## AQA

AQA Qualifications

# GCSE <br> Methods in Mathematics <br> (Linked Pair Pilot) 

93652H<br>Unit 2: Higher Tier<br>Mark Scheme

## 9365

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Version 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Glossary for Mark Schemes

GCSE examinations are marked in such a way as to award positive achievement wherever possible. Thus, for GCSE Mathematics papers, marks are awarded under various categories.
\(\left.\left.$$
\begin{array}{ll}\text { M } & \begin{array}{l}\text { Method marks are awarded for a correct method which could lead } \\
\text { to a correct answer. }\end{array} \\
\text { M dep } & \begin{array}{l}\text { A method mark dependent on a previous method mark being } \\
\text { awarded. }\end{array} \\
\text { A } & \begin{array}{l}\text { Accuracy marks are awarded when following on from a correct } \\
\text { method. It is not necessary to always see the method. This can be } \\
\text { implied. }\end{array} \\
\text { B } & \begin{array}{l}\text { Marks awarded independent of method. }\end{array} \\
\text { B dep mark that can only be awarded if a previous independent mark } \\
\text { has been awarded. }\end{array}
$$ \quad $$
\begin{array}{l}\text { Marks awarded for quality of written communication. }\end{array}
$$\right\} \begin{array}{l}Follow through marks. Marks awarded for correct working <br>

following a mistake in an earlier step.\end{array}\right]\)| Special case. Marks awarded for a common misinterpretation |
| :--- |
| which has some mathematical worth. |

## M2 Higher Tier

| Q Answer | Mark | Comments |
| :---: | :--- | :---: | :--- |
| $\mathbf{1 ( a )}$ $29.067 \ldots .$. B1 29.06782609 <br> $\mathbf{1 ( b )}$ 30 B1ft Ft their answer to (a) if given to at least 2 <br> sf. |  |  |


| 2 | 2 and 23, 3 and 23,5 and 23, 7 and 19, 11 and 17 | B2 | B1 for total between 24 and 30 with one prime plus an odd (non-prime) $\begin{aligned} & e g 2+25=27,2+27=29,3+25=28, \\ & 5+21=26,7+21=28,11+15=26, \\ & 13+15=28,9+17=26,9+19=28 \end{aligned}$ <br> B1 for $13+13=26$ <br> B1 for total of 24 or 30 using 2 primes $\begin{aligned} & (5+19=24,11+19=30,11+13=24 \\ & 7+23=30,7+17=24,13+17=30) \end{aligned}$ <br> B1 for list of primes with at most one error for every 5 correct primes <br> B1 for two sums of two primes seen |
| :---: | :---: | :---: | :---: |


| 3 | Any rectangle centred on M with an <br> area of $12 \mathrm{~cm}^{2}$ | B2 | B1 any rectangle centred on M <br> Allow $\pm 1 \mathrm{~mm}$ for any drawn not on <br> grid lines, eg $3 \times 4$ |
| :---: | :--- | :---: | :--- |
|  |  | B1 for rectangle area 12 not centred on M <br> approximately 3.5 |  |
| B1 for 4 corners that clearly show a <br> rectangle of area 12 (allow this mark for <br> badly drawn rectangles, ie if more than 1 <br> mm away from straight) |  |  |  |


