Paper Reference(s)

6667/01

Edexcel GCE

Further Pure Mathematics FP1
Bronze Level B2

Time: 1 hour 30 minutes

Materials required for examination papers	Items included with question
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Further Pure Mathematics FP1), the paper reference (6667), your surname, initials and signature.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 9 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
72	65	58	51	44	37

1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of x.

(4)

June 2013

2.

$$\mathbf{M} = \begin{pmatrix} 2a & 3 \\ 6 & a \end{pmatrix}, \text{ where a is a real constant.}$$

(*a*) Given that $a = 2$, find $\mathbf{M}^{-1}$.

(3)

(*b*) Find the values of a for which **M** is singular.

(2)

June 2010

3.

$$f(x) = x^3 - \frac{7}{x} + 2,\ x > 0.$$

(*a*) Show that $f(x) = 0$ has a root α between 1.4 and 1.5.

(2)

(*b*) Starting with the interval [1.4, 1.5], use interval bisection twice to find an interval of width 0.025 that contains α.

(3)

(*c*) Taking 1.45 as a first approximation to α, apply the Newton-Raphson procedure once to $f(x) = x^3 - \frac{7}{x} + 2,\ x > 0$ to obtain a second approximation to α, giving your answer to 3 decimal places.

(5)

June 2010

---

4. $$f(x) = x^3 + x^2 + 44x + 150.$$

Given that $f(x) = (x + 3)(x^2 + ax + b)$, where a and b are real constants,

(a) find the value of a and the value of b.

(2)

(b) Find the three roots of $f(x) = 0$.

(4)

(c) Find the sum of the three roots of $f(x) = 0$.

(1)

June 2010

5.

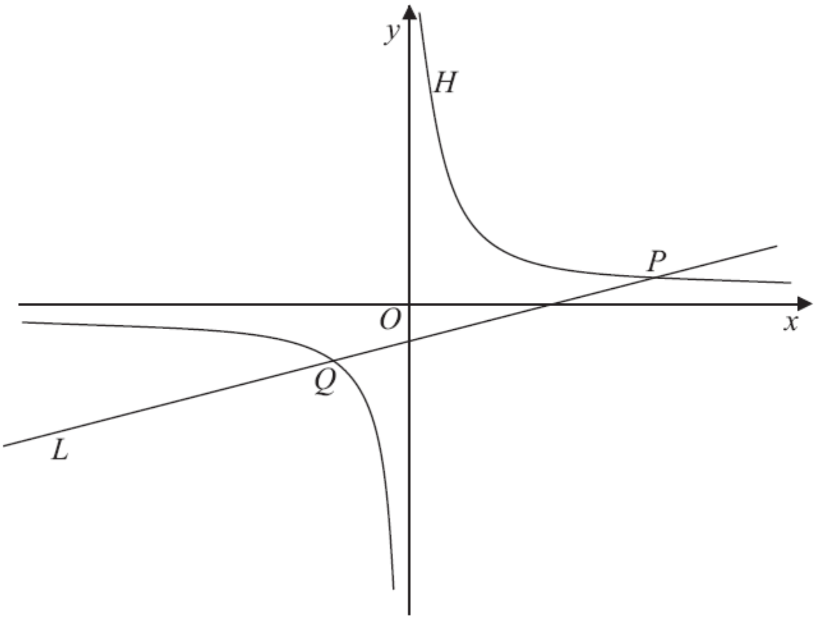

Figure 1

Figure 1 shows a rectangular hyperbola H with parametric equations

$$x = 3t, \quad y = \frac{3}{t}, \quad t \neq 0$$

The line L with equation $6y = 4x - 15$ intersects H at the point P and at the point Q as shown in Figure 1.

(a) Show that L intersects H where $4t^2 - 5t - 6 = 0$.

(3)

(b) Hence, or otherwise, find the coordinates of points P and Q.

(5)

June 2013 (R)

---

6. The parabola C has equation $y^2 = 16x$.

(a) Verify that the point $P(4t^2, 8t)$ is a general point on C.

(1)

(b) Write down the coordinates of the focus S of C.

