x^2 - 5x + 2)e^{-x}$.

(*a*) Find the coordinates of the point where C crosses the y-axis.

(1)

(*b*) Show that C crosses the x-axis at $x = 2$ and find the x-coordinate of the other point where C crosses the x-axis.

(3)

(*c*) Find $\dfrac{dy}{dx}$.

(3)

(*d*) Hence find the exact coordinates of the turning points of C.

(5)

June 2010

6. (*a*) Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(*b*) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

(2)

(*c*) Solve, for $0 < x < 2\pi$, the equation $3 \sin x + 2 \cos x = 1$, giving your answers to 3 decimal places.

(5)

June 2007

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7. (*a*) Express $4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta$ in terms of $\sin \theta$ and $\cos \theta$.

(2)

(*b*) Hence show that

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = \sec^2 \theta .$$

(4)

(*c*) Hence or otherwise solve, for $0 < \theta < \pi$,

$$4 \operatorname{cosec}^2 2\theta - \operatorname{cosec}^2 \theta = 4$$

giving your answers in terms of π.

(3)

June 2012

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8. (*a*) Starting from the formulae for $\sin(A + B)$ and $\cos(A + B)$, prove that

$$\tan (A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}.$$

(4)

(*b*) Deduce that

$$\tan \left(\theta + \frac{\pi}{6}\right) = \frac{1 + \sqrt{3} \tan \theta}{\sqrt{3} - \tan \theta}.$$

(3)

(*c*) Hence, or otherwise, solve, for $0 \leq \theta \leq \pi$,

$$1 + \sqrt{3} \tan \theta = (\sqrt{3} - \tan \theta) \tan (\pi - \theta).$$

Give your answers as multiples of π.

(6)

January 2012

TOTAL FOR PAPER: 75 MARKS

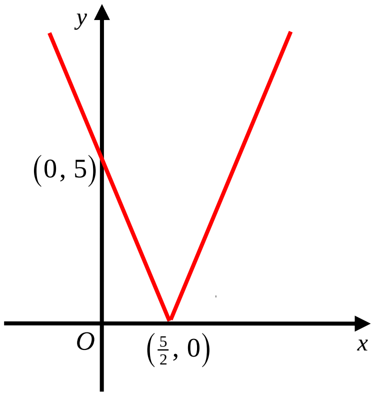
END

Question Number	*Scheme*	**Marks**
1. (*a*)	$e^{2x+1}=2$	
	$2x+1=\ln 2$	M1
	$x=\frac{1}{2}(\ln 2-1)$	A1 (2)
(*b*)	$\frac{dy}{dx}=8e^{2x+1}$	B1
	$x=\frac{1}{2}(\ln 2-1) \Rightarrow \frac{dy}{dx}=16$	B1
	$y-8=16\left(x-\frac{1}{2}(\ln 2-1)\right)$	M1
	$y=16x+16-8\ln 2$	A1 (4)
		(6 marks)

Question Number	Scheme	Marks
2.	At P, $y=\underline{3}$	B1
	$\frac{dy}{dx}=\underline{3(-2)(5-3x)^{-3}(-3)}$ $\left\{\text{or } \frac{18}{(5-3x)^3}\right\}$	M1<u>A1</u>
	$\frac{dy}{dx}=\frac{18}{(5-3(2))^3}$ $\{=-18\}$	M1
	$m(\mathbf{N})=\frac{-1}{-18}$ or $\frac{1}{18}$	M1
	N: $y-3=\frac{1}{18}(x-2)$	M1
	N: $\underline{x-18y+52=0}$	A1
		[7]

