Paper Reference(s)

6678/01

Edexcel GCE

Mechanics M2

Bronze Level B3

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
73	65	57	49	41	33

1. A particle P moves on the x-axis. The acceleration of P at time t seconds, $t \geq 0$, is $(3t + 5)$ m s^{-2} in the positive x-direction. When $t = 0$, the velocity of P is 2 m s^{-1} in the positive x-direction. When $t = T$, the velocity of P is 6 m s^{-1} in the positive x-direction.

Find the value of T.

(6)

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2. Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e,

(i) the speed of P immediately after the collision,

(ii) the speed of Q immediately after the collision.

(7)

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3. A truck of mass of 300 kg moves along a straight horizontal road with a constant speed of 10 m s^{-1}. The resistance to motion of the truck has magnitude 120 N.

(*a*) Find the rate at which the engine of the truck is working.

(2)

On another occasion the truck moves at a constant speed up a hill inclined at θ to the horizontal, where $\sin\theta = \frac{1}{14}$. The resistance to motion of the truck from non-gravitational forces remains of magnitude 120 N. The rate at which the engine works is the same as in part (*a*).

(*b*) Find the speed of the truck.

(4)

4.

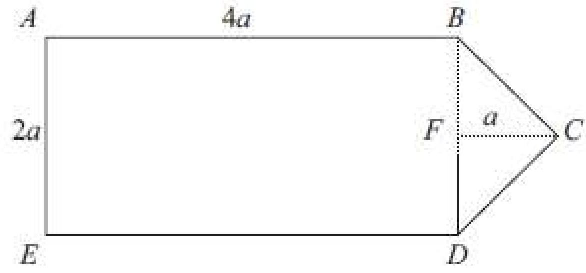

Figure 1

Figure 1 shows a uniform lamina *ABCDE* such that *ABDE* is a rectangle, $BC = CD$, $AB = 4a$ and $AE = 2a$. The point *F* is the midpoint of *BD* and $FC = a$.

(*a*) Find, in terms of *a*, the distance of the centre of mass of the lamina from *AE*.

(4)

The lamina is freely suspended from *A* and hangs in equilibrium.

(*b*) Find the angle between *AB* and the downward vertical.

(3)

5.

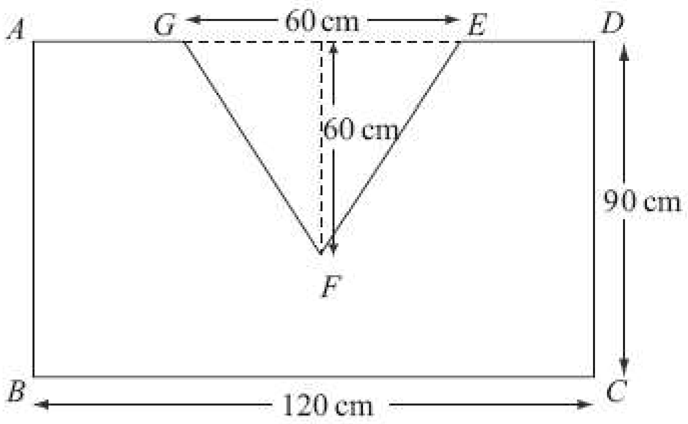

Figure 2

A shop sign *ABCDEFG* is modelled as a uniform lamina, as illustrated in Figure 2. *ABCD* is a rectangle with $BC = 120$ cm and $DC = 90$ cm. The shape *EFG* is an isosceles triangle with $EG = 60$ cm and height 60 cm. The mid-point of *AD* and the mid-point of *EG* coincide.

(*a*) Find the distance of the centre of mass of the sign from the side *AD*.

(5)

The sign is freely suspended from *A* and hangs at rest.

(*b*) Find the size of the angle between *AB* and the vertical.