(1)

(c) Show that the normal to C at P has equation

$$y + tx = 8t + 4t^3 .$$

(5)

The normal to C at P meets the x-axis at the point N.

(d) Find the area of triangle PSN in terms of t, giving your answer in its simplest form.

(4)

June 2009

---

7. The parabola C has equation $y^2 = 4ax$, where a is a positive constant.

The point $P(at^2, 2at)$ is a general point on C.

(a) Show that the equation of the tangent to C at $P(at^2, 2at)$ is

$$ty = x + at^2$$

(4)

The tangent to C at P meets the y-axis at a point Q.

(b) Find the coordinates of Q.

(1)

Given that the point S is the focus of C,

(c) show that PQ is perpendicular to SQ.

(3)

June 2013 (R)

---

8. The rectangular hyperbola H has equation $xy = c^2$, where c is a positive constant.

The point $P\left(ct, \frac{c}{t}\right)$, $t \neq 0$, is a general point on H.

(*a*) Show that an equation for the tangent to H at P is

$$x + t^2 y = 2ct.$$

(4)

The tangent to H at the point P meets the x-axis at the point A and the y-axis at the point B.

Given that the area of the triangle OAB, where O is the origin, is 36,

(*b*) find the exact value of c, expressing your answer in the form $k\sqrt{2}$, where k is an integer.

(4)

June 2012

---

9. (*a*) Prove by induction that

$$\sum_{r=1}^{n} r^2 = \frac{1}{6}n(n+1)(2n+1).$$

(6)

Using the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$,

(*b*) show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2 + an + b),$$

where a and b are integers to be found.

(5)

(*c*) Hence show that

$$\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}n(7n^2 + 27n + 26).$$

(3)

June 2010

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$	
	det**M** = $x(4x-11) - (3x-6)(x-2)$	M1
	$x^2 + x - 12$ (=0)	A1
	$(x+4)(x-3)$ (= 0)→ x = ...	M1
	$x = -4,\ x = 3$	A1
		[4]
2. (a)	$\mathbf{M} = \begin{pmatrix} 4 & 3 \\ 6 & 2 \end{pmatrix}$ Determinant: (8 – 18) = –10	B1
	$\mathbf{M}^{-1} = \frac{1}{-10}\begin{pmatrix} 2 & -3 \\ -6 & 4 \end{pmatrix} = \begin{pmatrix} -0.2 & 0.3 \\ 0.6 & -0.4 \end{pmatrix}$	M1 A1
		(3)
(b)	Setting Δ = 0 and using $2a^2 \pm 18 = 0$ to obtain a = .	M1
	$a = \pm 3$	A1 cao
		(2)
		[5]
3. (a)	f(1.4) = … and f(1.5) = … Evaluate both	M1
	f(1.4) = –0.256 (or $-\frac{32}{125}$), f(1.5) = 0.708… (or $\frac{17}{24}$)	A1
	Change of sign, ∴ root	
		(2)
(b)	f(1.45) = 0.221… or 0.2 [∴ root is in [1.4, 1.45]]	M1
	f(1.425) = –0.018… or –0.019 or –0.02	M1
	∴ root is in [1.425, 1.45]	A1cso
		(3)
(c)	$f'(x) = 3x^2 + 7x^{-2}$	M1 A1
	$f'(1.45) = 9.636\ldots$	A1ft
	(Special case: $f'(x) = 3x^2 + 7x^{-2} + 2$ then $f'(1.45) - 11.636\ldots$)	
	$x_1 = 1.45 - \frac{f(1.45)}{f'(1.45)} = 1.45 - \frac{0.221\ldots}{9.636\ldots} = 1.427$	M1 A1cao
		(5)
		[10]