| Q Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| 4 | Area of one square $=x^{2}$ <br> or area $\frac{1}{2}$ square $=\frac{1}{2} x^{2}$ <br> or area $\frac{1}{4}$ square $=\frac{1}{4} x^{2}$ <br> NB these may be written on diagram or stated in script | B1 | $\begin{aligned} & \text { eg } 16 \times x^{2}=16 x^{2} \\ & 11 \text { whole squares }=11 x^{2} \end{aligned}$ |
| :---: | :---: | :---: | :---: |
|  | 11 whole squares shown on diagram or stated. | B1 |  |
|  | $\frac{1}{2}\left(x^{2}\right)$ seen on diagram or stated for large triangle or $\frac{1}{4}\left(x^{2}\right)$ seen on diagram or stated for small triangle | B1 |  |
|  | Clear method showing that all individual squares and triangles sum to 16 , eg $\begin{aligned} & 11\left(x^{2}\right)+8 \times \frac{1}{2}\left(x^{2}\right)+4 \times \frac{1}{4}\left(x^{2}\right) \\ & \left(=16 x^{2}\right) \end{aligned}$ | Q1 | Strand (ii) <br> Not enough to show 16 squares on diagram, some clear statement must be made. |
| $\begin{gathered} 4 \\ \text { Alt } 1 \end{gathered}$ | $x^{2}+x^{2}=$ diagonal ${ }^{2}$ | M1 |  |
|  | Diagonal $=x \sqrt{ } 2$ | M1 |  |
|  | $4 x \sqrt{ } 2 \times 2 x \sqrt{ } 2$ | A1 |  |
|  | Clear method showing diagonal calculated using Pythagoras and lengths of sides calculated in terms of the diagonal, then multiplied and total 16. | Q1 | Strand (ii) |


| Q | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |


| 5 | Volume of water $=30 \times 20 \times 7.5$ or $30 \times 20 \times 10 \times 0.75(=4500)$ | B1 |  |
| :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & 30 \times 0.75(=22.5) \text { or } 20 \times 0.75(= \\ & 15) \end{aligned}$ | M1 |  |
|  | Using the value of height to show that $10 \times 20 \times 22.5=4500$ or $10 \times$ $30 \times 15=4500$ | M1 |  |
|  | Full method showing depth is threequarters for both faces | Q1 | Strand (ii) |
| 5 <br> Alt 1 | Volume of water $=30 \times 20 \times 7.5$ or $30 \times 20 \times 10 \times 0.75(=4500)$ | B1 |  |
|  | $\begin{aligned} & 4500 \div(20 \times 10)(=22.5) \\ & \text { or } 4500 \div(30 \times 10)(=15) \end{aligned}$ | M1 |  |
|  | Using the value of height to show that $22.5 \div 30=0.75$ (oe) <br> or $15 \div 20=0.75$ (oe) | M1 |  |
|  | Full method showing depth is threequarters for both faces | Q1 | Strand (ii) |
| $\begin{gathered} 5 \\ \text { Alt } 2 \end{gathered}$ | $\begin{aligned} & \text { Volume of water }=30 \times 20 \times 7.5 \\ & (=4500) \end{aligned}$ | B1 | Clear explanation that volume of water is 0.75 of total volume. |
|  | Volume of water $=(30 \times 20 \times 7.5) \div$ $(30 \times 20 \times 10)=0.75$ | B1 |  |
|  | Clear explanation using the proportion of water and the volume of prism formula, must mention constant cross-sectional area | Q2 | Strand (ii) |


| Q | Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| 6 | $12 y-4(=28)$ or $3 y-1=7$ | M 1 |  |
| :---: | :--- | :---: | :--- |
|  | Correctly rearranging their <br> expanded or divided equation to <br> get letters on one side, numbers on <br> the other. | M 1 | $12 y=32$ <br> $3 y=8$. <br> $32 \div 12$ is M2 |
|  | $2 \frac{2}{3}$ | A1ft | oe ft on $\mathrm{M} 1, \mathrm{M} 0$ or M0, M1 <br> SC 1 for $29 / 12(2.42$ or $2.416 .)$. |


| 7 | $3 x-5=x$ | M1 |  |
| :--- | :--- | :---: | :--- |
|  | $x=2.5$ | A1 |  |
|  | 13 | A1ft | If M1 awarded ft <br> $4 \times$ their $x+3$. <br> or $2 \times$ their $x+8$. <br> or $6 \times$ their $x-2$. |


| 8(a) |  |  |  |  |  |  |  |  | B2 | B1 for congruent triangle, with correct |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  |  |  |  |  |  |  |  | orientation, wholly within a rectangle |
|  |  |  |  |  |  |  |  |  |  | bounded by $y=-2, y=-5, x=1$ and $x=5$ |
|  |  |  | , |  |  |  |  |  |  |  |
|  |  | $\gamma$ |  |  |  |  |  |  |  | B1 for a congruent triangle wholly within |
|  |  |  |  |  |  |  |  |  |  | the same rectangle with two vertices on |
|  |  |  |  |  |  |  |  |  |  | coordinates $(2,-3)$ or $(2,-4)$ or $(4,-3)$ |
|  |  |  |  |  |  |  |  | $\rightarrow$ |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | - |  |  |  |
|  |  |  |  |  |  |  |  | $\square$ |  |  |
|  |  |  |  |  |  |  |  | $\square$ |  |  |