(4)

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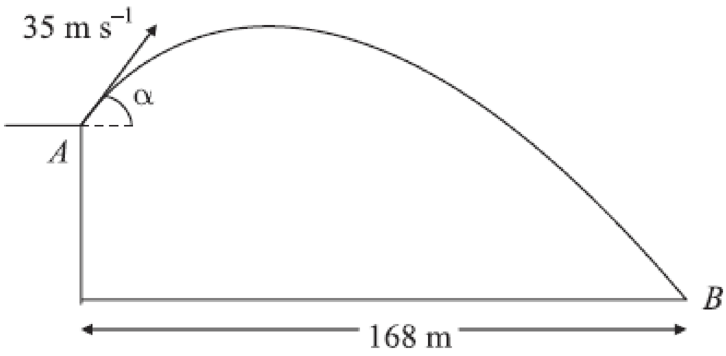
6. **Figure 4**



A golf ball P is projected with speed 35 m s^{-1} from a point A on a cliff above horizontal ground. The angle of projection is α to the horizontal, where tan $\alpha = \frac{4}{3}$. The ball moves freely under gravity and hits the ground at the point B, as shown in Figure 4.

(*a*) Find the greatest height of P above the level of A.

(3)

The horizontal distance from A to B is 168 m.

(*b*) Find the height of A above the ground.

(6)

By considering energy, or otherwise,

(*c*) find the speed of P as it hits the ground at B.

(3)

7. Three particles P, Q and R lie at rest in a straight line on a smooth horizontal table with Q between P and R. The particles P, Q and R have masses $2m$, $3m$ and $4m$ respectively. Particle P is projected towards Q with speed u and collides directly with it. The coefficient of restitution between each pair of particles is e.

(a) Show that the speed of Q immediately after the collision with P is $\frac{2}{5}(1 + e)u$.

(6)

After the collision between P and Q there is a direct collision between Q and R.

Given that $e = \frac{3}{4}$, find

(b) (i) the speed of Q after this collision,

(ii) the speed of R after this collision.

(6)

Immediately after the collision between Q and R, the rate of increase of the distance between P and R is V.

(c) Find V in terms of u.

(3)

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8. A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x \tan \alpha - \frac{gx^2}{2u^2 \cos^2 \alpha}.$$

(4)

A girl throws a ball from a point A at the top of a cliff. The point A is 8 m above a horizontal beach. The ball is projected with speed 7 m s^{-1} at an angle of elevation of 45°. By modelling the ball as a particle moving freely under gravity,

(b) find the horizontal distance of the ball from A when the ball is 1 m above the beach.

(5)

A boy is standing on the beach at the point B vertically below A. He starts to run in a straight line with speed v m s^{-1}, leaving B 0.4 seconds after the ball is thrown.

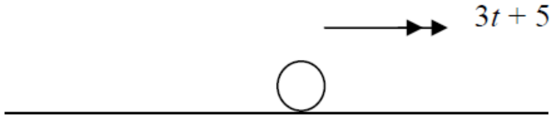
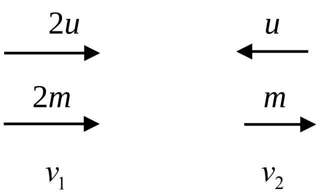
He catches the ball when it is 1 m above the beach.

(c) Find the value of v.

(4)

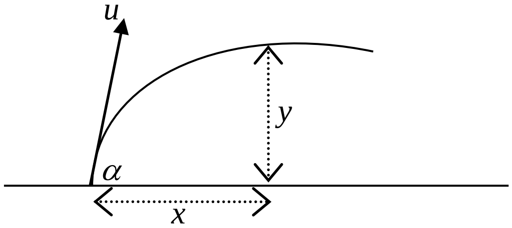
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	 $\frac{dv}{dt} = 3t + 5$	
	$v = \int(3t + 5)dt$	**M1***
	$v = \frac{3}{2}t^2 + 5t\ (+ c)$	**A1**
	$t = 0\ v = 2 \Rightarrow c = 2$	**B1**
	$v = \frac{3}{2}t^2 + 5t + 2$	
	$t = T \quad 6 = \frac{3}{2}T^2 + 5T + 2$	**DM1***
	$12 = 3T^2 + 10T + 4$ $3T^2 + 10T - 8 = 0$	
	$(3T - 2)(T + 4) = 0$	**M1**
	$T = \frac{2}{3} \quad (T = -4)$	
	$\therefore T = \frac{2}{3}$ (or 0.67)	**A1**
		[6]
2.	 	
	CLM: $4mu - mu = 2mv_1 + mv_2$ i.e. $3u = 2v_1 + v_2$	M1 A1
	NIL: $3eu = -v_1 + v_2$	M1 A1
	$v_1 = u(1 - e)$	DM1 A1
	$v_2 = u(1 + 2e)$	A1
		[7]

