6665/01

Edexcel GCE

Core Mathematics C3
Bronze Level B2

Time: 1 hour 30 minutes

Materials required for examination papers	**Items included with question**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

Write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Core Mathematics C3), the paper reference (6665), your surname, initials and signature.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper. The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner. Answers without working may gain no credit.

Suggested grade boundaries for this paper:

A*	A	B	C	D	E
72	66	60	53	48	41

1.   Differentiate with respect to x

   (*a*) $\ln (x^2 + 3x + 5)$,

   (2)

   (*b*) $\dfrac{\cos x}{x^2}$.

   (3)

   June 2011

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2. $f(x) = \dfrac{2x+3}{x+2} - \dfrac{9+2x}{2x^2+3x-2}, \quad x > \tfrac{1}{2}.$

   (*a*) Show that $f(x) = \dfrac{4x-6}{2x-1}$.

   (7)

   (*b*) Hence, or otherwise, find $f'(x)$ in its simplest form.

   (3)

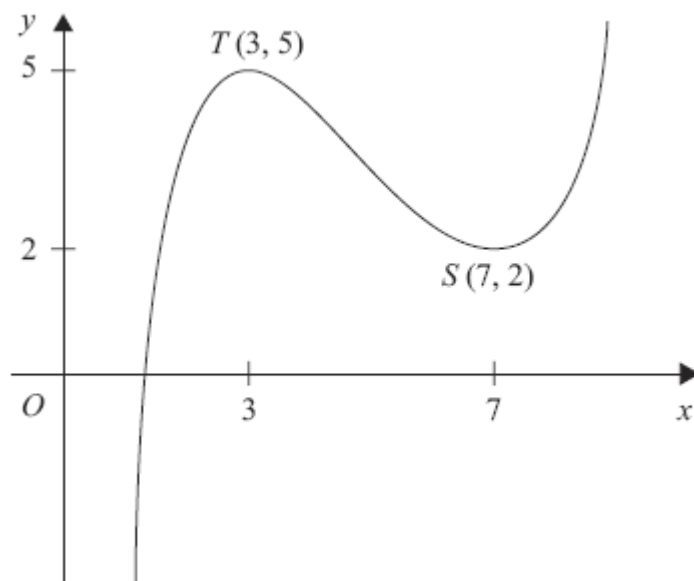   **June 2007**

3.

Figure 1

Figure 1 shows the graph of $y = f(x)$, $1 < x < 9$.

The points $T(3, 5)$ and $S(7, 2)$ are turning points on the graph.

Sketch, on separate diagrams, the graphs of

(a) $y = 2f(x) - 4$,

(3)

(b) $y = |f(x)|$.

(3)

Indicate on each diagram the coordinates of any turning points on your sketch.
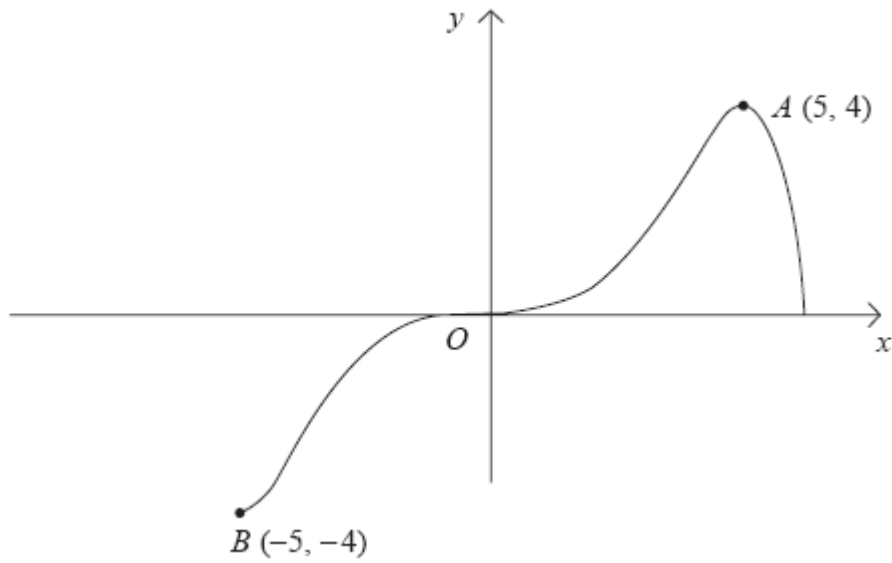
January 2009

4.