| $\mathbf{Q}$ | Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |



| 9(a) |  | B2 | B1 for at least one correct region |
| :---: | :---: | :---: | :---: |


| 9(b) | Lists of multiples for either 12 or 9 | M1 |  |
| :---: | :--- | :---: | :--- |
|  | 36 | A1 | Sc1 any multiple of $36>36$ |
| 9(b) <br> Alt $\mathbf{1}$ | Appropriate lists for either of the two <br> largest numbers in set B | M1 |  |
|  | LCM from their list | A1 | Sc1 any multiple of their LCM $>$ LCM. |


| $\mathbf{Q}$ | Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| $\mathbf{1 0 ( a )}$ | $a$ and $p$ or $b$ and $q$ or $c$ and $r$ or $d$ <br> and $s$ | B1 |  |
| :--- | :--- | :---: | :--- |
|  | $a$ and $d$ or $c$ and $b$ or $r$ and $q$ or $p$ <br> and $s$ | B1 |  |
|  | $c$ and $p$ or $d$ and $q$ | B1 |  |


| 10(b) | $360 \div 45$ | M1 | $6 \times 180=1080$ <br> and $1080 \div 8=135$ <br> and 135 stated or shown as interior angle |
| :---: | :--- | :---: | :--- |
|  | 8 | A1 | Allow octagon if working seen |


| 11(a) | $-7,-4,-1,2,5$ | B2 |
| :--- | :--- | :--- |
| B1 for 4 correct or $-10,-7,-4,-1,2$ |  |  |
| $\mathbf{1 1 ( b )}$ $-8 n+98$ B2 oe B1 for $-8 n$ |  |  |


| 11(c) | Shows for any rectangles that the area is height $\times$ height +1 or width $\times$ width -1 eg $2 \times 1,3 \times 2,6 \times 5$ etc | B1 | Can draw a square and rectangle on diagram |
| :---: | :---: | :---: | :---: |
|  | Shows that the sides are $n$ and $n+1$ | B1 | eg a diagram showing $n$ and $n+1$ as sides of a rectangle. <br> Shows the area of the square is $n^{2}$ and the rectangle is $n$ |
|  | Full method showing area is $n \times$ $(n+1)$ | Q1 | Strand (ii) |
| 11(c) <br> Alt 1 | Differencing to get second difference 2 | M1 |  |
|  | Obtaining 'linear' part after subtracting $n^{2}$ is $1,2,3,4$, | M1 |  |
|  | Full method showing area is $n \times$ $(n+1)$ | Q1 | Strand (ii) |


| Q | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 11(c) <br> Alt 2 | Table of differencing to get second difference 2 | M1 |  |
|  | Working table backwards correctly to get the 'zero'th term' | M1 | oe Difference between $1^{\text {st }}$ and $2^{\text {nd }}$ term is $3 a+b$ and first term is $a+b+c$ |
|  | Identifying $c=0, a=1$ and $b=1$ in $a x^{2}+b x+c$ to give $n^{2}+n$ | Q1 | Strand (ii) |


| 12(a) | $22^{2}$ and $38^{2}$ seen added or <br> subtracted. | M1 | 1928 is M1 |
| :---: | :--- | :---: | :--- |
|  | Vtheir $\left(22^{2}+38^{2}\right)$ | M1Dep | oe |
|  | $[43.9,44]$ | A1 | 44 with working, SC1 31 or 30.98... |


| 12(b) | Sight of sin used | M1 |  |
| :--- | :--- | :---: | :--- |
|  | $19 \div \sin 36$ | M1Dep |  |
|  | $[32,32.35]$ | A1 | 32 with working |


| 13 | $3 \times 9(=27)$ | M1 |  |
| :---: | :--- | :---: | :--- |
|  | $6 \times x=$ their 27 | M1Dep |  |
|  | 4.5 | A1 |  |