Question Number	Scheme	Marks
4. (a)	$a = -2, \quad b = 50$	B1, B1 (2)
(b)	-3 is a root	B1
	Solving 3-term quadratic $x = \dfrac{2 \pm \sqrt{4 - 200}}{2}$ or $(x-1)^2 - 1 + 50 = 0$	M1
	$= 1 + 7\mathrm{i}, \quad 1 - 7\mathrm{i}$	A1 A1ft (4)
(c)	$(-3) + (1 + 7\mathrm{i}) + (1 - 7\mathrm{i}) = -1$	B1ft (1) **[7]**
	Ignore part labels and mark part (a) and part (b) together	
5. (a)	$H: \ x = 3t, \ y = \dfrac{3}{t}, \quad L: 6y = 4x - 15$	
	$H = L \ \Rightarrow \ 6\left(\dfrac{3}{t}\right) = 4(3t) - 15$	M1 A1
	$\Rightarrow 18 = 12t^2 - 15t \ \Rightarrow \ 12t^2 - 15t - 18 = 0$	
	$\Rightarrow 4t^2 - 5t - 6 = 0$ *	A1 **cso** (3)
(b)	$(t-2)(4t+3) \{= 0\}$	M1
	$\Rightarrow t = 2, -\frac{3}{4}$	A1
	When $t = 2$,	
	$x = 3(2) = 6, \ y = \dfrac{3}{2} \ \Rightarrow \left(6, \dfrac{3}{2}\right)$	M1
	When $t = -\dfrac{3}{4}$,	A1
	$x = 3\left(-\dfrac{3}{4}\right) = -\dfrac{9}{4}, \ y = \dfrac{3}{\left(-\frac{3}{4}\right)} = -4 \ \Rightarrow \left(-\dfrac{9}{4}, -4\right)$	A1
		(5) **[8]**

Question Number	Scheme	Marks
6. (a)	$y^2 = (8t)^2 = 64t^2$ and $16x = 16\times 4t^2 = 64t^2$	
	Or identifies that $a = 4$ and uses general coordinates $(at^2, 2at)$	B1 (1)
(b)	(4, 0)	B1 (1)
(c)	$y = 4x^{\frac{1}{2}}$ $\quad \frac{\mathrm{d}y}{\mathrm{d}x} = 2x^{-\frac{1}{2}}$	B1
	Replaces x by $4t^2$ to give **gradient** $\quad [\,2(4t^2)^{-\frac{1}{2}} = \frac{2}{2t} = \frac{1}{t}\,]$	M1,
	Uses Gradient of normal is $-\frac{1}{\text{gradient of curve}}$ $\quad [-t]$	M1
	$y - 8t = -t(x - 4t^2) \quad \Rightarrow \quad y + tx = 8t + 4t^3$ (*)	M1 A1cso (5)
(d)	At N, $y = 0$, so $x = 8 + 4t^2$ or $\frac{8t + 4t^3}{t}$	B1
	Base $SN = (8 + 4t^2) - 4 \;\; (= 4 + 4t^2)$	B1ft
	Area of $\Delta\, PSN = \frac{1}{2}(4 + 4t^2)(8t) = 16t(1 + t^2)$ or $16t + 16t^3$ for $t > 0$	M1 A1
		(4) **[11]**

Question Number	Scheme	Marks
7. (a)	$y^2 = 4ax$, at $P(at^2, 2at)$.	
	$y = 2\sqrt{a}\,x^{\frac{1}{2}} \Rightarrow \dfrac{dy}{dx} = \sqrt{a}\,x^{-\frac{1}{2}}$	M1
	or (implicitly) $2y\dfrac{dy}{dx} = 4a$	
	or (chain rule) $\dfrac{dy}{dx} = 2a \times \dfrac{1}{2at}$	
	When $x = at^2$, $\dfrac{dy}{dx} = \dfrac{\sqrt{a}}{\sqrt{at^2}} = \dfrac{\sqrt{a}}{\sqrt{a}\,t} = \dfrac{1}{t}$	A1
	or $\dfrac{dy}{dx} = \dfrac{4a}{2(2at)} = \dfrac{1}{t}$	
	T: $y - 2at = \dfrac{1}{t}(x - at^2)$	M1
	T: $ty - 2at^2 = x - at^2$	
	T: $ty = x + at^2$	A1 **cso***
		(4)
(b)	At Q, $x = 0 \Rightarrow y = \dfrac{at^2}{t} = at \Rightarrow Q(0, at)$	B1
		(1)
(c)	$S(a, 0)$	
	$\text{m}(PQ) = \dfrac{at - 2at}{0 - at^2} = \dfrac{-at}{-at^2} = \dfrac{1}{t}$	M1
	$\text{m}(SQ) = \dfrac{at - 0}{0 - a} = \dfrac{at}{-a} = -t$	A1
	$\text{m}(PQ)\times\text{m}(SQ) = \dfrac{1}{t} \times -t = -1 \Rightarrow PQ \perp SQ$	A1 **cso**
		(3)
		[8]

