Paper Reference(s)

6665/01

Edexcel GCE

Core Mathematics C3
Bronze Level B4

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
72	66	59	51	45	40

1. Express

$$\frac{2(3x+2)}{9x^2-4} - \frac{2}{3x+1}$$

as a single fraction in its simplest form.

(4)

June 2012

2.

$$g(x) = e^{x-1} + x - 6$$

(*a*) Show that the equation $g(x) = 0$ can be written as

$$x = \ln(6 - x) + 1, \qquad x < 6.$$

(2)

The root of $g(x) = 0$ is α.

The iterative formula

$$x_{n+1} = \ln(6 - x_n) + 1, \qquad x_0 = 2.$$

is used to find an approximate value for α.

(*b*) Calculate the values of x_1, x_2 and x_3 to 4 decimal places.

(3)

(*c*) By choosing a suitable interval, show that $\alpha = 2.307$ correct to 3 decimal places.

(3)

January 2013

3.

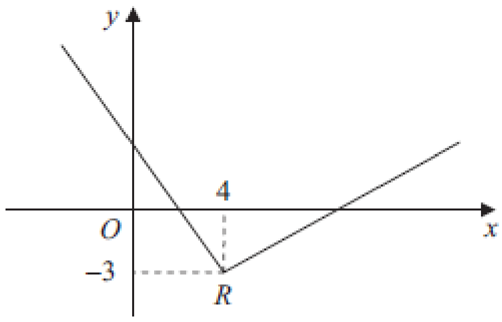

Figure 1

Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R\ (4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x + 4)$,

(3)

(b) $y = |\mathrm{f}(-x)|$.

(3)

On each diagram, show the coordinates of the point corresponding to R.

June 2011

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4. (i) Differentiate with respect to x

(a) $x^2 \cos 3x$,

(3)

(b) $\dfrac{\ln(x^2+1)}{x^2+1}$.

(4)

(ii) A curve C has the equation

$$y = \sqrt{(4x+1)}, \quad x > -\tfrac{1}{4}, \quad y > 0.$$

The point P on the curve has x-coordinate 2. Find an equation of the tangent to C at P in the form $ax + by + c = 0$, where a, b and c are integers.

(6)

June 2009

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5. (*a*) Given that $\sin^2 \theta + \cos^2 \theta \equiv 1$, show that $1 + \cot^2 \theta \equiv \operatorname{cosec}^2 \theta$.

(2)

(*b*) Solve, for $0 \leq \theta < 180°$, the equation

$$2 \cot^2 \theta - 9 \operatorname{cosec} \theta = 3,$$

giving your answers to 1 decimal place.

(6)

June 2008

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6. (*a*) Express $3 \sin x + 2 \cos x$ in the form $R \sin (x + \alpha)$ where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

(4)

(*b*) Hence find the greatest value of $(3 \sin x + 2 \cos x)^4$.

(2)

(*c*) Solve, for $0 < x < 2\pi$, the equation

$$3 \sin x + 2 \cos x = 1,$$

giving your answers to 3 decimal places.

(5)

June 2007

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7. The function f is defined by

$$\mathrm{f} : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, \ x > \frac{1}{2}.$$

(*a*) Show that $\mathrm{f}(x) = \frac{1}{2x-1}$.

(4)

(*b*) Find $\mathrm{f}^{-1}(x)$.

(3)

(*c*) Find the domain of f^{-1}.

(1)

$$\mathrm{g}(x) = \ln (x + 1).$$

(*d*) Find the solution of $\mathrm{fg}(x) = \frac{1}{7}$, giving your answer in terms of e.

(4)

January 2012

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8. The functions f and g are defined by

$$\mathrm{f} : x \mapsto 1 - 2x^3, \quad x \in \mathbb{R}.$$

$$\mathrm{g} : x \mapsto \frac{3}{x} - 4, \quad x > 0, \ x \in \mathbb{R}.$$

(*a*) Find the inverse function f^{-1}.

(2)

(*b*) Show that the composite function gf is

$$\mathrm{gf} : x \mapsto \frac{8x^3 - 1}{1 - 2x^3}.$$

(4)

(*c*) Solve gf $(x) = 0$.

(2)

(*d*) Use calculus to find the coordinates of the stationary point on the graph of $y = \mathrm{gf}(x)$.

