Paper Reference(s)

6664/01

Edexcel GCE

Core Mathematics C2
Gold Level G2

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C2), the paper reference (6664), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
64	56	48	40	32	24

1.
$$f(x) = 2x^3 - 3x^2 - 39x + 20$$

    (*a*) Use the factor theorem to show that $(x + 4)$ is a factor of f (x).

                                                                              (2)

    (*b*) Factorise f (x) completely.

                                                                              (4)

                                                                          June 2008

2.
$$y = \frac{x}{\sqrt{(1+x)}}$$

    (*a*) Complete the table below with the value of *y* corresponding to $x = 1.3$, giving your
        answer to 4 decimal places.

                                                                                    (1)

x	1	1.1	1.2	1.3	1.4	1.5
y	0.7071	0.7591	0.8090		0.9037	0.9487

    (*b*) Use the trapezium rule, with all the values of *y* in the completed table, to obtain an
        approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{(1+x)}}\ dx$$

    giving your answer to 3 decimal places.

    You must show clearly each stage of your working.

                                                                                  (4)

                                                                           May 2013 (R)

3.
$$y = x^2 - k\sqrt{x}, \quad \text{where } k \text{ is a constant.}$$

    (*a*) Find $\dfrac{dy}{dx}$.

                                                                                    (2)

    (*b*) Given that *y* is decreasing at $x = 4$, find the set of possible values of *k*.

                                                                                   (2)

                                                                          June 2010
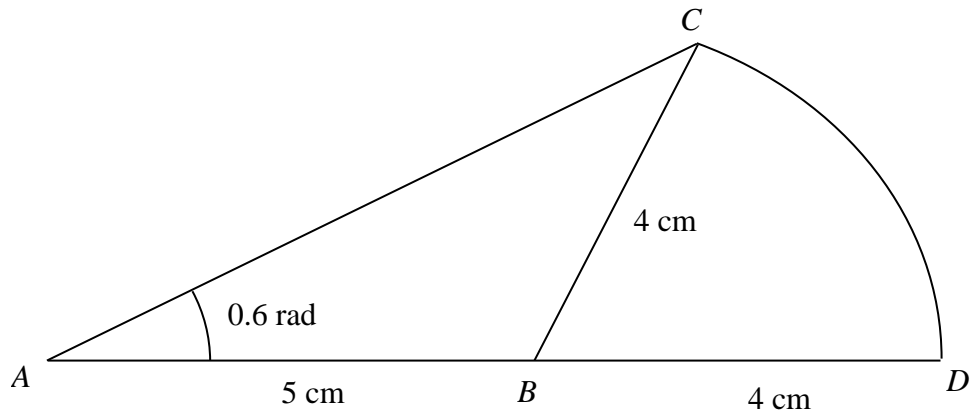

4.

Figure 1

An emblem, as shown in Figure 1, consists of a triangle *ABC* joined to a sector *CBD* of a circle with radius 4 cm and centre *B*. The points *A*, *B* and *D* lie on a straight line with *AB* = 5 cm and *BD* = 4 cm. Angle *BAC* = 0.6 radians and *AC* is the longest side of the triangle *ABC*.

(*a*) Show that angle *ABC* = 1.76 radians, correct to three significant figures.

(4)

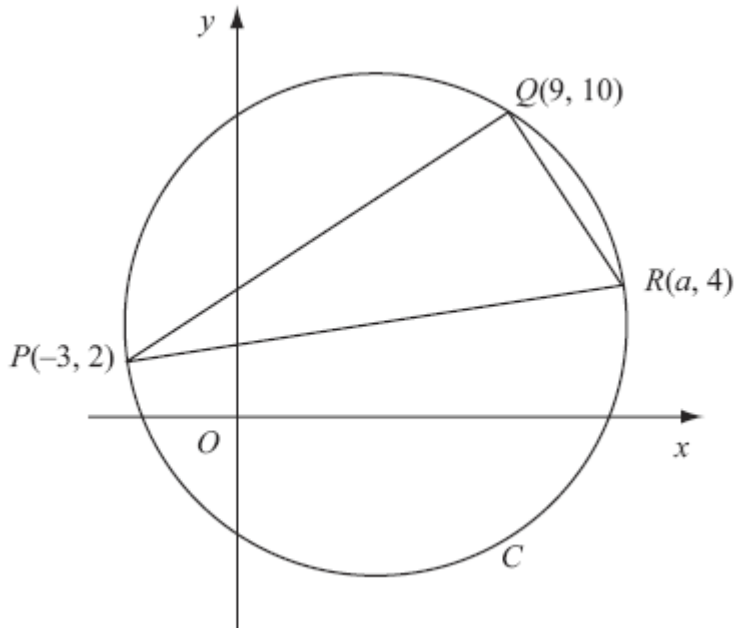(*b*) Find the area of the emblem.

