AQA

GCSE

Methods in Mathematics (Linked Pair)

Higher Tier Unit 2 Geometry and Algebra
Mark scheme

9365
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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.

M	Method marks are awarded for a correct method which could lead to a correct answer.
M dep	A method mark dependent on a previous method mark being awarded.
A	Accuracy marks are awarded when following on from a correct method. It is not necessary to always see the method. This can be implied.
B	Marks awarded independent of method.
B dep	A mark that can only be awarded if a previous independent mark has been awarded.
Q	Marks awarded for quality of written communication.
ft	Follow through marks. Marks awarded for correct working following a mistake in an earlier step.
SC	Special case. Marks awarded for a common misinterpretation which has some mathematical worth.
oe	Or equivalent. Accept answers that are equivalent. eg accept 0.5 as well as $\frac{1}{2}$
[*a*, *b*]	Accept values between *a* and *b* inclusive.
25.3 …	Allow answers which begin 25.3 e.g. 25.3, 25.31, 25.378.
Use of brackets	It is not necessary to see the bracketed work to award the marks.

Examiners should consistently apply the following principles

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Responses which appear to come from incorrect methods

Whenever there is doubt as to whether a candidate has used an incorrect method to obtain an answer, as a general principle, the benefit of doubt must be given to the candidate. In cases where there is no doubt that the answer has come from incorrect working then the candidate should be penalised.

Questions which ask candidates to show working

Instructions on marking will be given but usually marks are not awarded to candidates who show no working.

Questions which do not ask candidates to show working

As a general principle, a correct response is awarded full marks.

Misread or miscopy

Candidates often copy values from a question incorrectly. If the examiner thinks that the candidate has made a genuine misread, then only the accuracy marks (A or B marks), up to a maximum of 2 marks are penalised. The method marks can still be awarded.

Further work

Once the correct answer has been seen, further working may be ignored unless it goes on to contradict the correct answer.

Choice

When a choice of answers and/or methods is given, mark each attempt. If both methods are valid then M marks can be awarded but any incorrect answer or method would result in marks being lost.

Work not replaced

Erased or crossed out work that is still legible should be marked.

Work replaced

Erased or crossed out work that has been replaced is not awarded marks.

Premature approximation

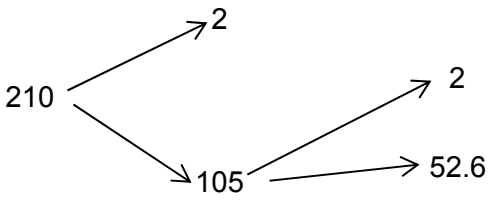
Rounding off too early can lead to inaccuracy in the final answer. This should be penalised by 1 mark unless instructed otherwise.

Continental notation

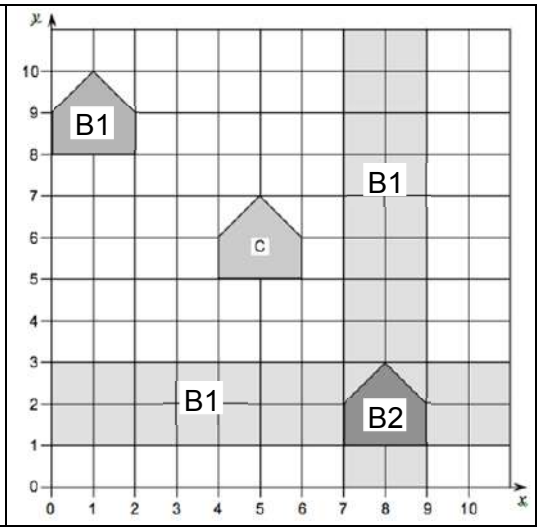
Accept a comma used instead of a decimal point (for example, in measurements or currency), provided that it is clear to the examiner that the candidate intended it to be a decimal point.