Figure 2

Figure 2 shows a sketch of the curve with equation $y = f(x)$.

The curve passes through the origin O and the points $A(5, 4)$ and $B(-5, -4)$.

In separate diagrams, sketch the graph with equation

(a) $y = |f(x)|$,

(3)

(b) $y = f(|x|)$,

(3)

(c) $y = 2f(x + 1)$.

(4)

On each sketch, show the coordinates of the points corresponding to A and B.

January 2008

5. The radioactive decay of a substance is given by

$$R = 1000e^{-ct}, \quad t \geq 0.$$

where R is the number of atoms at time t years and c is a positive constant.

(a) Find the number of atoms when the substance started to decay.

(1)

It takes 5730 years for half of the substance to decay.

(b) Find the value of c to 3 significant figures.

(4)

(c) Calculate the number of atoms that will be left when $t = 22\,920$.

(2)

(d) Sketch the graph of R against t.

(2)

January 2008

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6. (i) Use an appropriate double angle formula to show that

$$\operatorname{cosec} 2x = \lambda \operatorname{cosec} x \sec x,$$

and state the value of the constant λ.

(3)

(ii) Solve, for $0 \leq \theta < 2\pi$, the equation

$$3\sec^2 \theta + 3 \sec \theta = 2 \tan^2 \theta$$

You must show all your working. Give your answers in terms of π.

(6)

June 2013 (R)

7.
$$\mathrm{f}(x) = \frac{4x - 5}{(2x + 1)(x - 3)} - \frac{2x}{x^2 - 9}, \qquad x \neq \pm 3, \ x \neq -\frac{1}{2}.$$

(a) Show that

$$\mathrm{f}(x) = \frac{5}{(2x + 1)(x + 3)}.$$

(5)

The curve C has equation $y = \mathrm{f}(x)$. The point $P\left(-1, -\dfrac{5}{2}\right)$ lies on C.

(b) Find an equation of the normal to C at P.
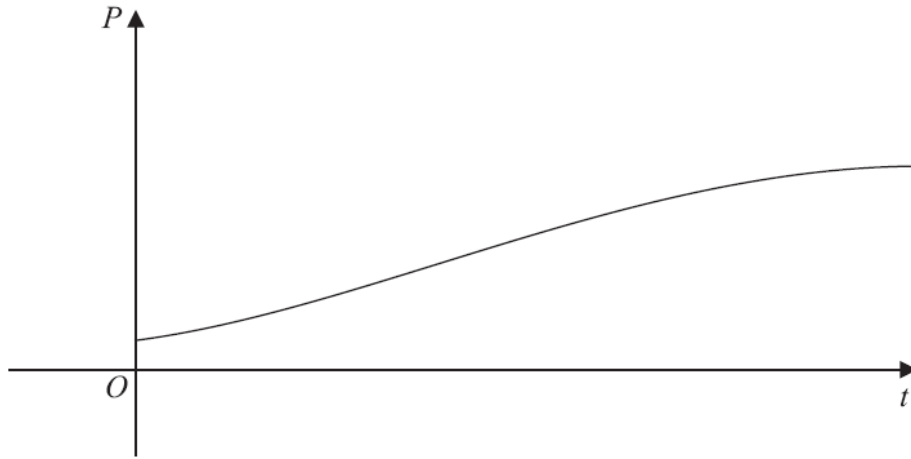
(8)

June 2011

8.

Figure 3

The population of a town is being studied. The population P, at time t years from the start of the study, is assumed to be

$$P = \frac{8000}{1 + 7e^{-kt}}, \qquad t \geq 0$$

where k is a positive constant.

The graph of P against t is shown in Figure 3.

Use the given equation to

(*a*) find the population at the start of the study,

(2)

(*b*) find a value for the expected upper limit of the population.

(1)

Given also that the population reaches 2500 at 3 years from the start of the study,

(*c*) calculate the value of k to 3 decimal places.