(3)

January 2010

5.

Figure 2

The points $P(-3, 2)$, $Q(9, 10)$ and $R(a, 4)$ lie on the circle C, as shown in Figure 2.

Given that PR is a diameter of C,

(a) show that $a = 13$,

(3)

(b) find an equation for C.

(5)

January 2009

---

6. Given that $\log_3 x = a$, find in terms of a,

(a) $\log_3 (9x)$

(2)

(b) $\log_3 \left(\dfrac{x^5}{81} \right)$

(3)

giving each answer in its simplest form.

(c) Solve, for x,

$$\log_3 (9x) + \log_3 \left(\frac{x^5}{81} \right) = 3$$

giving your answer to 4 significant figures.

(4)

May 2013 (R)

---

7. (i) Solve, for $-180° \leq x < 180°$,

$$\tan(x - 40°) = 1.5,$$

giving your answers to 1 decimal place.

(3)

(ii) (*a*) Show that the equation

$$\sin \theta \tan \theta = 3 \cos \theta + 2$$

can be written in the form

$$4 \cos^2\theta + 2 \cos \theta - 1 = 0.$$

(3)

(*b*) Hence solve, for $0 \leq \theta < 360°$,

$$\sin \theta \tan \theta = 3 \cos \theta + 2,$$

showing each stage of your working.
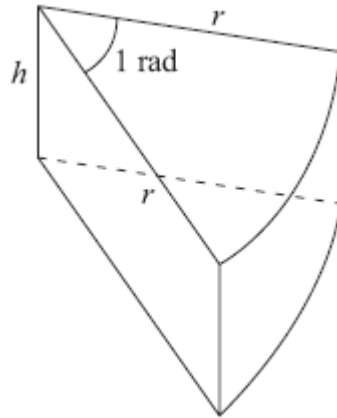
(5)

May 2013

8.

Figure 3

Figure 3 shows a closed box used by a shop for packing pieces of cake. The box is a right prism of height h cm. The cross section is a sector of a circle. The sector has radius r cm and angle 1 radian.

The volume of the box is 300 cm^3.

(*a*) Show that the surface area of the box, S cm^2, is given by

$$S = r^2 + \frac{1800}{r}.$$

(5)

(*b*) Use calculus to find the value of r for which S is stationary.

(4)

(*c*) Prove that this value of r gives a minimum value of S.

(2)

(*d*) Find, to the nearest cm^2, this minimum value of S.

(2)

June 2009

9. A solid right circular cylinder has radius r cm and height h cm.

The total surface area of the cylinder is 800 cm^2 .

(a) Show that the volume, V cm^3 , of the cylinder is given by

$$V = 400r - \pi r^3.$$

(4)

Given that r varies,

(b) use calculus to find the maximum value of V, to the nearest cm^3.

(6)

(c) Justify that the value of V you have found is a maximum.

(2)