Q	Answer	Mark	Comments
1	**Alternative method 1**		
	$6 \times 6 \times \pi$	M1	oe
	36π or [113, 113.112]	A1	
	mm^2	B1	
	Alternative method 2		
	$0.6 \times 0.6 \times \pi$	M1	oe
	0.36π or [1.13, 1.13112]	A1	
	cm^2	B1	
2	**Alternative method 1**		
	0.37 or 1.37 seen	M1	oe
	460 × their 1.37 or 460 × their 0.37 or 170.2	M1	oe
	630.2	A1	
	Alternative method 2		
	10% = 46, 1% = 4.6	M1	Any valid percentage stated, eg 5% = 23
	460 + 3 × Their 10% + 7 × their 1%	M1	Any valid combination, eg 460 + 4 × their 10% – 3 × their 1%
	630.2	A1	
3 (a)	They are alternate angles	B1	
3 (b)	$a + b = 180$	B1	

Q	Answer	Mark	Comments
4 (a)	60	B1	

4 (b)	Any product, or division, of 210 that involves a prime number, eg 2×105, 5×42, $210 \div 7 = 30$ etc	M1	
	$2 \times 3 \times 5 \times 7$	A1	
	Additional Guidance		
	Product may be implied for M1 by a prime factor tree, or values written as pairs, eg (2, 105)		
	Do not award M1 if there is a clear indication of misunderstanding, eg  		

5 (a)	Rotation	B1	Any combined transformation is B0
	90° clockwise or 270°	B1	–90° or +270°
	(5, 4)	B1	

5 (b)	 	B2	B2 correct translation. B1 a translation of $\begin{pmatrix} -4 \\ 3 \end{pmatrix}$ B1 a translation of $\begin{pmatrix} 3 \\ a \end{pmatrix}$, $a \neq -4$ B1 a translation of $\begin{pmatrix} b \\ -4 \end{pmatrix}$, $b \neq 3$ ie correct shape in correct orientation in the light grey areas.

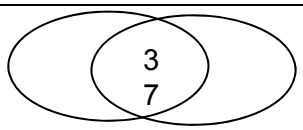
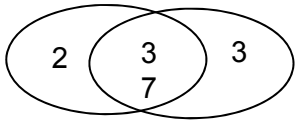
Q	Answer	Mark	Comments
6	5 **and** 4.5	B1	oe
	18 × their 5 × their 5 **or** 20 × their 4.5 × their 4.5	M1	their 5 or 4.5 must come from 20 ÷ 4 or 18 ÷ 4
	450 **and** 405	A1	
	Correct conclusion based on their volumes if one volume correct.	Q1ft	Strand (iii) ft their volumes.
	Additional Guidance		
	Volume A = 18 × 4 × 4 Volume B = 20 × 4.5 × 4.5 A = 288 and B = 405 B has the greater volume		B0 M1 A1 Q1
	Volume A = 18 × 5 × 5 Volume B = 20 × 4.5 × 4.5 A = 180 and B = 405 B has the greater volume		B1 M1 A0 Q1
	Volume A = 18 × 5 × 5 Volume B = 30 × 4.5 × 4.5 A = 450 and B = 607.5 B has the greater volume	Take 30 as misread. Loses A mark.	B1 M1 A0 Q1
	Volume A = 18 × 5 × 5 Volume B = 20 × 4.5 × 4.5 A = 180 and B = 180 Volumes same	Correct conclusion but neither volume correct	B1 M1 A0 Q0
7	$\frac{x}{2} = -1$ or $x + 10 = 8$	M1	
	–2	A1	SC1 3 from $x + 5 = 8$ SC1 –6 from $x + 10 = 4$
8	$5n + 4$	B2	Do not accept $n5$, but $n5 + 4$ scores B1 B1 for $5n$ Accept $5 \times n$ or $n \times 5$

Q	Answer	Mark	Comments
9	$6x^2 + 15x – 4x – 10$	M1	Allow one sign or arithmetic error but must have 4 terms. One in x^2, two in x and a constant term.
	$6x^2 + 11x – 10$	A1	
	Additional Guidance		
	$6x + 15x – 4x – 10$ $16x – 10$	M0, A0	
	$6x^2 + 15x + 4x – 10$ $6x^2 + 19x – 10$	M1, A0	
	$5x^2 + 15x + 4x – 10$ $5x^2 + 19x – 10$	M0, A0	