(5)

Using this value for k,

(*d*) find the population at 10 years from the start of the study, giving your answer to 3 significant figures.
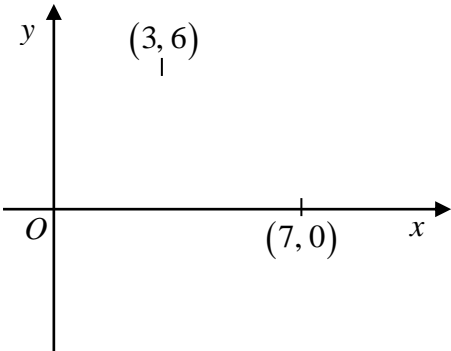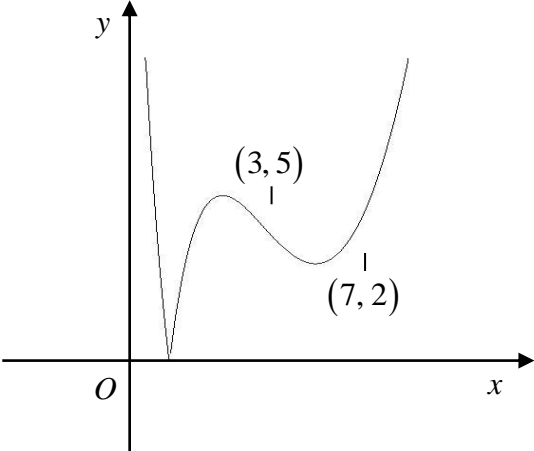
(2)

(*e*) Find, using $\dfrac{\mathrm{d}P}{\mathrm{d}t}$, the rate at which the population is growing at 10 years from the start of the study.

(3)
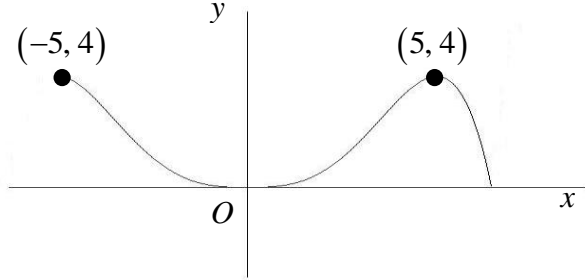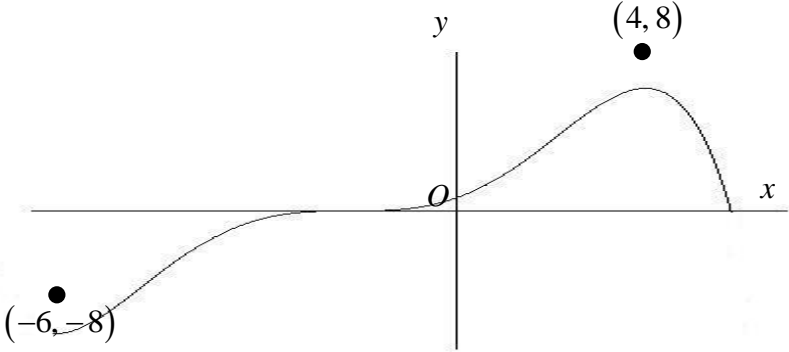
June 2013 (R)
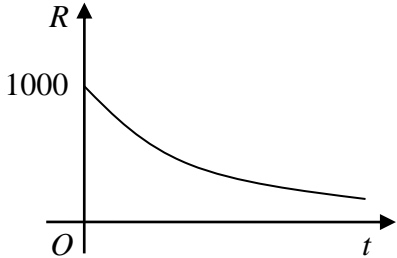
TOTAL FOR PAPER: 75 MARKS

END

Question Number	Scheme	Marks
1. (a)	$\dfrac{1}{(x^2+3x+5)} \times \ldots = \dfrac{2x+3}{(x^2+3x+5)}$	M1, A1 (2)
(b)	Applying $\dfrac{vu'-uv'}{v^2}$ $\dfrac{x^2 \times -\sin x - \cos x \times 2x}{\left(x^2\right)^2} = \dfrac{-x^2\sin x - 2x\cos x}{x^4} = \dfrac{-x\sin x - 2\cos x}{x^3}$ oe	M1 A2,1,0 (3) **[5]**
2. (a)	$2x^2 + 3x - 2 = (2x-1)(x+2)$ at any stage $f(x) = \dfrac{(2x+3)(2x-1)-(9+2x)}{(2x-1)(x+2)}$ f.t. on error in denominator factors (need not be single fraction) Simplifying numerator to quadratic form Correct **numerator** $= \dfrac{4x^2+2x-12}{[(2x-1)(x+2)]}$ Factorising numerator, **with** a denominator $= \dfrac{2(2x-3)(x+2)}{(2x-1)(x+2)}$ o.e. $= \dfrac{4x-6}{2x-1}$ (✱)	B1 M1, A1√ M1 A1 M1 A1 cso (7)
(b)	Complete method for $f'(x)$; e.g $f'(x) = \dfrac{(2x-1)\times 4-(4x-6)\times 2}{(2x-1)^2}$ o.e $= \dfrac{8}{(2x-1)^2}$ or $8(2x-1)^{-2}$ Not treating f^{-1} (for f') as misread	M1 A1 A1 (3) **[10]**