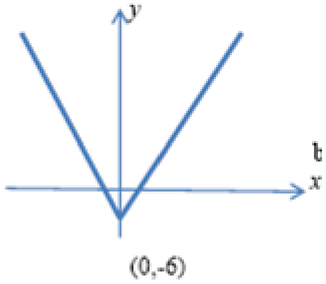
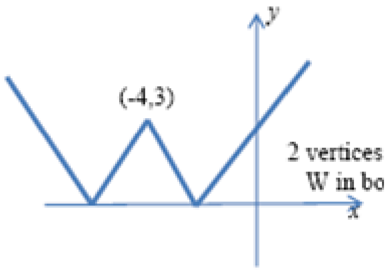
(5)

January 2008

TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1.	$9x^2-4=(3x-2)(3x+2)$ At any stage	B1
	Eliminating the common factor of (3x + 2) at any stage $\dfrac{2\cancel{(3x+2)}}{(3x-2)\cancel{(3x+2)}}=\dfrac{2}{3x-2}$	B1
	Use of a common denominator $\dfrac{2(3x+2)(3x+1)}{(9x^2-4)(3x+1)}-\dfrac{2(9x^2-4)}{(9x^2-4)(3x+1)}$ or $\dfrac{2(3x+1)}{(3x-2)(3x+1)}-\dfrac{2(3x-2)}{(3x+1)(3x-2)}$	M1
	$\dfrac{6}{(3x-2)(3x+1)}$ or $\dfrac{6}{9x^2-3x-2}$	A1 **[4]**
2. (a)	$0=e^{x-1}+x-6\Rightarrow x=\ln(6-x)+1$	M1A1* (2)
(b)	Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1\Rightarrow x_1=2.3863$ AWRT 4 dp. $x_2=2.2847$ $x_3=2.3125$	M1, A1 A1 (3)
(c)	Chooses interval [2.3065,2.3075]	M1
	g(2.3065)=-0.0002(7), g(2.3075)=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1 (3) **[8]**

Question Number	Scheme	Marks
3. (a)	 V shape	B1
	Vertex on *y* axis & both branches of graph cross *x* axis	B1
	'*y*' coordinate of R is –6	B1
		(3)
(b)	 W shape	B1
	2 vertices on the negative *x* axis. W in both quad 1 & quad 2	B1 dep
	R' = (–4, 3)	B1
		(3)
		[6]

Question Number	Scheme	Marks
4. (i)(a)	$y = x^2\cos 3x$	
	Apply product rule: $\begin{cases} u = x^2 & v = \cos 3x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = 2x & \frac{\mathrm{d}v}{\mathrm{d}x} = -3\sin 3x \end{cases}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x\cos 3x - 3x^2\sin 3x$	A1A1 (3)
(i)(b)	$y = \frac{\ln(x^2+1)}{x^2+1}$	
	$u = \ln(x^2+1) \Rightarrow \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2x}{x^2+1}$	M1A1
	Apply quotient rule: $\begin{cases} u = \ln(x^2+1) & v = x^2+1 \\ \frac{\mathrm{d}u}{\mathrm{d}x} = \frac{2x}{x^2+1} & \frac{\mathrm{d}v}{\mathrm{d}x} = 2x \end{cases}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\left(\frac{2x}{x^2+1}\right)\left(x^2+1\right) - 2x\ln(x^2+1)}{\left(x^2+1\right)^2}$	A1
(ii)	$y = \sqrt{4x+1},\ x > -\frac{1}{4}$	
	At P, $y = \sqrt{4(2)+1} = \underline{\sqrt{9}} = \underline{3}$	B1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \underline{\frac{1}{2}(4x+1)^{-\frac{1}{2}}(4)}$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4x+1)^{\frac{1}{2}}}$	A1
	At P, $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2}{(4(2)+1)^{\frac{1}{2}}}$	M1
	Hence m(**T**) = $\frac{2}{3}$	
	Either **T**: $y - 3 = \frac{2}{3}(x-2)$;	M1
	T: $3y - 9 = 2(x-2)$;	
	T: $3y - 9 = 2x - 4$	
	T: $\underline{2x - 3y + 5 = 0}$	A1 (6)
		[13]