14

| $4(x+1)+3(x+5)$ | M1 |  |
| :--- | :---: | :--- |
| $7 x+19$ | A1 |  |
| Their $7 x+19=12$ | M1 |  |
| -1 | A1ft | ft on one error and both Ms awarded. <br> sc 1 answer with no valid algebra shown. |


| Q | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 15 | $45 \div 3(=15)$ | M1 | Can be embedded in a calculation |
|  | $7 \times$ their 15 | M1Dep | $2 \times$ their 15 and $5 \times$ their 15 or 30 and 75 seen even if not added eg $30: 75$ or $75-30$ |
|  | 105 | A1 |  |
| $\begin{gathered} 15 \\ \text { Alt } 1 \end{gathered}$ | Chooses a multiple of 7 and divides in ratio $2: 5$ then calculates the difference | M1 | eg $70 \Rightarrow 20: 50 \Rightarrow 50-20=30$ |
|  | Chooses a second multiple of 7 closer to 105 and divides in ratio 2 : 5 then calculates the difference | M1Dep | eg $91 \Rightarrow 26: 65 \Rightarrow 65-26=39$ |
|  | 105 | A1 |  |
| $\begin{gathered} 15 \\ \text { Alt } 2 \end{gathered}$ | $x: x+45=2: 5 \text { or } \frac{x}{x+45}=\frac{2}{5}$ | M1 |  |
|  | $x=30$ | M1Dep |  |
|  | 105 | A1 |  |


| 16 | $\frac{5 \pm \sqrt{(-5)^{2}-4(3)(-3)}}{2(3)} \text { or }\left(x-\frac{5}{6}\right)$ | M1 | Allow one error for M1 from <br> Wrong sign for $-b$ <br> Wrong sign for $-b^{2}$ <br> Wrong sign for $-4 a c$ <br> But not 2 on bottom or not dividing whole top by $2 a$ or wrong formula |
| :---: | :---: | :---: | :---: |
|  | $\frac{5 \pm \sqrt{61}}{6} \text { or }\left(x-\frac{5}{6}\right)^{2}-\frac{25}{12}-3$ | A1 |  |
|  | 2.14 and -0.47 | A1ft | ft on wrong sign for $-b: 0.47$ and -2.14 ft on wrong sign for $-b^{2}$ : 1.39 and 0.28 |


| 17(a) | $x \div \sin 39^{\circ}=21 \div \sin 124^{\circ}$ | M1 | oe |
| :--- | :--- | :--- | :--- |
|  | $x=\left(21 \times \sin 39^{\circ}\right) \div \sin 124^{\circ}$ | A1 |  |
|  | $[15.9,16]$ | A1 | 16 with working |


| Q | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 17(b) | $\begin{aligned} & \cos y=\left(19^{2}+14^{2}-28^{2}\right) \div \\ & (2 \times 19 \times 14) \end{aligned}$ | M1 | $28^{2}=19^{2}+14^{2}-2 \times 14 \times 19 \times \cos y$ |
|  | $\cos y=-0.42669 \ldots$ | A1 |  |
|  | [115, 115.3] | A1 | 115 with working |


| 18(a) | ( $h=$ ) $5 \times \tan 67.5$ or $5 \div \tan 22.5$ | M1 |  |
| :---: | :---: | :---: | :---: |
|  | Area $=0.5 \times 10 \times$ their height | M1Dep |  |
|  | $5 \times 12.07106781=60.35$. | A1 | oe Must calculate a value that rounds to 60.4 but it is not necessary to state $60.35 \approx$ 60.4 <br> NB if height calculated using 60.4 this is MO unless the height is then use with an angle and a trig ratio to relate to 10 or 5 , in which case it would be M2. Be careful of premature rounding as 12.1 leads to 12.1 $\times \tan 22.5=5.011$ whereas $12.07 \times$ $\tan (22.5)$ leads to 4.999. If answer is not within range [4.999, 5.001] do not award the last A1. |
| 18(a) <br> Alt 1 | Find side of isosceles triangle $x=5 / \sin 22.5 \text { or } 5 / \cos 67.5$ | M1 | Hypotenuse $=13.065 \ldots$ |
|  | Area $=1 / 2$ their $13.066^{2} \times \sin 45$ or Area $=1 / 2 \times$ their $13.066 \times 10 \times$ $\sin 67.5$ | M1Dep |  |
|  | Area $=60.35$. | A1 | Must calculate a value that rounds to 60.4 but it is not necessary to state $60.35 \approx 60.4$ |