Question Number	Scheme	Marks
8. (a)	$xy = c^2$ at $\left(ct, \frac{c}{t}\right)$.	
	$y = \frac{c^2}{x} = c^2x^{-1} \Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x} = -c^2x^{-2} = -\frac{c^2}{x^2}$	
	$xy = c^2 \Rightarrow x\frac{\mathrm{d}y}{\mathrm{d}x} + y = 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{dy}{dt}.\frac{dt}{dx} = -\frac{c}{t^2}.\frac{1}{c}$	
	$\frac{dy}{dx} = -c^2x^{-2}$ *or* $x\frac{dy}{dx} + y = 0$ *or* $\frac{dy}{dx} = \frac{-c}{t^2}.\frac{1}{c}$ (or equivalent expressions)	A1
	So, $m_T = \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{t^2}$	
	$y - \frac{c}{t} = -\frac{1}{t^2}(x - ct)$ $(\times t^2)$	M1
	$x + t^2y = 2ct$ (Allow $t^2y + x = 2ct$)	A1 *
		(4)
(b)	$y = 0 \Rightarrow x = 2ct \Rightarrow A(2ct, 0)$.	B1
	$x = 0 \Rightarrow y = \frac{2ct}{t^2} \Rightarrow B\left(0, \frac{2c}{t}\right)$.	B1
	Area $OAB = 36 \Rightarrow \frac{1}{2}(2ct)\left(\frac{2c}{t}\right) = 36$	M1
	$\Rightarrow 2c^2 = 36 \Rightarrow c^2 = 18 \Rightarrow c = 3\sqrt{2}$	A1
		(4) **[8]**

Question Number	Scheme	Marks
9. (a)	If n = 1, $\sum_{r=1}^{n} r^2 = 1$ and $\frac{1}{6}n(n+1)(2n+1) = \frac{1}{6}\times 1\times 2\times 3 = 1$, so true for n = 1.	B1
	Assume result true for $n = k$	M1
	$\sum_{r=1}^{k+1} r^2 = \frac{1}{6}k(k+1)(2k+1) + (k+1)^2$	M1
	$= \frac{1}{6}(k+1)(2k^2+7k+6)$ or $= \frac{1}{6}(k+2)(2k^2+5k+3)$ or $= \frac{1}{6}(2k+3)(k^2+3k+2)$	A1
	$= \frac{1}{6}(k+1)(k+2)(2k+3) = \frac{1}{6}(k+1)(\{k+1\}+1)(2\{k+1\}+1)$ oe	dM1
	True for $n = k + 1$ if true for $n = k$, (and true for n = 1) so true by induction for all n.	A1cso
		(6)
(b)	$\sum_{r=1}^{n}(r^2+5r+6) = \sum_{r=1}^{n} r^2 + 5\sum_{r=1}^{n} r + \left(\sum_{r=1}^{n} 6\right)$	M1
	$\frac{1}{6}n(n+1)(2n+1) + \frac{5}{2}n(n-1)$, $+ 6n$	A1, B1
	$= \frac{1}{6}n\left[(n+1)(2n+1) + 15(n+1) + 36\right]$	M1
	$\frac{1}{6}n\left[2n^2+18n+52\right] = \frac{1}{3}n(n^2+9n+26)$ or a = 9, b = 26	A1
		(5)
(c)	$\sum_{r=n+1}^{2n}(r+2)(r+3) = \frac{1}{3}2n(4n^2+18n+26) - \frac{1}{3}n(n^2+9n+26)$	M1 A1ft
	$\frac{1}{3}n(8n^2+36n+52-n^2-9n-26) = \frac{1}{3}n(7n^2+27n+26)$ (*)	A1cso
		(3)
		[14]

Examiner reports

Question 1

The majority of candidates understood what was meant by a singular matrix and gained full marks for this question. The usual sign errors caused some to lose marks and some factorised incorrectly. There were a significant number of candidates who wrote down the solutions to their quadratic with no working shown, possibly using a graphic calculator. Some put the determinant equal to 1 and some went on to find the inverse matrix.