January 2009

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	Attempt to find f(–4) or f(4). $\left(f(-4) = 2(-4)^3 - 3(-4)^2 - 39(-4) + 20\right)$	M1
	$\left(= -128 - 48 + 156 + 20\right) = 0$, so $(x + 4)$ is a factor.	A1
		(2)
(b)	$2x^3 - 3x^2 - 39x + 20 = (x + 4)(2x^2 - 11x + 5)$	M1 A1
	$.....(2x - 1)(x - 5)$ oe	M1 A1cso
		(4)
		[6]
2. (a)	$\{ x = 1.3 \}$ $y = 0.8572$ (only)	B1 cao
		(1)
(b)	$\dfrac{1}{2} \times 0.1.........$	B1
	$\{0.7071 + 0.9487 + 2(0.7591 + 0.8090 + "0.8572" + 0.9037)\}$	M1
	$...\{0.7071 + 0.9487 + 2(0.7591 + 0.8090 + "0.8572" + 0.9037)\}$	A1ft
	$\{0.05(8.3138)\} = 0.41569 = $ awrt 0.416	A1
		(4)
		[5]
3. (a)	$\left(\dfrac{dy}{dx} = \right)\ 2x - \dfrac{1}{2}kx^{-\frac{1}{2}}$	M1 A1
		(2)
(b)	Substituting $x = 4$ into their $\dfrac{dy}{dx}$ and 'compare with zero'	M1
	$8 - \dfrac{k}{4} < 0$ $k > 32$ (or $32 < k$)	A1
		(2)
		[4]
4. (a)	$\dfrac{\sin(A\hat{C}B)}{5} = \dfrac{\sin 0.6}{4}$	M1
	$\therefore\ A\hat{C}B = \arcsin(0.7058...)$	M1
	$= [0.7835..\ \text{ or }\ 2.358]$	
	$A\hat{B}C = \pi - 0.6 - A\hat{C}B$	M1
	$A\hat{B}C = 1.76$ (*)(3sf)	A1
		(4)
(b)	$\left[C\hat{B}D = \pi - 1.76 = 1.38\right]$ Sector area $= \frac{1}{2} \times 4^2 \times (\pi - 1.76) = [11.0 \sim 11.1]$	M1
	Area of $\triangle ABC = \frac{1}{2} \times 5 \times 4 \times \sin(1.76) = [9.8]$	M1
	Required area = awrt 20.8 or 20.9 or 21.0	A1
		(3)
		[7]

Question Number	Scheme	Marks
5. (a)	PQ: $m_1 = \dfrac{10-2}{9-(-3)}$ $\left(= \tfrac{2}{3}\right)$ **and** QR: $m_2 = \dfrac{10-4}{9-a}$	M1
	$m_1 m_2 = -1$: $\quad \dfrac{8}{12} \times \dfrac{6}{9-a} = -1 \qquad a = 13 \qquad (*)$	M1 A1
		(3)
(b)	Centre is at $(5,\ 3)$	B1
	$\left(r^2 =\right) (10-3)^2 + (9-5)^2$ o.e., or $\left(d^2 =\right) (13-(-3))^2 + (4-2)^2$	M1 A1
	$(x-5)^2 + (y-3)^2 = 65 \qquad$ or $\ x^2 + y^2 - 10x - 6y - 31 = 0$	M1 A1
		(5)
		[8]
6. (a)	$\log_3(9x) = \log_3 9 + \log_3 x$	M1
	$= 2 + a$	A1
		(2)
(b)	$\log_3\left(\dfrac{x^5}{81}\right) = \log_3 x^5 - \log_3 81$	M1
	$\log x^5 = 5\log x$ or $\log 81 = 4\log 3$ or $\log 81 = 4$	M1
	$= 5a - 4$	A1 cso
		(3)
(c)	$\log_3(9x) + \log_3\left(\dfrac{x^5}{81}\right) = 3$	
	$\Rightarrow 2 + a + 5a - 4 = 3$	M1
	$\Rightarrow a = \dfrac{5}{6}$	A1
	$\Rightarrow x = 3^{\frac{5}{6}}$ or $\log_{10} x = a\log_{10} 3$ so $x =$	M1
	$x = 2.498$ or awrt	A1
		(4)
		[9]

Question Number	Scheme	Marks
7. (i)	$\left(\lvert\alpha\rvert = 56.3099...\right)$	
	$x = \{\alpha + 40 = 96.309993...\} = $ **awrt 96.3**	B1
	$x - 40^\circ = -180 + "56.3099"...$ $\qquad$ or $\quad x - 40^\circ = -\pi + "0.983"...$	M1
	$x = \{-180 + 56.3099... + 40 = -83.6901...\} = $ **awrt -83.7**	A1
		(3)
(ii)(a)	$\sin\theta\left(\dfrac{\sin\theta}{\cos\theta}\right) = 3\cos\theta + 2$	M1
	$\left(\dfrac{1-\cos^2\theta}{\cos\theta}\right) = 3\cos\theta + 2$	dM1
	$1 - \cos^2\theta = 3\cos^2\theta + 2\cos\theta \;\Rightarrow\; 0 = 4\cos^2\theta + 2\cos\theta - 1 \qquad *$	A1 cso
		(3)
(b)	$\cos\theta = \dfrac{-2 \pm \sqrt{4 - 4(4)(-1)}}{8}$ $\quad$ or $\;4(\cos\theta \pm \frac{1}{4})^2 \pm q \pm 1 = 0,$	M1
	or $\;(2\cos\theta \pm \frac{1}{2})^2 \pm q \pm 1 = 0, \; q \neq 0 \;$ so $\cos\theta = ...$	
	One solution is 72° or $\;144^\circ$, $\qquad$ Two solutions are 72° and $\;144^\circ$	A1 A1
	$\theta = \{72, 144, 216, 288\}$	M1 A1
		(5)
		[11]