Question Number	Scheme	Marks
3. **(a)**	Constant v $\Rightarrow$ driving force = resistance $\Rightarrow$ F=120 (N) $\Rightarrow$ P=120 × 10 = 1200W	M1 M1 (2)
(b)	Resolving parallel to the slope, zero acceleration: $\frac{P}{v} = 120 + 300g\sin\theta (= 330)$ $\Rightarrow$ v = $\frac{1200}{330} = 3.6$ (ms^{-1})	M1A1A1 A1 (4) **[6]**
4. **(a)**		*ABDE*	*BCD*	Lamina | Mass ratio: $8a^2\rho$, $a^2\rho$, $9a^2\rho$ 8, 1, 9 Dist of C of M From AE: 2a, $4\frac{1}{3}a$, $\bar{x}$ $8\times 2a + 1\times\frac{13}{3}a = 9\bar{x}$ $\bar{x} = \frac{61}{27}a$ (2.26a)	B1 B1 M1 A1 (4)
(b)	$\tan\phi = \frac{a}{\frac{61}{27}a} = \frac{27}{61}$ $\phi = 23.87... = 24°$ (accept 23.9), 0.417 radians	M1 A1 ft A1 (3) **7**
5. **(a)**	Ratio of areas triangle:sign:rectangle = 1 : 5 : 6 (1800:9000:10800) Centre of mass of the triangle is 20cm down from *AD* (seen or implied) $\Rightarrow$ $6\times 45 - 1\times 20 = 5\times\bar{y}$ $\bar{y} = 50cm$	B1 B1 M1A1 A1 (5)
(b)	Distance of centre of mass from *AB* is 60cm Required angle is $\tan^{-1}\frac{60}{50}$ (their values) = 50.2° (0.876 rads)	B1 M1A1ft A1 (4) **[9]**

6. **(a)**	$0 = (35 \sin \alpha)^2 - 2gh$ $h =$ <u>40 m</u>	M1 A1 A1 (3)
(b)	$x = 168 \Rightarrow 168 = 35 \cos \alpha \,.\, t$ ($\Rightarrow$ t = 8s)	M1 A1
	At $t = 8$, $y = 35 \sin \alpha \times t - \frac{1}{2} g t^2$ (= $28.8 - ½.g.8^2 = -89.6$ m)	M1 A1
(c)	Hence height of A = <u>89.6 m</u> or 90 m	DM1 A1 (6)
	$½mv^2 = 1/2.m.35^2 + mg.89.6$ $\Rightarrow v =$ <u>54.6 or 55 m s^{-1}</u>	M1 A1 A1 (3)
7 **(a)**	CLM: $2mu = 2mv + 3mw$	M1 A1
	Impact: $w - v = eu$	M1 A1
	Subst $v = w - eu$: $2u = 2(w - eu) + 3w = 5w - 2eu$	DM1
	$w = \frac{2}{5}(1+e)u$ ***Answer Given***	A1 (6)
(b)	$w = \frac{7u}{10}$	B1
	CLM: $3mw = 3mx + 4my$ and Impact: $y - x = \frac{3w}{4}$	M1A1
	Subst: $3w = 3x + 4\left(x + \frac{3}{4}w\right)$	DM1
	$x = 0$,	A1
	$y = \frac{3}{4}w = \frac{21}{40}u$	A1 (6)
(c)	$v = -\frac{u}{20}$	B1
	Speed of separation = $\frac{u}{20} + \frac{21u}{40} = \frac{23u}{40}$	M1 A1 (3) [15]