Question Number	Scheme		Marks
3 (a)		Shape $(3,6)$ $(7,0)$	B1 B1 B1 (3)
(b)		Shape $(3,5)$ $(7,2)$	B1 B1 B1 (3)
			[6]

8

Question Number	Scheme	Marks
4.	(a) y $(-5, 4)$ $(5, 4)$ O x Shape $(5, 4)$ $(-5, 4)$	 B1 B1 B1 (3)
	(b) For the purpose of marking this paper, the graph is identical to (a) Shape $(5, 4)$ $(-5, 4)$	 B1 B1 B1 (3)
	(c) y $(4, 8)$ O x $(-6, -8)$ General shape – unchanged Translation to left $(4, 8)$ $(-6, -8)$	 B1 B1 B1 B1 (4)
	In all parts of this question ignore any drawing outside the domains shown in the diagrams above.	**[10]**

Question Number	Scheme	Marks
5. (a)	1000	B1 (1)
(b)	$1000\,\mathrm{e}^{-5730c} = 500$	M1
	$\mathrm{e}^{-5730c} = \dfrac{1}{2}$	A1
	$-5730c = \ln\dfrac{1}{2}$	M1
	$c = 0.000121$ cao	A1 (4)
(c)	$R = 1000\,\mathrm{e}^{-22920c} = 62.5$ Accept 62-63	M1 A1 (2)
(d)	Shape 1000	B1 B1 (2)

[9]

Question Number	Scheme	Marks
6. (i)	$\operatorname{cosec}2x = \dfrac{1}{\sin 2x}$	M1
	$= \dfrac{1}{2\sin x\cos x}$	M1
	$= \dfrac{1}{2}\operatorname{cosec}x\sec x \Rightarrow \ \lambda = \dfrac{1}{2}$	A1 **(3)**
(ii)	$3\sec^2\theta + 3\sec\theta = 2\tan^2\theta \Rightarrow 3\sec^2\theta + 3\sec\theta = 2(\sec^2\theta - 1)$	M1
	$\sec^2\theta + 3\sec\theta + 2 = 0$	
	$(\sec\theta + 2)(\sec\theta + 1) = 0$	M1
	$\sec\theta = -2, -1$	A1
	$\cos\theta = -0.5, -1$	M1
	$\theta = \dfrac{2\pi}{3}, \dfrac{4\pi}{3}, \pi$	A1A1 **(6)**

[9]

Question Number	Scheme	Marks
7. (a)	$x^2 - 9 = (x+3)(x-3)$	B1
	$\dfrac{4x-5}{(2x+1)(x-3)} - \dfrac{2x}{(x+3)(x-3)}$	
	$= \dfrac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \dfrac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$= \dfrac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$= \dfrac{5(x-3)}{(2x+1)(x-3)(x+3)} = \dfrac{5}{(2x+1)(x+3)}$	A1*
		(5)
(b)	$f(x) = \dfrac{5}{2x^2 + 7x + 3}$	
	$f'(x) = \dfrac{-5(4x+7)}{\left(2x^2 + 7x + 3\right)^2}$	M1M1A1
	$f'(-1) = -\dfrac{15}{4}$	M1A1
	Uses $m_1 m_2 = -1$ to give gradient of normal $= \dfrac{15}{4}$	M1
	$\dfrac{y - \left(-\frac{5}{2}\right)}{\left(x - -1\right)} = their \ \dfrac{15}{4}$	M1
	$y + \dfrac{5}{2} = \dfrac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		[13]