Question Number	Scheme	Marks
8. (a)	(Arc length =) $r\theta = r \times 1 = r$.	B1
	(Sector area =) $\dfrac{1}{2}r^2\theta = \dfrac{1}{2}r^2 \times 1 = \dfrac{r^2}{2}$.	B1
	Surface area = 2 sectors + 2 rectangles + curved face	M1
	$\qquad (= r^2 + 3rh)$	
	Volume = $300 = \frac{1}{2}r^2h$	B1
	$S = r^2 + 3 \times \dfrac{600}{r} = r^2 + \dfrac{1800}{r}$ $\qquad\qquad$ (*)	A1cso
		(5)
(b)	$\dfrac{dS}{dr} = 2r - \dfrac{1800}{r^2}$ or $2r - 1800r^{-2}$ or $2r + -1800r^{-2}$	M1A1
	$\dfrac{dS}{dr} = 0 \implies r^3 = ...,\quad r = \sqrt[3]{900},\qquad$ or awrt 9.7	M1A1
		(4)
(c)	$\dfrac{d^2 S}{dr^2} = ...$ and consider sign	M1
	$\dfrac{d^2 S}{dr^2} = 2 + \dfrac{3600}{r^3} > 0$ so point is a minimum	A1ft
		(2)
(d)	$S_{min} = (9.65...)^2 + \dfrac{1800}{9.65...}$	M1
	$\qquad = 279.65...$	A1
		(2)
		[13]
9. (a)	$2\pi rh + 2\pi r^2 = 800$	B1
	$h = \dfrac{400 - \pi r^2}{\pi r},\qquad V = \pi r^2\left(\dfrac{400 - \pi r^2}{\pi r}\right) = 400r - \pi r^3$ $\quad$ (*)	M1 M1 A1
		(4)
(b)	$\dfrac{dV}{dr} = 400 - 3\pi r^2$	M1 A1
	$400 - 3\pi\ r^2 = 0 \qquad r^2 = ...,\qquad r = \sqrt{\dfrac{400}{3\pi}}\qquad (= 6.5\ (2\ \text{s.f.}))$	M1 A1
	$V = 400r - \pi r^3 = 1737 = \dfrac{800}{3}\sqrt{\dfrac{400}{3\pi}}\ (\text{cm}^3)$	M1 A1
		(6)
(c)	$\dfrac{d^2 V}{dr^2} = -6\pi r$, Negative, $\therefore$ maximum	M1 A1
		(2)
		[12]

Examiner reports

Question 1

Part (a) of this question required the use of the factor theorem (rather than long division) and most candidates were able to show f(–4) = 0. As in previous papers, a simple conclusion was expected. Many candidates failed to provide this.

The most popular strategy in part (b) was to use long division, dividing the cubic expression by $(x + 4)$ to find the quadratic factor. Some candidates stopped at that stage and so could only gain a maximum of two marks, but of those who reached $2x^2 – 11x + 5$ and went on to factorise this, the vast majority gained full marks. Less formal approaches to the division, including 'division by inspection', were occasionally seen and usually effective.

Candidates who solved $2x^2 – 11x + 5 = 0$ gained neither of the final two marks until they produced the relevant factors, and then one of the factors was often left as $\left(x - \dfrac{1}{2} \right)$, which lost the final mark unless the factor 2 was included.

Some candidates went on to give 'solutions' $x = –4$, $x = 5$ $x = \dfrac{1}{2}$, suggesting confusion over the meaning of 'factorise'.

Question 2

In part (a) almost all achieved the first mark for 0.8572. The value 0.8571 was seen rarely, as was 0.857. These answers did not get this mark.

In part (b) the main error was in calculating the strip width; $\frac{1}{12}$ coming from doing the calculation $(1.5 – 1) \div 6$.