Question Number	Scheme	Marks
3. (*a*)	$\frac{dy}{dx} = x^2e^x + 2xe^x$	M1,A1,A1 (3)
(*b*)	If $\frac{dy}{dx} = 0$, $e^x(x^2 + 2x) = 0$ setting (*a*) = 0 $[e^x \neq 0]$ $x(x + 2) = 0$ $(x = 0)$ $x = -2$ $x = 0, y = 0$ **and** $x = -2, y = 4e^{-2}$ (= 0.54…)	M1 A1 A1 √ (3)
(*c*)	$\frac{d^2y}{dx^2} = x^2e^x + 2xe^x + 2xe^x + 2e^x$ $\left[= (x^2 + 4x + 2)e^x\right]$	M1, A1 (2)
(*d*)	$x = 0$, $\frac{d^2y}{dx^2} > 0$ (=2) $x = -2$, $\frac{d^2y}{dx^2} < 0$ [$= -2e^{-2}$ (= –0.270…)] M1: Evaluate, or state sign of, candidate's (c) for at least one of candidate's x value(s) from (b) $\therefore$ minimum $\therefore$ maximum	M1 A1 (cso) (2) **(10 marks)**

Question Number	Scheme	Marks
Q4 (i)	$y = \dfrac{\ln(x^2+1)}{x}$	
	$u = \ln(x^2+1) \Rightarrow \dfrac{du}{dx} = \dfrac{2x}{x^2+1}$	M1 A1
	Apply quotient rule: $\left\{\begin{array}{ll} u = \ln(x^2+1) & v = x \\ \dfrac{du}{dx} = \dfrac{2x}{x^2+1} & \dfrac{dv}{dx} = 1 \end{array}\right\}$	
	$\dfrac{dy}{dx} = \dfrac{\left(\dfrac{2x}{x^2+1}\right)(x) - \ln(x^2+1)}{x^2}$	M1 A1
		(4)
	$\left\{\dfrac{dy}{dx} = \dfrac{2}{(x^2+1)} - \dfrac{1}{x^2}\ln(x^2+1)\right\}$	
(ii)	$x = \tan y$	
	$\dfrac{dx}{dy} = \sec^2 y$	M1* A1
	$\dfrac{dy}{dx} = \dfrac{1}{\sec^2 y}$ $\{= \cos^2 y\}$	dM1*
	$\dfrac{dy}{dx} = \dfrac{1}{1+\tan^2 y}$	dM1*
	Hence, $\dfrac{dy}{dx} = \dfrac{1}{1+x^2}$, (as required)	A1 **AG**
		(5)
		[9]

Question Number	Scheme	Marks
5. (a)		M1A1 (2)
(b)	$\underline{x = 20}$ $2x - 5 = -(15 + x)$; $\Rightarrow$ $\underline{x = -\frac{10}{3}}$	B1 M1;A1 oe. (3)
(c)	$\text{fg}(2) = \text{f}(-3) = \lvert 2(-3) - 5 \rvert; = \lvert -11 \rvert = 11$	M1;A1 (2)
(d)	$\text{g}(x) = x^2 - 4x + 1 = (x-2)^2 - 4 + 1 = (x-2)^2 - 3$. Hence $\text{g}_{\text{min}} = -3$ Either $\text{g}_{\text{min}} = -3$ or $\text{g}(x) \ldots -3$ or $\text{g}(5) = 25 - 20 + 1 = 6$ $\underline{-3 \,„\, \text{g}(x) \,„\, 6}$ or $\underline{-3 \,„\, y \,„\, 6}$	M1 B1 A1 (3) **[10]**

Question Number	Scheme	Marks
6. (*a*)	Complete method for *R*: e.g. $R\cos\alpha = 3$, $R\sin\alpha = 2$, $R = \sqrt{(3^2 + 2^2)}$ $R = \sqrt{13}$ or 3.61 (or more accurate) Complete method for $\tan\alpha = \frac{2}{3}$ [Allow $\tan\alpha = \frac{3}{2}$] $\alpha = 0.588$ (Allow 33.7°)	M1 A1 M1 A1 (4)
(*b*)	Greatest value = $\left(\sqrt{13}\right)^4 = 169$	M1, A1 (2)
(*c*)	$\sin(x + 0.588) = \frac{1}{\sqrt{13}}$ $(= 0.27735\ldots)$ $\sin(x + \text{their } \alpha) = \frac{1}{\text{their } R}$ $(x + 0.588)$ $= 0.281(03\ldots)$ or 16.1° $(x + 0.588)$ $= \pi - 0.28103\ldots$ Must be $\pi -$ their 0.281 or 180° – their 16.1° or $(x + 0.588)$ $= 2\pi + 0.28103\ldots$ Must be $2\pi +$ their 0.281 or 360° + their 16.1° $x = 2.273$ or $x = 5.976$ (awrt) Both (radians only) If 0.281 or 16.1° not seen, correct answers imply this A mark	M1 A1 M1 M1 A1 (5) **(11 marks)**