Q	Answer	Mark	Comments
10	$3x - 3 = x + 4$	M1	
	$2x = 7$	A1	
	$(x =)$ 3.5	A1ft	ft their equation if of the form $2x = a$ where a is not –3, 3 or 4 or of the form $bx = 7$ where b is not 1 or 3
	(Side =) their $x + 4$ or 7.5 or 56.25	A1ft	ft their $x + 4$ only if no further errors
	56	B1ft	ft their $(x + 4)^2$ rounded to the nearest whole number only if no further errors. ft (their x)^2 and rounded to nearest whole number, eg $x = 3.5$ answer 12 SC1 $3x^2 + 9x - 12$ if seen and no other marks awarded.
	Additional Guidance		
	NB follow through allowed on at most one error. After 2nd error no further marks		
	$3x - 3 = x + 4$ $2x = 1$ $(x =)$ 0.5 $4.5^2 = 20.25$ 20	First error	M1 A0 A1ft A1ft B1ft
	$3x - 3 = x + 4$ $2x = 1$ $(x =)$ 2 $6^2 = 36$ 36	First error Second error	M1 A0 A0ft A0ft B0ft
	$3x - 3 = x + 4$ $4x = 7$ $(x =)$ 1.75 $5.75^2 = 33.0625$ 33	First error	M1 A0 A1ft A1ft B1ft
	$3x - 3 = x + 4$ $2x = 7$ $(x =)$ 3.5 $3.5^2 = 12.25$ 12	First error	M1 A1 A1 A0ft B1ft
	$3x - 3 = x + 4$ $x = 1$ $(x =)$ 3.5 $3.5^2 = 12.25$ 12	2 errors	M1 A0 A0ft A0ft B0ft
	$3x - 3 = x + 4$ $2x = 7$ $(x =)$ 4.5 $4.5^2 = 20.25$ 20	First error Second error	M1 A1 A0 A0ft B0ft

Q	Answer	Mark	Comments
11 (a)	**Alternative method 1**		
	$42^2 - 18^2$ or 1440 or $42 + 18^2$ or 2088	M1	
	$\sqrt{1440}$	M1dep	
	[37.9, 38] or $12\sqrt{10}$	A1	
	Alternative method 2		
	$\cos^{-1}(18 \div 42)$ or 64.6… **and** 42 × sin their 64.6… **or** 18 × tan their 64.6…	M2	$\sin^{-1}(18 \div 42)$ or 25.3… **and** 42 × cos their 25.3.. **or** 18 ÷ tan their 25.3..
	[37.9, 38] or $12\sqrt{10}$	A1	Answers outside this range due to premature rounding score A0
11 (b)	**Alternative method 1**		
	cos used with 32 and 28 even if wrong	M1	$\frac{y}{\sin 90} = \frac{32}{\sin 62}$
	32 ÷ cos 28	M1dep	32 ÷ sin 62
	[36, 36.25]	A1	36 with working
	Alternative method 2		
	(side) = 32 × tan 28 (= 17. …) **and** $\sqrt{32^2 + 17^2}$	M2	NB M0 for 28 × tan 32 = 17.49… 32 ÷ tan 62 (= 17. …)
	[36, 36.25]	A1	Answers outside this range due to premature rounding score A0
12 (a)	AAA	B1	

Q	Answer	Mark	Comments
12 (b)	**Alternative method 1**		
	AD = *BC* or *AB* = *CD*	B1	
	AC is common	B1	
	AD = *BC* (equal sides of a rectangle) and *AB* = *CD* (equal sides of a rectangle) and *AC* is common and SSS	B1	
	Alternative method 2		
	AD = *BC* or *AB* = *CD*	B1	
	AC is common	B1	
	AD = *BC* (equal sides of a rectangle) or *AB* = *CD* (equal sides of a rectangle) and *AC* is common and angles *B* and *D* are 90° and RHS	B1	
	Alternative method 3		
	AD = *BC* or *AB* = *CD*	B1	
	AC is common or angle *BAC* = angle *CAB* or angle *ACB* = angle *CAD*	B1	
	AD = *BC* (equal sides of a rectangle) or *AB* = *CD* (equal sides of a rectangle) and *AC* is common and angle *BAC* = angle *CAB* or angle *ACB* = angle *CAD* (alternate angles) and SAS	B1	
	Alternative method 4		
	AC is common	B1	
	angle *BAC* = angle *CAB* or angle *ACB* = angle *CAD*	B1	
	AC is common and angle *BAC* = angle *CAB* and angle *ACB* = angle *CAD* (alternate angles) and ASA	B1	