The common bracketing error, with brackets omitted, appeared relatively frequently. This usually led to errors in the calculation. There were occasional, but rare, errors with extra values repeated in the innermost bracket, or 0 included as the first value. There were some who tried to integrate to produce an answer (but got nowhere) and others who split it up into several integrals to attempt to evaluate, again with little success.

Usually, however, this question was answered correctly.

Question 3

The differentiation in part (a) of this question was usually completed correctly, although the $k\sqrt{x}$ term sometimes caused problems, with k being omitted or $\sqrt{x}$ misinterpreted.

It was clear in part (b), however, that many candidates did not know the condition for a function to be decreasing. Some substituted $x = 4$ into y rather than $\dfrac{dy}{dx}$ and some used the second derivative. Even those who correctly used $\dfrac{dy}{dx}$ were usually unable to proceed to a correct solution, either making numerical mistakes (often being unable to find the correct value of $4^{-\frac{1}{2}}$) or failing to deal correctly with the required inequality. The answer $k = 32$ was commonly seen instead of $k > 32$.

Question 4

Part (a) This was a discriminating question, as the method required two stages of solution. Candidates could either find the angle ACB using a correct form of the sine rule, then use angles of a triangle, or they could first find the length AC, then use the sine rule. Finding length AC was complicated (requiring a correct cosine rule and use of a quadratic formula) and the former method was easier. Weaker candidates tried to use Pythagoras, despite the triangle not being right-angled, or used the sine rule wrongly and manipulated their answer to give the printed solution. Others assumed the printed answer and attempted verification, but this sometimes resulted in circular arguments and frequently the verification was not conclusive due to the angle being given correct to 3sf. This verification method could earn a maximum of 2 out of 4 marks. Some candidates converted in and out of degrees, often successfully.

Part (b) Good candidates found the area of the triangle *ABC* and the area of the sector *BCD* and added these to give a correct answer. Weak candidates assumed that the emblem was a sector of radius 9 cm and angle 0.6 radians. Some made errors in their use of formulae and included pi erroneously, or neglected the ½ factor. A few used the wrong angle in their formulae or indeed used the wrong formula, confusing arc length or area of a segment with area of a sector.

Question 5

Part (a) caused much more of a problem than part (b). A large number of solutions did not really provide an adequate proof in the first part of this question. The original expected method, involving gradients, was the least frequently used of the three successful methods. Finding the three lengths and using Pythagoras was quite common although successful in a limited number of cases – there were many instances of equations being set up but abandoned when the expansion of brackets started to cause problems. Finding the gradient of QR as –3/2 and substituting to find the equation of the line for QR before using y = 4 to get a, was usually well done. Some used verification but in many cases this led to a circular argument.

In part (b) the centre was often calculated as (8, 3) or (8, 1) indicating errors with negative signs. There were several instances of (5, 4) arising from (4 + 2)/2 being thought to be 4 – maybe cancelling the 2's? The length of PQ was usually correct but frequently thought to be the radius rather than the diameter. The equation of a circle was well known but weaker candidates in some cases took points on the circumference as the centre of the circle in their equation, showing lack of understanding.

Question 6

This question was well answered by most candidates. In part (a), almost all candidates used the addition rule of logs to separate the terms and were able then to write the given expression in terms of *a*. Candidates who tried to change base introduced extra complications. There were some weaker candidates who did not know the log laws well. Some did not know how to deal with the 9 or 81 and some simply replaced *x* with *a* giving common incorrect answers of

$9a$ and $\dfrac{a^5}{81}$.

In part (b), again, most candidates used the subtraction law correctly and spotted that the power law was the next step; almost all achieved the correct answer. In parts (a) and (b), almost all candidates used the first method, "way 1" on the mark scheme.

Unusually, part (c) was answered by some candidates who had not been able to answer parts (a) and (b). Most answered part (c) using the first method on the scheme, although it was disappointing how many achieved the value for *a*, without going on to find a value for *x*. Those who used the alternative method tended to be less successful, making errors in 'undoing' the log and in multiplying and dividing by powers of 3 correctly. Those using method 2 sometimes gave the "extra false solution" of –2.498 losing the last mark.