Question Number	Scheme	Marks
5. (a)	$\sin^2\theta+\cos^2\theta=1$	
	$\div\sin^2\theta \qquad \dfrac{\sin^2\theta}{\sin^2\theta}+\dfrac{\cos^2\theta}{\sin^2\theta}=\dfrac{1}{\sin^2\theta}$	M1
	$1+\cot^2\theta=\text{cosec}^2\theta$ ∗	A1 cso (2)
(b)	$2\left(\text{cosec}^2\theta-1\right)-9\,\text{cosec}\,\theta=3$	M1
	$2\,\text{cosec}^2\theta-9\,\text{cosec}\,\theta-5=0$ *or* $5\sin^2\theta+9\sin\theta-2=0$	M1
	$\left(2\,\text{cosec}\,\theta+1\right)\left(\text{cosec}\,\theta-5\right)=0$ *or* $\left(5\sin\theta-1\right)\left(\sin\theta+2\right)=0$	M1
	$\text{cosec}\,\theta=5$ *or* $\sin\theta=\dfrac{1}{5}$	A1
	$\theta=11.5°, 168.5°$	A1A1 (6)
		[8]
6. (a)	Complete method for R: e.g. $R\cos\alpha=3$, $R\sin\alpha=2$, $R=\sqrt{(3^2+2^2)}$	M1
	$R=\sqrt{13}$ or 3.61 (or more accurate)	A1
	Complete method for $\tan\alpha=\dfrac{2}{3}$ [Allow $\tan\alpha=\dfrac{3}{2}$]	M1
	α = 0.588 (Allow 33.7°)	A1 (4)
(b)	Greatest value = $\left(\sqrt{13}\right)^4$ = 169	M1, A1 (2)
(c)	$\sin(x+0.588)=\dfrac{1}{\sqrt{13}}$ (= 0.27735…) sin(x + their α) = $\frac{1}{\text{their } R}$	M1
	(x + 0.588) = 0.281(03…) or 16.1°	A1
	(x + 0.588) = π – 0.28103…	M1
	Must be π – their 0.281 or 180° – their 16.1°	
	or (x + 0.588) = 2π + 0.28103…	M1
	Must be 2π + their 0.281 or 360° + their 16.1°	
	x = 2.273 or x = 5.976 (awrt) Both (radians only)	A1 (5)
	If 0.281 or 16.1° not seen, correct answers imply this A mark	**[11]**

Question Number	Scheme	Marks
7. (a)	$2x^2 + 7x - 4 = (2x-1)(x+4)$	B1
	$\dfrac{3(x+1)}{(2x-1)(x+4)} - \dfrac{1}{(x+4)} = \dfrac{3(x+1)-(2x-1)}{(2x-1)(x+4)}$	M1
	$= \dfrac{x+4}{(2x-1)(x+4)}$	M1
	$= \dfrac{1}{2x-1}$	A1* (4)
(b)	$y = 1/(2x-1)$ ($y(2x-1) = 1$ ($2xy - y = 1$	
	$x = \dfrac{1+y}{2y}$	M1 M1
	$y \; OR \; f^{-1}(x) = \dfrac{1+x}{2x}$	A1 (3)
(c)	$x > 0$	B1 (1)
(d)	$\dfrac{1}{2\ln(x+1) - 1} = \dfrac{1}{7}$	M1
	$\ln(x+1) = 4$	A1
	$x = e^4 - 1$	M1 A1 (4) **[12]**

Question Number	Scheme	Marks
8.	(a) $x = 1 - 2y^3 \Rightarrow y = \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ *or* $\sqrt[3]{\frac{1-x}{2}}$	M1 A1 (2)
	$f^{-1} : x \mapsto \left(\frac{1-x}{2}\right)^{\frac{1}{3}}$ Ignore domain	
	(b) $gf(x) = \frac{3}{1-2x^3} - 4$	M1 A1
	$= \frac{3-4(1-2x^3)}{1-2x^3}$	M1
	$= \frac{8x^3-1}{1-2x^3}$ ∗ cso	A1 (4)
	$gf : x \mapsto \frac{8x^3-1}{1-2x^3}$ Ignore domain	
	(c) $8x^3 - 1 = 0$ Attempting solution of numerator = 0	M1
	$x = \frac{1}{2}$ Correct answer and no additional answers	A1 (2)
	(d) $\frac{dy}{dx} = \frac{(1-2x^3)\times 24x^2 + (8x^3-1)\times 6x^2}{(1-2x^3)^2}$	M1 A1
	$= \frac{18x^2}{(1-2x^3)^2}$	A1
	Solving their numerator = 0 and substituting to find *y*.	M1
	$x = 0,\ y = -1$	A1 (5) **[13]**

Examiner reports

Question 1

This question provided most candidates with a confidence boosting start to the paper. The majority of candidates scoring either full marks, or losing just a single mark as a result of incorrectly expanding the brackets.