Question Number	Scheme	Marks
8. (a)	$t = 0 \Rightarrow P = \dfrac{8000}{1+7} = 1000$ cao	M1A1
		(2)
(b)	$t \to \infty \quad P \to \dfrac{8000}{1} = 8000$	B1
		(1)
(c)	$t = 3, P = 2500 \Rightarrow 2500 = \dfrac{8000}{1+7e^{-3k}}$	B1
	$e^{-3k} = \dfrac{2.2}{7} = (0.31..)$ oe	M1,A1
	$k = -\dfrac{1}{3}\ln\left(\dfrac{2.2}{7}\right) = $ awrt 0.386	M1A1
		(5)
(d)	Sub t=10 into $P = \dfrac{8000}{1+7e^{-0.386t}} \Rightarrow P = 6970$ cao	M1A1
		(2)
(e)	$\dfrac{dP}{dt} = -\dfrac{8000}{(1+7e^{-kt})^2} \times -7ke^{-kt}$	M1,A1
	Sub t=10 $\left.\dfrac{dP}{dt}\right|_{t=10} = 346$	A1
		(3)
		[13]

Examiner reports

Question 1

The majority of candidates knew the rules for differentiation and were able to apply them accurately.

Part (a) was generally well done with candidates scoring both marks. In those situations where candidates did not achieve the correct answer, $\dfrac{1}{x}$ or $\dfrac{1}{(2x+3)}$ or 1 over some other function of x was seen instead of $\dfrac{1}{(x^2+3x+5)}$ whilst others omitted the $(2x+3)$.

In a minority of cases it was disappointing to see some candidates believing that $\ln(a+b) = \ln a + \ln b$ and applying this before attempting to differentiate. Surprisingly, many who failed in part (a) went on to succeed in part (b). The quotient rule was generally well known, but it should be noted that candidates should quote the formula before proceeding. Even though a version of the formula is given in the formula book some candidates still stated it incorrectly. Solutions were sometimes given without appropriate bracketing, multiplication signs (or dots), the presence of which make for better clarity and helps to avoid subsequent errors. The product rule, when attempted, was often correctly done, but on occasions the original expression was not adapted to $\cos x \times x^{-2}$ before differentiating.

Question 2

The majority of candidates scored very well in this question. Those candidates who did not factorise the quadratic expression in the denominator in part (a) clearly had more work to do and usually only gained three marks, but they were a small minority and most candidates scored at least five marks here. The most common source of error was to omit the bracket around $(9+2x)$ in the numerator, so that $f(x) = \dfrac{4x^2+6x-12}{(x+2)(2x-5)}$ frequently occurred.

In part (b) the vast majority of candidates attempted to use the quotient rule, and again any errors were through poor use of, or lack of, brackets; $-2(4x-6) = -8x-12$ leading to $\dfrac{-16}{(2x-1)^2}$, being the most common.

It was surprising, this long into the Specification, to see how often $f'(x)$ was interpreted as $f^{-1}(x)$.

Question 3

The principles of transforming graphs were well understood. Part (a) was generally well done and almost all candidates recognised that the transformation left the shape and the x-coordinates of the stationary points unchanged. The y-coordinates, however, were often given incorrectly. Part (b) was very well done and the majority reflected the correct part of the curve in the x-axis and it was pleasing to note that almost all candidates knew they had to draw a cusp and not round off the curve. A few drew the graph of $y = f(|x|)$ instead of $y = |f(x)|$.

Question 4

Most candidates scored well on this question. The mark scheme allowed a fair degree of tolerance so far as quality of sketching was concerned. An occasional problem in marking occurred when candidates used intermediate graphs, all on one set of axes, on their way to their final sketch. When this approach is adopted, it is important to label which curve is the final answer.

In part (a), very few errors were seen. Those seen were largely slips with regard to the signs of coordinates. In part (b), the commonest error was to attempt to reflect into the fourth quadrant rather than the second.

Part (c) caused the candidates a few more problems. They usually got the shape correct but a significant proportion of candidates failed to translate the curve to the left. The evaluation of the image coordinates proved demanding in this part and errors in applying the scaling factor were relatively common. It was surprising that there were substantial numbers of candidates who had both coordinates correct but produced a diagram contradicting this by not translating the curve.

Question 5

This question was well answered by a high proportion of the candidates and completely correct solutions to parts (a), (b) and (c) were common. Virtually all candidates gained the mark in part (a). A few thought the answer was 1000e.

Those who failed to get the correct answer in part (b) often substituted $R = \dfrac{1}{2}$ instead of $R = 500$. Logs were usually taken correctly but it was perhaps disappointing that a not insignificant number of candidates lost the final mark by failing to give the answer correctly to 3 significant figures. In part (c), most candidates scored the method mark even if they had struggled with part (b). Very few who had a correct value of c failed to go on to gain the second mark.