Q	Answer	Mark	Comments
13	**Alternative method 1**		
	LCM = 2 x 3 x 3 x 7 **or** HCF = 3 x 7	M1	oe
	Place their values correctly into Venn diagram	M1	
	42 **and** 63	A1	SC1 21 = 3 × 7 **and** 126 = 2 x 3 x 3 x 7
	Alternative method 2		
	LCM = 2 x 3 x 3 x 7 **or** HCF = 3 x 7	M1	2 x 3^2 x 7
	Cancel prime factors to leave 2 and 3	M1	
	42 **and** 63	A1	
	Alternative method 3		
		M1	
		M1	
	42 **and** 63	A1	
	Alternative method 4		
	Multiplies 21 by 2 and divides 126 by 2 or multiplies 21 by 3 and divided 126 by 3	M2	
	42 **and** 63	A1	

Q	Answer	Mark	Comments
14	**Alternative method 1**		
	List or table of numbers of cubes for 1, 2, 3, 4 steps and calculation of second difference of 3 3 9 18 30 6 9 12 3 3	M1	oe
	Subtraction of $1\frac{1}{2}n^2$ from quadratic sequence $1\frac{1}{2}$ 3 $4\frac{1}{2}$ 6	M1dep	
	(Linear sequence) $1\frac{1}{2}n$	A1	
	$1\frac{1}{2}n^2 + 1\frac{1}{2}n$	A1ft	oe ft $1\frac{1}{2}n^2$ plus their linear if both Ms awarded.
	Alternative method 2		
	Sets up table of differences n: 0 1 2 3 4 c: 0 3 9 18 30 $a+b$: 3 6 9 12 $2a$: 3 3 3	M1	
	Extends table back to $n = 0$	M1	
	Identifies rows as $2a$, $a + b$ and c	A1	
	$1\frac{1}{2}n^2 + 1\frac{1}{2}n$	A1	oe

Q	Answer	Mark	Comments
14 (cont)	**Alternative method 3**		
	$a + b + c = 3$ **and** $4a + 2b + c = 9$ **and** $9a + 3b + c = 18$	M1	
	$3a + b = 6$ **and** $5a + b = 9$	M1	
	$2a = 3$ or $a = 1\frac{1}{2}$	A1	
	$1\frac{1}{2}n^2 + 1\frac{1}{2}n$	A1	oe
	Alternative method 4		
	Writes terms as $3 \times 1, 3 \times 3, 3 \times 6, 3 \times 10, \ldots$	M1	
	Identifies 1, 3, 6, 10 as triangle number sequence	M1	
	nth term of TNS = $\frac{1}{2}n^2 + \frac{1}{2}n$	A1	oe
	$1\frac{1}{2}n^2 + 1\frac{1}{2}n$	A1	oe
15	sf of 4 identified	B1	Accept 'increased by 4' but not 4 '4 more'
	$4x + 20 = 46$ or $x + 5 = 11.5$	M1	
	6.5	A1	
16 (a)	48	B1	
	Angles on the same arc are equal or angles in the same segment are equal	Q1	oe strand (i)
	Additional Guidance		
	Do not accept colloquial terms such as 'bow-tie' angles.		
	Further exemplars to be provided after standardisation.		

Q	Answer	Mark	Comments
16 (b)	$2y + y + 30 = 180$	M1	oe Allow $2y + y + 30 + BAD + DCB = 360$ if $BAD + DCB = 180$, for example both 90°
	50	A1	
16 (c)	**Alternative method 1**		
	CAB = 40 (alt segment)	B1	
	ACB = 70 (isosceles triangle)	B1	
	ACP = 70 (angles on straight line) **and** *ACP* = *ACB* or 70 is half of 140	Q1	Strand (ii)
	Alternative method 2		
	ACB = *ABC* (angles in isosceles triangle)	B1	
	PCA = *ABC* (Alt segment)	B1	
	ACB = *PCA* (equal) so *AC* bisects *PCB*	Q1	Strand (ii)
	Additional Guidance		
	Penalise first omission of reason only.		
	If assumption is made that *AC* bisects *PCB* then the scheme above is 'reversed'. Do not award Q mark.		