Question 2

Generally this was a very accessible question where the vast majority of candidates gained full marks. They had a clear understanding of the process to find the inverse matrix and were able to apply it successfully in most cases. There were some arithmetical errors in finding the determinant and some candidates could not deal with the structure of the inverse matrix. A few did not substitute the given value for a. A much longer method involving simultaneous equations was much more prone to errors and fortunately not seen very often. In part (b) most understood the definition of a singular matrix and were able to solve their quadratic equation to give two accurate values for a. Some rejected the negative solution however, losing the last mark.

Question 3

Part (a) was straightforward and generally well done. Very few errors were made in the numerical evaluations (we do need to see these). There were, however, a minority of candidates who did not give the required conclusion, which ideally requires the sign change to be noted and a statement made of the interval for the root. In part (b) a few candidates attempted linear interpolation, maybe indicating a lack of practice with interval bisection. The numerical evaluations of f(1.45) and f(1.425), which are required, were well done by the majority of candidates. A noticeable number, however, did not produce a correctly stated conclusion – commonly no statement at all or just a single *x* value. Candidates should be made aware that, if the interval notation is used, the smaller number should be first – in this case [1.425, 1.45]. Part (c) was generally very well done with the vast majority knowing the Newton-Raphson iteration. The most common error was not differentiating the constant (+2). A few had problems differentiating $\frac{1}{x}$ and a small number continued beyond one iteration.

Candidates should be encouraged to evaluate and write down intermediate values in their working, so that if a slip is made the examiner can see where. Failure to do so will have lost a mark here for some candidates.

Candidates should also be encouraged to check that their answers throughout a question are consistent.

Question 4

In part (a) long division and comparing coefficients were each used to good effect and errors were rare. Part (b) resulted in many good answers. Some however felt that 3, and even $x + 3$, was a root and others omitted the real root completely. Some confused roots with factors. It was disappointing at this level to see many candidates failing to solve a quadratic correctly. Candidates should be advised to quote the quadratic formula before using it to ensure that they earn the method mark. Completion of the square was often more successful in this question than use of the formula. In part (c) some included an x in their answer and others found a product instead of a sum. The vast majority earned this follow through mark however.

Question 5

Most candidates were able to make correct substitutions in part (a) and rearrange to obtain the correct quadratic in t. Some candidates obtained equations in x or y first, but usually worked through to the given answer with valid working. The majority of candidates reached the desired formula by convincing methods and only small mathematical slips led to them dropping the final accuracy mark. In part (b) the responses typically showed valid attempts to solve the quadratic to obtain correct coordinates.

Question 6

In part (a) candidates either substituted $x = 4t^2$ and $y = 8t$ into the equation $y^2 = 16x$, thereby verifying that the point lies on the parabola, or they compared the coordinates of P with the general point $(at^2, 2at)$ on the general parabola and identified that $a = 4$.

Most wrote down the correct focus as coordinates in part (b).

Part (c) required candidates to show that the equation of the normal to the parabola at the point P was $y + tx = 8t + 4t^3$. Some seemed to have learned the gradient of a parabola is $1/t$ and began with this result. This does not constitute a complete proof however and the gradient should have been established by differentiation.

Part (d) was found challenging by some and a simple sketch would have helped. Typical problems included making N the point where the normal crossed the y-axis, and finding the area of the wrong triangle OPN. The simplest method for finding the area was to use half base times height for the whole triangle but there were a variety of methods presented which usually led to the correct answer. It was common to divide the triangle into two right-angled triangles, for example. Others used Pythagoras to determine other lengths and used an indirect method, and some used determinant methods to find the area. Those who understood and correctly executed their method frequently made algebraic slips simplifying the final answer, and the final mark was often lost.