| 18(b) | Area octagon $=8 \times$ their 60.4 <br> $(=483.2)$ | B1 | If a square drawn round the octagon, sides <br> are 24.14 $\ldots$, so area is 582.7396, the 4 <br> corners (area 50 each) must be subtracted |
| :---: | :--- | :---: | :---: |
|  | $100 \div$ their 483.2 | M1 |  |
|  | $[20.5,21]$ | A1 |  |


| Q | Answer | Mark | Comments |
| :---: | :---: | :---: | :---: |
| 19(a) | $(2 x+3)(2 x+3)=5(3 x+2)$ | M1 | $4 x^{2}+6 x+6 x+9$ and $15 x+10$ |
|  | $4 x^{2}+6 x+6 x+9=15 x+10$ | A1 | Equality can be implied by subtraction |
|  | Terms rearranged so that all are on LHS and an indication of cancelling. Minimum is $4 x^{2}+6 x+6 x+9-15 x-10=0$ | A1 | Must have $=0$ <br> If brackets expanded and terms rearranged into a quadratic of form $a x^{2}+b x+c=0$ |


| 19(b) | $(4 x+1)(x-1)=0$ | B1 | $\frac{3 \pm \sqrt{25}}{8}$ <br> If no working in (b) this mark can be <br> awarded if seen in (a) |
| :--- | :--- | :---: | :--- |
|  | -0.25 | Q1 | Strand (i) Do not award if 1 given as a <br> value. Must make the decision to choose <br> the value that gives a rectangle. |


| 20 | Angle $A=$ angle $B=90$ | B1 |  |
| :---: | :---: | :---: | :---: |
|  | $B N=A M$ (given) | B1 | Given need not be stated |
|  | $M L=M N$ and sides of (same) square | B1 | oe, eg ML $=M N(M N Q L$ is a square $)$ |
|  | Congruent due to RHS | B1 | Reason can be in words but must be clearly RHS, eg right angle triangle with a side and the hypotenuse |
| $\begin{gathered} 20 \\ \text { Alt } 1 \end{gathered}$ | Angle $B M N=$ Angle $A L M$ | B1 | These must be explained using 180 on a straight line and 180 in a triangle |
|  | Angle $B N M=$ Angle $A M L$ | B1 |  |
|  | $B N=A M$ (given) | B1 | Given need not be stated |
|  | Congruent due to ASA | B1 | Reason can be in words but must be clearly ASA, eg two angles and the side between them |


| Q Answer | Mark | Comments |
| :--- | :---: | :---: | :---: |


| $\begin{gathered} 20 \\ \text { Alt } 2 \end{gathered}$ | $M L=M N$ and sides of (same) square | B1 | NB Pythagoras stated or used is a reason for third side. |
| :---: | :---: | :---: | :---: |
|  | $B M=A L$ and both sides of larger square - $A M$ or $D L$ | B1 | This must be justified |
|  | $B N=A M$ (given) | B1 | Given need not be stated |
|  | Congruent due to SSS | B1 | Reason can be in words but must be clearly SSS, eg all sides equal |
| $\begin{gathered} 20 \\ \text { Alt } 3 \end{gathered}$ | Angle $A=$ angle $B=90$ | B1 |  |
|  | $B M=A L$ and both sides of larger square - $A M$ or $D L$ | B1 | This must be justified |
|  | $B N=A M$ (given) | B1 | Given need not be stated |
|  | Congruent due to SAS (Do not accept ASS) | B1 | Reason can be in words but must be clearly SAS, eg two sides and the angle between them |