Q	Answer	Mark	Comments
17	**Alternative method 1**		
	$\frac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-3)}}{2(5)}$	M1	Allow one error from: wrong sign for $-b$; Wrong sign for $(-b)^2$; wrong sign for $4ac$. Do not allow division of square root only or division by 2 only
	$\frac{7 \pm \sqrt{109}}{10}$	A1	
	1.74 and –0.34	A1ft	ft on wrong sign for b only, eg –1.74 and 0.34
	Alternative method 2		
	$\left(x - \frac{7}{10}\right)^2 - \frac{49}{100}$	M1	
	$x = \frac{7}{10} \pm \sqrt{\frac{109}{100}}$	A1	
	1.74 and –0.34	A1	Strand (ii)
18	**Alternative method 1**		
	A + B = 90 or B + C = 210	M1	
	A + 2B + C = 300 or C = 132 or A = 12	A1	
	78	A1	
	Alternative method 2		
	Chooses a value for A, **and** calculates B as 90 – A **and** calculates C as 210 – B	M1	
	Calculates A + B + C correctly	A1	
	78	A1	

Q	Answer	Mark	Comments
19	$(5x - 3)(5x + 3)$	B1	
	$(ax \pm c)(bx \pm d)$	M1	$ab = 10$, $cd = 3$
	$(5x - 3)(2x + 1)$	A1	
	$\frac{5x + 3}{2x + 1}$	A1ft	ft their factorisations if cancelling possible Do not award if further contradictory work
	Additional Guidance		
	$(5x - 3)(5x - 3)$ $(5x - 3)(2x + 1)$ $\frac{5x - 3}{2x + 1}$	B0 M1 A0 A1ft	
	$(5x - 3)(5x + 3)$ $(5x + 3)(2x - 1)$ $\frac{5x - 3}{2x - 1}$	B1 M1 A0 A1ft	
	$(5x - 3)(5x - 3)$ $(5x + 3)(2x - 1)$ $\frac{(5x - 3)(5x - 3)}{(2x - 1)(5x + 3)}$	B0 M1 A0 A0ft	
	$(5x - 3)(5x + 3)$ $(5x - 3)(2x + 1)$ $\frac{5x + 3}{2x + 1} = 1$ $5x + 3 = 2x + 1$ $3x = -2$ $x = -\frac{2}{3}$	B1 M1 A1 A0	
	$(5x - 3)(5x + 3)$ $(5x - 3)(2x + 1)$ $\frac{5x + 3}{2x + 1} = 3x + 2$	B1 M1 A1 A0	

Q	Answer	Mark	Comments
20	**Alternative method 1**		
	$AF = 10$	B1	
	Use of tan with 10 and 12 or use of sin with 10 and their AG or use of cos with 12 and their AG	M1	$AG = \sqrt{(12^2 + 6^2 + 8^2)}$ or $2\sqrt{61}$ or [15.6, 15.62]
	$\tan^{-1}(10 \div 12)$ or $\sin^{-1}(10 \div AG)$ or $\cos^{-1}(12 \div AG)$	M1dep	
	[39.8, 40]	A1	Values outside this range due to premature rounding score A0 unless correct value seen and subsequently rounded.
	Alternative method 2		
	$AF = 10$	B1	
	$AG = \sqrt{(12^2 + 6^2 + 8^2)}$ or $2\sqrt{61}$ or [15.6, 15.62]	M1	
	$\cos AGF = \dfrac{12^2 + AG^2 - 10^2}{12 \times 12 \times AG}$	M1dep	
	[39.8, 40]	A1	Values outside this range due to premature rounding score A0 unless correct value seen and subsequently rounded.

Q	Answer	Mark	Comments
21	**Alternative method 1**		
	90 = ½ × x^2 × sin 60	M1	oe
	x^2 = 180 ÷ sin 60 or 14.416….	M1dep	oe
	3 × their $\sqrt{180 \div \sin 60}$	M1dep	
	[43.25, 43.3]	A1	
	Alternative method 2		
	height = $\sqrt{(2x)^2 - x^2}$	M1	
	90 = $x \times \sqrt{3}x$	M1dep	
	$\sqrt{90 \div \sqrt{3}}$ or 7.208…	M1 dep	
	[43.25, 43.3]	A1	Values outside this range due to premature rounding score A0 unless correct value seen and subsequently rounded.
22	$9m \times 2m = 10n \times 5n$	M1	
	$18m^2 = 50n^2$ or $m^2 = \frac{50}{18}n^2$	M1dep	oe
	$m = \frac{5}{3}n$	M1dep	
	$\frac{m}{n} = \frac{5}{3}$	A1	