The graph in part (d) was generally less well done. A common error was to draw a curve of the type $y = \dfrac{1}{x}$ and this lost both marks. Occasionally candidates had the curve meeting the positive x-axis rather than approaching it asymptotically.

Question 6

Many candidates achieved a completely correct response to this question.

In part (a) nearly all candidates used the identities correctly, but a significant number, after reaching $\dfrac{1}{2 \sin x \cos x}$ then gave their answer as 2 cosec x sec x.

In part (b) a variety of methods were seen, some fairly laborious, to achieve a quadratic equation in either sec x or cos x. After solving this, a few candidates wrote sec x as $\dfrac{1}{\sin x}$, and some omitted some solutions of the equation or gave incorrect secondary values. Solutions in degrees were rare. Nevertheless many fully correct solutions were seen.

Question 7

The level of accuracy in part (a) was pleasing with a significant majority of candidates scoring full marks. Nearly all factorised $x^2 - 9$ correctly, although a few left the factorisation till further on in the question, making it more difficult. A few ended up with quartic functions but most of these candidates managed to reach the required result. Where there were errors, the main causes were the failure to include all necessary brackets leading to poor products, casual miswriting of signs part way through calculations and only occasionally showing an inability to deal with factors. A large proportion of candidates answered part (b) correctly and efficiently. Nearly all knew the required set of steps to reach the answer, with algebraic and arithmetical errors rather the cause of dropped marks. Other non-standard methods were seen, including splitting f(x) into partial fractions, writing f(x) as $5(2x + 1)^{-1}(x + 3)^{-1}$ then using the product rule and perhaps more elegantly writing the function as $\frac{5}{y}(2x + 1)(x + 3)$ and then differentiating implicitly. Those who multiplied out the brackets, followed by an application of the quotient or the chain rule differentiated most successfully. Errors arising from differentiating 5 to give 1, careless slips with the derivative of the denominator, missing brackets leading to $-20x + 35$ were seen in equal measure.

Once the gradient had been found most candidates proceeded to find the equation of the normal without further problems. There was some evidence of numerical inaccuracy in substituting
$x = -1$ into the gradient function, but nearly all candidates modified their tangent gradient to obtain the normal. One common error in calculating the gradient was a failure to square the denominator resulting in a value of $\frac{15}{2}$ which became $-\frac{2}{15}$ for the normal. Most candidates were able to find a normal gradient from a tangent gradient and use point P to form a straight line equation.

Question 8

Most candidates seemed to have sufficient time to attempt all parts of this question.

Part (a) was usually answered correctly, but many candidates did not know how to attempt part (b), frequently leaving it blank.

Part (c) was a procedure most candidates have now mastered, and was usually completely correct. After rearranging the equation there were a few errors in the log work; very few candidates used $\log_{10}$ rather than ln, but there were numerical errors.

Part (d) was generally approached correctly, although incorrect values of k sometimes led to an incorrect answer. However many failed to achieve the second mark because they did not round the answer to 3 significant figures as required.

Part (e) was challenging. A minority of candidates managed to differentiate completely correctly, although there were some attempts with only a numerical factor missing. The use of the chain rule on a rather complicated function was one of the techniques shown to be lacking in a number of candidates.

Statistics for C3 Practice Paper Bronze Level B2

Qu	Max score	Modal score	Mean %	Mean score for students achieving grade:							
				ALL	A*	A	B	C	D	E	U
1	5		86	4.31	4.95	4.82	4.61	4.31	3.77	2.87	1.59
2	10		83	8.33		9.43	8.82	8.17	7.27	6.03	3.87
3	6		86	5.15		5.75	5.30	4.99	4.52	3.96	2.81
4	10		84	8.35		9.16	8.34	7.85	7.15	6.17	4.59
5	9		79	7.08		8.50	7.46	6.52	5.95	4.39	2.82
6	9		82	7.37	8.83	8.36	7.49	6.51	4.64	2.37	0.91
7	13		80	10.35	12.72	11.93	11.06	9.99	8.42	6.54	3.88
8	13		79	10.30	12.38	11.12	9.70	9.24	8.33	7.01	4.80
	75		82	61.24		69.07	62.78	57.58	50.05	39.34	25.27