For example the error $\dfrac{2(3x+1)-2(3x-2)}{(3x-2)(3x+1)}=\dfrac{-2}{(3x-2)(3x+1)}$ was commonplace.

The easiest and most direct way to achieve the correct answer involved writing $(9x^2 - 4)$ as $(3x + 2)(3x - 2)$, cancelling the common factor of $(3x + 2)$ and then adding two linear fractions. However, those who did not recognise the difference of two squares in the denominator, resulting in not being able to cancel, only rarely coped with the complicated algebra needed to achieve an answer. Frequently these candidates produced lengthy, but unsuccessful solutions and only scored the single mark for an attempt to combine two fractions. A few candidates, having factorised the $(9x^2 - 4)$ term, retained all 3 linear terms as the common denominator. Most were able to cancel later on in their solution and hence score all of marks.

Question 2

Many candidates achieved full marks on this question.

In Q2(a) some candidates did not set $g(x) = 0$ at the start. Candidates who set $g(x) = 0$ generally proceeded correctly to the required solution. The ‘ln’ work was mostly well done, and very few made errors such as $\ln 6 - \ln x = \ln(6 - x)$. Very few candidates started with the given expression, and attempted to work backwards to $g(x) = 0$. A small number of candidates failed to put the correct brackets around their ‘ln’ work.

In Q2(b) the majority of candidates gained full marks. There were very few errors, with marks being generally lost for incorrect rounding or rounding to an incorrect number of decimal places.

In Q2(c) almost all candidates chose a suitable interval, usually [2.3065, 2.3075], and proceeded to the correct answer. Errors were seen where candidates substituted incorrectly, into the wrong function, or omitted the $\times 10^{-3}$ or $\times 10^{-4}$ when expressing answers in standard form. A smaller number of candidates tried further iterations. Candidates mostly provided appropriate reasons and minimal conclusions, although a few failed to do this, omitting to mention “change in sign” or “hence root” or equivalent.

Question 3

This question tested the candidates’ ability in transforming graphs. In part (a), the majority of candidates achieved all three marks. A few candidates applied only one of the two transformations resulting in R at (0, –3) or (4, –6). Others applied a scale factor of $\frac{1}{2}$ instead of 2 resulting in R at (0, –1.5). Both branches of the V were required to cross the x-axis and a few candidates lost a mark because of this. Wrong notation for (0, –6) was a little too frequently seen, but the position on the graph often clarified the students’ intention.

Part (b) was more demanding. Candidates were generally conversant with the modulus function and sketched a ‘W’ shape. The most common mistake was to have the W shifted to the right so that at least one of the vertices was on the positive x-axis. Some candidates gave incorrect coordinates even though their drawing suggested that their R was correct. Even when the diagram was correct, R was at times seen labeled as (–3, 4), (3, –4) or (4, 3).

Question 4

Part (i)(a) was well answered by the majority of candidates. The most common error was incorrectly differentiating $\cos 3x$ to either $3\sin 3x$ or $-\sin 3x$. A few candidates lost the final accuracy mark for simplification errors such as simplifying $(\cos 3x)(2x)$ to $\cos 6x^2$.

In (i)(b), the quotient rule was generally well applied in most candidates' working. A significant number of candidates, however, struggled to differentiate $\ln(x^2+1)$ correctly. $\frac{1}{x^2+1}$, $\frac{2}{x^2+1}$ or even $\frac{1}{x}$ were common incorrect outcomes. Those candidates who decided to use the product rule in this part were less successful in gaining some or all of the marks.

Again part (ii) was generally well attempted by candidates of all abilities. The most common error was incorrectly differentiating $\sqrt{(4x+1)}$ although a few candidates failed to attempt to differentiate this. A few candidates found the equation of the normal and usually lost the final two marks. Also, a number of candidates failed to write the equation of the tangent in the correct form and so lost the final accuracy mark.