Question 7

Although the majority of candidates managed to find the first solution 96.3 in part (i), many struggled with the second solution. Clearly the limits of –180 to +180 were challenging for many candidates, who preferred to give positive answers which were outside the required range. The angle 56.3 was usually found but then it was often subtracted from 180 rather than the other way round. Some candidates, after correctly stating $x - 40 = 56.3$, subtracted 40 to give an answer of 16.3. Just a few thought that tan $(x - 40)$ was equivalent to tan x – tan 40.

Part (ii)(a) was generally well answered with the correct substitutions made, although there were some instances of incorrect identities such as $\tan \theta = \dfrac{\cos \theta}{\sin \theta}$ and sin $\theta = 1 - \cos \theta$.

Mistakes were due more to errors with the basic manipulation of the equation than a lack of knowledge of the identities.

A common mistake came in multiplying the right-hand side by cos θ to give 3 cos^2 θ + 2 instead of 3 cos^2 θ + 2 cos θ.

In part (ii)(b) the quadratic formula was usually quoted and used correctly leading to at least one correct answer $\theta = 72$. Those who tried to complete the square often made mistakes, especially in dealing with the coefficient 4. Candidates who attempted to factorise usually ended up with answers such as 60, 90 or 180 and gained no more than one method mark for attempting $360 - \theta$. The quadratic formula yielded most success.

Some problems occurred with candidates rounding answers too early and therefore losing accuracy in later steps. Most knew they had to subtract their initial solution from 360 to find other solutions, but some appeared to be randomly adding and subtracting 180, 270 and 360.

Question 8

Many candidates had difficulty in their attempts to establish the given result for the surface area in part (a) of this question. Solutions often consisted of a confused mass of formulae, lacking explanation of whether expressions represented length, area or volume. Formulae for arc length and sector area usually appeared at some stage, but it was often unclear how they were being used and at which point the substitution $\theta = 1$ was being made. It was, however, encouraging to see well-explained, clearly structured solutions from good candidates.

Having struggled with part (a), some candidates disappointingly gave up. The methods required for the remainder of the question were, of course, more standard and should have been familiar to most candidates.

In part (b), most candidates successfully differentiated the given expression then formed an equation in r using $\dfrac{dS}{dr} = 0$. While many solved $2r - \dfrac{1800}{r^2} = 0$ successfully, weaker candidates were sometimes let down by their algebraic skills and could not cope correctly with the negative power of r. A common slip was to proceed from $r^3 = 900$ to $r = 30$.

In part (c), the majority of candidates correctly considered the sign of the second derivative to establish that the value of S was a minimum, although occasionally the second derivative was equated to zero.

Those who proceeded as far as part (d) were usually able to score at least the method mark, except when the value of r they substituted was completely inappropriate, such as the value of the second derivative.

Question 9

Part (a) required a proof. Common mistakes in the formula for the surface areas were to omit either one or both ends. Algebraic mistakes caused problems with rearranging to make h the subject and some candidates did not know the volume formula. This part was often not attempted or aborted at an early stage.

Parts (b) and (c) were answered well. Most candidates knew that they should differentiate and equate to zero although many could not manage to correctly evaluate r (poor calculator work) and it was common to forgot to evaluate V. Part (c) was often incorporated in (b) (and vice versa!), but generally contained all the elements necessary to score both marks. Most solutions used the second derivative here and there were relatively few of the alternative methods of determining a maximum point. Only a few candidates were unsure of the procedures for establishing the nature of stationary points.

Statistics for C2 Practice Paper Gold Level G2

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	6		78	4.68		5.70	5.43	5.10	4.60	3.77	1.98
2	5		89	4.47	5.00	4.85	4.74	4.40	4.14	3.86	2.38
3	4		64	2.54	3.55	3.16	2.75	2.57	2.40	2.18	1.55
4	7		57	3.99		5.90	4.33	3.32	2.20	1.61	0.73
5	8		51	4.09		6.53	4.31	3.21	1.97	1.26	0.54
6	9		80	7.19	8.78	8.60	7.29	6.34	6.29	4.25	3.05
7	11		49	5.42	10.48	9.15	7.08	5.48	4.07	2.66	0.99
8	13		45	5.91		11.21	7.78	5.05	2.82	1.37	0.31
9	12		51	6.13		10.18	6.65	4.37	2.76	1.73	0.56
	75		59	44.42		65.28	50.36	39.84	31.25	22.69	12.09