Question Number	Scheme	Marks
8. **(a)**		
	Horiz: $x = u\cos\alpha t$	B1
	Vert: $y = u\sin\alpha t - \frac{1}{2}gt^2$	M1
	$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}g \times \frac{x^2}{u^2\cos^2\alpha}$	DM1
	$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ **	A1
		(4)
(b)	$y = -7:\quad -7 = \tan 45x - \frac{gx^2}{2\times 7^2\cos^2 45}$	M1 A1
	$-7 = x - \frac{9.8x^2}{7^2}$	
	$-7 = x - \frac{x^2}{5}$	M1
	$x^2 - 5x - 35 = 0$	
	$x = \frac{5 \pm \sqrt{25 + 4\times 35}}{2}$	M1
	$x = 8.92$ or 8.9	A1
		(5)
(c)	Time to travel 8.922 m horizontally $= \frac{8.922}{7\cos 45} \quad = 1.802...\text{s}$	M1
	$v = \frac{8.922}{1.402}$	M1 A1 ft
	$= 6.36$ or 6.4 $\left(\text{m s}^{-1}\right)$	A1
		(4) **13**

Examiner reports

Question 1

This question proved very accessible and gave most candidates a confident start to the paper. There were very few incorrect answers, with the overwhelming majority integrating the given acceleration correctly. Any errors in the integration were mostly when $3t^2$ was not divided by 2. There was some confusion about the constant of integration in a few cases, often taken in error to be zero. Nearly all candidates set their velocity expressions equal to 6 and attempted to solve the resulting quadratic equation. There were some basic algebraic or arithmetical slips resulting in incorrect equations. A method was not always shown in the solution of a quadratic. This should be discouraged as credit can be given for correct working if it is seen.

There were a small number of candidates who tried to apply "*suvat*" to the motion, losing 5 out of the 6 marks available.

Question 2

This question was very well answered by the majority of candidates. The momentum and impact equations were often correct - the most common error was lack of consistency in signs between their equations. A surprising number of candidates did not draw a diagram which possibly made it more difficult to avoid these sign errors. Only a small number of candidates quoted the impact law the wrong way round.

A significant number of candidates had P going to the left after the collision, obtaining a velocity of $u(e - 1)$. However, they almost always failed to realise that this was a negative answer, ignoring the fact that the question had asked for speed.

A number of candidates, who started with two correct equations, went on to lose marks at the end due to algebraic errors in solving the simultaneous equations.

Question 3

Most candidates dealt with this question very well; a great proportion of candidates being able to score full marks for both parts. Part (a) was invariably correct, but a small number of candidates added a mass/weight to the resistance given in the question.

In part (b) the usual approach was to find the force of 330 N separately and then to substitute into $P=Fv$. Some candidates used the power from (a) as a force so scored nothing. Some wrote a correct equation for the total resistance to motion but forgot to include the 120 when they completed the calculation; a few included the 120 twice in some way - either adding or subtracting from 330.

A significant minority lost the final A mark through over-accuracy following the use of an approximate value for g or incorrect rounding.

Question 4

Centre of mass is a topic which is well understood by nearly all candidates and this question was a good source of marks for most, with many fully correct responses seen.

Many candidates completed part (a) successfully, but it was disappointing to find a number of basic errors made in calculating areas. There was also some confusion over the centre of mass of the triangle; some candidates found the distance from *BD*, but not from *AE*, some knew that they we using $\frac{1}{3}$ of the median, but measured from the vertex rather than from the base, and there were several candidates using $\frac{1}{4}$ rather than $\frac{1}{3}$. Having established their values a moments equation was almost always formed.

Most candidates were able to identify the required angle correctly in part (b) and follow through appropriately on their previous values. It was disappointing to see several candidates needing to work through to find the distance of the centre of mass of the lamina from *AB*, not appreciating the significance of the symmetry of the lamina. Without working, a common error was to use $2a$ rather than a for this distance.

Question 5

This question was generally well answered by most candidates. It was pleasing to see many correct evaluations of the centre of mass of the triangle and many completely correct solutions to part (a). A few candidates attempted breaking up the lamina into rectangles and triangles rather than subtracting the moment of the triangle from the moment of the rectangle and so made the question much more difficult. Of those who calculated the areas of the rectangle and triangle correctly some failed to subtract these and added them instead. Most seemed to be happy with the use of large numbers for areas and only a few reduced these to a ratio. A very small number of candidates tried to replace the lamina by a framework of rods.

In part (b) a significant number of candidates failed to recognise that the lamina was symmetrical and wasted time in finding the distance to the centre of mass from *AB* using the same method, rather than using symmetry to write it down.