Question 5

This was the best done question on the paper and full marks were very common. There were a great many concise solutions to part (a), the great majority started from $\sin^2\theta + \cos^2\theta = 1$ and divided all the terms by $\sin^2\theta$. Those who started from $1 + \cot^2\theta$ often produced longer solutions but both marks were usually gained. The majority of candidates saw the connection between parts (a) and (b) and usually obtained both correct solutions. Candidates who substituted $\tan\theta = \frac{\sin\theta}{\cos\theta}$ could achieve the same results by a longer method but sometimes made errors in multiplying their equation by $\sin^2\theta$. A significant number who used a quadratic in cosec θ obtained cosec $\theta = 5$ and got no further, seemingly deciding that it was not possible to solve this.

Question 6

Although some candidates had no idea how to proceed, most candidates were able to gain some credit in part (a). Usually finding R was not a problem, but mistakes such as $\cos\alpha = 3$ and $\sin\alpha = 2$, and $\tan\alpha = \frac{3}{2}$ were reasonably common and lost the mark for α.

Good candidates realised what was required for part (b), but in general this was poorly answered.

In the final part, again there were many complete and concise answers but marks (often the third M mark, and consequently the final A mark) were lost for not realising that there was a second solution. Many candidates worked in degrees throughout, which was fine if the final answers were then converted to radians, but often, in this case, the final mark was lost. There were very few approaches other than the one in the main mark scheme, and any there were had limited success.

Question 7

Part (a): This was done very well by the majority of candidates. Most candidates were able to factorise the quadratic and hence find the common denominator of $(2x-1)(x+4)$ without resorting to cubic denominators. The most common error was the ‘invisible bracket’ leading to a numerator of $x + 2$ instead of $x + 4$. When they made this mistake candidates were tempted to fudge their working in order to reach the required result.

Part (b): Most candidates did this part of the question quite well with only the occasional sign slip in evidence. Some candidates misread the question and attempted to differentiate to find $f'(x)$, or found $\dfrac{1}{f(x)}$.

A few candidates failed to write their answer in terms of x and therefore lost the final mark. Others made errors when dividing their algebraic fraction by 2.

Part (c): Very few candidates scored this mark. Answers such as $x \neq 0$, $x > \dfrac{1}{2}$, $x \neq \dfrac{1}{2}$ were frequently seen. There seemed to be little awareness that the restriction on the domain of $f(x)$ has an effect on the domain of $f^{-1}(x)$.

Part (d): Generally an attempt at fg was put equal to $\dfrac{1}{7}$ although the positioning of the brackets and ‘–1’ was often incorrect. A few candidates attempted to calculate x leading from $gf(x) = \dfrac{1}{7}$ whilst others substituted $\dfrac{1}{7}$ for x in $fg(x)$. After that initial step there were frequent arithmetic or sign errors in reaching $\ln(x + 1) =$ constant. Most were able to follow on with correct ln work to produce an answer for x in terms of e, occasionally again with sign errors.

Question 8

Part (a) was generally well done although a few made errors in changing the subject and some left their answers in terms of y. Part (b) was also well done. Very few attempted to combine the functions in the wrong order and most were able to perform the necessary algebra to obtain the printed algebra. In both parts (c) and (d), it was disturbing to see a relatively high proportion of candidates proceeding from $\dfrac{\text{numerator}}{\text{denominator}} = 0$ numerator = denominator. This usually lost 4 marks in these parts. If this error was avoided, part (c) was usually well done, although additional spurious answers, such as $-\dfrac{1}{2}$ or answers deriving from $1 - 2x^3 = 0$, were sometimes seen.

Statistics for C3 Practice Paper Bronze Level B4

				Mean score for students achieving grade:							
Qu	**Max score**	**Modal score**	**Mean %**	**ALL**	**A***	**A**	**B**	**C**	**D**	**E**	**U**
1	4		83	3.33	3.92	3.71	3.45	3.16	2.85	2.48	1.84
2	8		89	7.09	7.81	7.60	7.28	6.88	6.30	5.54	3.70
3	6		82	4.94	5.83	5.50	5.12	4.70	4.27	3.66	2.91
4	13		78	10.17		12.26	11.10	9.78	7.70	5.64	2.58
5	8		77	6.12		7.65	6.98	5.84	4.23	2.47	1.06
6	11		62	6.84		9.40	7.47	5.74	3.99	2.44	0.99
7	12		79	9.48	11.59	10.84	10.12	9.55	8.62	7.86	4.94
8	13		72	9.41		12.29	10.90	9.91	8.63	7.13	5.25
	75		**77**	**57.38**		**69.25**	**62.42**	**55.56**	**46.59**	**37.22**	**23.27**