Question 7

Many of the candidates were well rehearsed for this question and the majority were able to show that the equation of the tangent was as stated in part (a) and in part (b) the vast majority of answers were correct. In part (c) many candidates found the gradient of PQ again, but some candidates just quoted the equation. This part of the question was usually correct, however some of the responses lacked a conclusion.

Question 8

Part (a) was answered well by the majority of candidates. They were very few cases where the gradient was quoted rather than showing a full calculus method.

Part (b) was more challenging and a diagram was useful for some candidates. Those with correct coordinates for *A* and *B* could often proceed to a correct value for *c* although some left the answer as $\pm 3\sqrt{2}$. Some candidates did not appreciate what was required and failed to find the points *A* and *B* and simply used the *x* and *y* coordinates of *P* for the dimensions of the triangle.

Question 9

Showing the result in part (a) was fairly standard bookwork, and many answers were therefore surprisingly disappointing. The solution to a proof by induction question should be complete and precise. Often far too much was left to the examiner's imagination.

At the first sep when $n = 1$, we expect to see both sides evaluated and shown to be equal, followed by a conclusion – usually "true for $n = 1$". In this case there was often no reference to the LHS and there was a lack of conclusion.

"Assume true for $n = k$ " is an essential step and should always be there in a proof by induction. This was often reduced to simply "$n = k$", which is not enough.

Adding the next term to check for $n = k + 1$ was generally clear but a few added 1^2 or just $(k + 1)$ instead of $(k+1)^2$. There nearly always is a common factor at this stage, which makes the algebra fairly straightforward. Too many candidates expanded completely to find a cubic, which not all were able to factorise convincingly, many moving straight from a cubic to three linear factors in one step.

A candidate then needs to show clearly that they have the required result with $(k + 1)$ in place of *n*. A large number did not satisfactorily show this, just leaving the expression as $\frac{1}{6}(k + 1)(k + 2)(2k + 3)$ without any reference to substituting $k + 1$.

A final conclusion is always required and should state that the result "by induction is true for all positive integers *n*". This conclusion was completely missing at times and incomplete on others.

In contrast part (b) was generally well done. A fairly common error at the beginning was to use 6 instead of $\sum 6$, causing an error of a factor of *n*. Very few candidates failed to expand $(r + 2)(r + 3)$ correctly.

Many candidates made errors when taking $\frac{n}{6}$ or $\frac{n}{3}$ out as a factor – most commonly leaving an n in the last term. This lack of care did create problems in part (c).

In part (c) a majority of candidates were successful but quite a few problems were evident. Some candidates chose to ignore the word "hence" which condemned them straight away. Others only used the result from (b) for the sum from 1 to *n*, failing to see that they could also use it for the sum from 1 to $2n$ – thus creating a lot of extra work for themselves. The result for part (c) was given and a number of candidates missed out essential working on the way to this solution. Many candidates having obtained the wrong answer to (b) were convinced they had found the given solution. A small number having failed to get this answer correct went back and corrected their working in (b) – which is what we would hope to see. 30% of the candidates scored full marks on this question indicating the quality of the candidature on this paper.

Statistics for FP1 Practice Paper Bronze Level B2

				Mean score for students achieving grade:							
Qu	**Max Score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4	4	90	3.58	3.96	3.90	3.74	3.55	3.27	3.03	1.89
2	5		88	4.39	4.93	4.81	4.53	4.26	3.99	3.45	2.48
3	10		89	8.90	9.71	9.52	9.04	8.73	8.45	7.66	6.13
4	7		92	6.44	6.95	6.82	6.65	6.39	6.19	5.66	4.44
5	8		95	7.61	8.00	7.90	7.90	7.63	7.98	7.35	6.29
6	11		80	8.80		10.15	8.91	8.27	7.28	5.77	3.80
7	8		87	6.99	7.94	7.91	7.57	6.78	6.18	6.09	4.04
8	8		81	6.46	7.88	7.62	6.99	6.13	4.87	3.60	2.15
9	14		72	10.06	13.28	12.39	10.44	8.65	6.95	5.59	3.10
	75		**84**	**63.23**		**71.02**	**65.77**	**60.39**	**55.16**	**48.20**	**34.32**