Many candidates lost the last two marks by finding the angle between *AD* and the vertical instead of the angle required.

Question 6

Part (a) There were several instances of a possible misread of tan α = $\frac{3}{4}$ in this question, although it was not always possible to tell whether the error was a misread or use of an incorrect expression tan = $\frac{\cos\alpha}{\sin\alpha}$.Most candidates used the formula $v^2 = u^2 + 2as$ to find the maximum height. This was often found correctly, but a common error was to forget to square their value for u. Some candidates made the task more difficult than necessary by adopting a method with two, or more, stages.

Part (b) Having found the time to travel a horizontal distance of 168m it is possible to find the vertical distance in one step, but many candidates elected either to find the time from the highest point to the ground, or to find the time from when the ball returns to the level of A until it reaches B. Candidates choosing one of these longer alternatives did not always match up their value for time taken with an appropriate value for the initial vertical speed. Having reached a value for vertical distance there was then some confusion about whether or not to add or subtract their answer from part (a).

Part (c) Whether using energy or an alternative approach, the most common error in this part of the question was to concentrate on the vertical speed and to ignore or omit the horizontal component. This resulted in many candidates scoring no marks here.

Question 7

Most candidates were successful in forming correct equations for conservation of momentum and for the impact law in part (a) and using these to deduce the given answer. A few made the task more difficult by finding the velocity of *P* first and then finding the speed of *Q*. There were a few sign errors and arithmetic errors which resulted in attempts to fudge the answer rather than find the source of the error.

Those candidates who started by substituting the given value for *e* in part (b) tended to be more successful in forming and solving equations to find the speeds of *Q* and *R* after the collision. Some candidates did not use the value of *e* at all. Solutions were often spoiled by careless algebra.

In part (c) a lot of candidates did not realise that the rate of increase of distance is the same thing as speed of separation. There was also an element of confusion over which speeds should be used, with many using the speed of *Q* after the first collision rather than the speed of *P*.

Question 8

There were some excellent and succinct solutions to this question. However some candidates try to use the same (very long) method for all projectile situations instead of trying to assess the most efficient route to the required answer.

The bookwork required in part (a) was usually completed very confidently, but some candidates clearly had little idea of how to proceed, despite similar questions on recent past papers.

In part (b) many candidates ignored the given equation from (a), instead using the longer route of $s = ut + \frac{1}{2}at^2$ vertically and then horizontally. A few candidates made this even longer by finding the time taken to reach the maximum height and then the time taken to fall to the point 1 m above the beach. Unfortunately, this was a very error prone method, usually accumulating rounding errors.

Choosing the correct value of y proved very problematic. Some of the wrong values chosen lead to a quadratic equation with no real roots although this did not seem to ring alarm bells very often. It is a pity that candidates do not always show their working when solving a quadratic equation and marks were therefore lost here.

Part (c) was answered well by those candidates who realised that the boy was running at a constant speed (and hence had zero acceleration) and that the ‘0.4 seconds later’ meant that the time taken was reduced. A few added the 0.4 instead of subtracting it. Some candidates thought that the boy was starting from rest and accelerating to speed v at the instant when he caught the ball.

Statistics for M2 Practice Paper Bronze 3

				Mean average scored by candidates achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	6		91.0	5.46	5.92	5.83	5.62	5.34	4.93	4.26	2.88
2	7		82.1	5.75		6.12	5.42	4.38	3.98	2.94	1.83
3	6		85.8	5.15		5.73	5.34	4.87	4.28	3.32	1.92
4	7		79.9	5.59	6.61	6.27	5.63	4.83	4.12	3.26	1.94
5	9		77.4	6.97		8.05	6.96	6.18	5.26	4.19	2.51
6	12		74.3	8.91		10.61	9.14	8.05	6.42	5.17	2.78
7	15	15	71.5	10.73	13.21	12.24	10.87	9.72	8.14	6.30	3.36
8	13		69.1	8.98	11.76	10.77	8.76	6.71	4.74	3.13	1.88
	75		**76.7**	**57.54**		**65.62**	**57.74**	**50.08**	**41.87**	**32.57**	**19.10**