Question Number	Scheme	Marks
7. (a)	$4\operatorname{cosec}^2 2\theta - \operatorname{cosec}^2\theta = \dfrac{4}{\sin^2 2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta}$	B1 B1
		(2)
(b)	$\dfrac{4}{(2\sin\theta\cos\theta)^2} - \dfrac{1}{\sin^2\theta} = \dfrac{4}{4\sin^2\theta\cos^2\theta} - \dfrac{1}{\sin^2\theta}$	
	$= \dfrac{1}{\sin^2\theta\cos^2\theta} - \dfrac{\cos^2\theta}{\sin^2\theta\cos^2\theta}$	M1
	Using $1-\cos^2\theta = \sin^2\theta$ $\quad = \dfrac{\sin^2\theta}{\sin^2\theta\cos^2\theta}$	M1
	$= \dfrac{1}{\cos^2\theta} = \sec^2\theta$	M1A1*
		(4)
(c)	$\sec^2\theta = 4 \Rightarrow \sec\theta = \pm 2 \Rightarrow \cos\theta = \pm\dfrac{1}{2}$	M1
	$\theta = \dfrac{\pi}{3}, \dfrac{2\pi}{3}$	A1,A1
		(3)
		(9 marks)

Question number	Scheme	Marks
8. (a)	$\tan(A+B)=\dfrac{\sin(A+B)}{\cos(A+B)}=\dfrac{sinAcosB+cosAsinB}{cosAcosB-sinAsinB}$	M1A1
	$=\dfrac{\frac{sinA}{cosA}+\frac{sinB}{cosB}}{1-\frac{sinAsinB}{cosAcosB}}$	M1
	$(\div cosAcosB)$	
	$=\dfrac{tanA+tanB}{1-tanAtanB}$	A1 * (4)
(b)	$\tan\left(\theta+\dfrac{\pi}{6}\right)=\dfrac{tan\theta+\frac{\tan\pi}{6}}{1-\frac{tan\theta\tan\pi}{6}}$	M1
	$=\dfrac{tan\theta+\frac{1}{\sqrt{3}}}{1-tan\theta\frac{1}{\sqrt{3}}}$	M1
	$=\dfrac{\sqrt{3}tan\theta+1}{\sqrt{3}-tan\theta}$	A1 * (3)
(c)	$\tan\left(\theta+\dfrac{\pi}{6}\right)=\tan(\pi-\theta).$	M1
	$\left(\theta+\dfrac{\pi}{6}\right)=(\pi-\theta)$	M1
	$\theta=\dfrac{5}{12}\pi$	M1 A1
	$\tan\left(\theta+\dfrac{\pi}{6}\right)=\tan(2\pi-\theta)$	M1
	$\theta=\dfrac{11}{12}\pi$	A1 (6) **(13 marks)**

Statistics for C3 Practice Paper G1

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade: ALL	A*	A	B	C	D	E	U
1	6		69	4.15		5.31	4.48	3.77	2.97	2.12	1.01
2	7		76	5.30	6.72	6.24	5.74	5.12	4.22	2.99	1.55
3	10		73	7.34		9.12	7.87	6.78	5.51	4.09	2.18
4	9		60	5.38		7.70	6.17	4.83	3.74	2.42	1.21
5	10		61	6.05	8.91	7.66	6.22	4.88	3.75	2.77	1.66
6	11		62	6.84		9.40	7.47	5.74	3.99	2.44	0.99
7	9		57	5.09	8.65	6.98	5.04	3.59	2.44	1.60	0.76
8	13		51	6.63	12.08	9.66	7.53	5.97	4.35	3.19	1.60
	75		**62**	**46.78**		**62.07**	**50.52**	**40.68**	**30.97**	**21.62**	**10.96**