

AS MATHEMATICS 7356/1

Paper 1

Mark scheme

June 2020

Version: 1.0 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
R	mark is for reasoning
Α	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
'their'	Indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

AS/A-level Maths/Further Maths assessment objectives

Α	0	Description					
	AO1.1a	Select routine procedures					
AO1	AO1.1b	Correctly carry out routine procedures					
	AO1.2	Accurately recall facts, terminology and definitions					
	AO2.1	Construct rigorous mathematical arguments (including proofs)					
	AO2.2a	Make deductions					
AO2	AO2.2b	Make inferences					
AUZ	AO2.3	Assess the validity of mathematical arguments					
	AO2.4	Explain their reasoning					
	AO2.5	Use mathematical language and notation correctly					
	AO3.1a	Translate problems in mathematical contexts into mathematical processes					
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes					
	AO3.2a	Interpret solutions to problems in their original context					
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems					
AO3	AO3.3	Translate situations in context into mathematical models					
	AO3.4	Use mathematical models					
	AO3.5a	Evaluate the outcomes of modelling in context					
	AO3.5b	Recognise the limitations of models					
	AO3.5c	Where appropriate, explain how to refine models					

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to students showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the student to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

Q	Marking Instructions	AO	Marks	Typical Solution
1	Ticks correct box	1.1b	B1	The gradient is positive and decreasing
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
2	Ticks correct box	2.2a	B1	f(2x) = 10 when x = 2
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
3(a)	Explains that Jia should not have cancelled by $\cos\theta$ OE Or obtains solution $\cos\theta = 0$	2.3	E1	Jia should not have cancelled by $\cos\theta$ Jia has forgotten a second solution to
	Explains that Jia has omitted the other solution to $\cos\theta = 0.5$ OE	2.3	E1	$\cos\theta = 0.5$
	Subtotal		2	
3(b)	Obtains any two correct solutions. Accept answer written in part (a)	1.1a	M1	θ = ±60° and ±90°
	Obtains four correct solutions and no extra ones	1.1b	A1	
	Subtotal		2	
	Question Total		4	

Q	Marking Instructions	AO	Marks	Typical Solution
4	Expands with correct binomial coefficients for at least the two terms required. Accept ⁴ C ₂ etc. PI	1.1a	M1	$(\sqrt{3})^4 + 4(\sqrt{3})^3\sqrt{2} + 6(\sqrt{3})^2(\sqrt{2})^2 + 4\sqrt{3}(\sqrt{2})^3 + (\sqrt{2})^4$
	Correctly simplifies or evaluates at least one of the irrational terms PI by 20√6	1.1b	B1	$4 \times 3\sqrt{3} \times \sqrt{2} - 4 \times \sqrt{3} \times 2\sqrt{2}$ $= 12\sqrt{6} - 8\sqrt{6}$
	Obtains ± correct answer Accept AWRT 9.8	1.1b	A1	= 4√6
	Total		3	

Q	Marking Instructions	AO	Marks	Typical Solution
5	Uses correct formula and notation for this function; must have substituted $(x + h)$ correctly	1.1a	M1	$\lim_{h \to 0} \frac{4(x+h)^2 + (x+h) - (4x^2 + x)}{h}$
	Multiplies out $4(x + h)^2$ correctly	1.1b	B1	$\lim_{h \to 0} \frac{4x^2 + 8xh + 4h^2 + x + h - 4x^2 - x}{h}$
	Obtains numerator with no x^2 or x terms PI	1.1b	A1	$\lim_{h\to 0} \frac{8xh + 4h^2 + h}{h}$
	Completes rigorous argument, including dividing by <i>h</i> and correctly using limit	2.1	R1	$\lim_{h \to 0} (8x + 4h + 1)$ $= 8x + 1$
	Total		4	

Q	Marking Instructions	AO	Marks	Typical Solution
6(a)	Substitutes $x = 2$ into function	1.1a	M1	$f(2) = 2^3 - 2^2 + 2 - 6$
	Completes reasoned argument to explain that $f(2) = 0$ shows $(x - 2)$ is a factor	2.1	R1	f(2) = 0 which shows that (x, y) is a factor
	is a factor			which shows that $(x - 2)$ is a factor
	Subtotal		2	
6(b)	Obtains correct factor	1.1b	B1	$x^2 + x + 3$
	Subtotal		1	
6(c)	Calculates discriminant for their quadratic OE	3.1a	M1	$x^2 + x + 3 = 0$
				Discriminant = $1^2 - 4 \times 1 \times 3$
	States that there are no real solutions from the quadratic	2.1	A1	= -11 < 0
	Coldients from the quadratic			So no real solutions to the quadratic
	Deduces that there is only one	2.2a	B1	
	solution coming from factor $(x-2)$			Therefore $x = 2$ is the only solution
	Subtotal		3	
6(d)	Expresses equation as a cubic in a single different variable or in terms of e^x	3.1a	M1	$y^3 - y^2 + y - 6 = 0$ where $y = e^x$
	$(e^x)^3 - (e^x)^2 + (e^x) - 6 = 0$			Solution $y = 2$
	Obtains solution $e^x = 2$	1.1b	A1	$e^x = 2$
		1.1b	A1	x = ln 2
	Obtains In 2. ISW			
	Subtotal		3	
	Question Total		9	

Q	Marking Instructions	AO	Marks	Typical Solution
7	Identifies transformed functions as $(x \pm 3)^2$ and $2x$ or $\frac{1}{2}x$, at least one correct.	3.1a	M1	C ₁ has equation $y = (x - 3)^2$ L ₁ has equation $y = \frac{1}{2}x$
	Forms correct equation	1.1b	A1	$(x-3)^2 = \frac{1}{2}x$
	Solves their quadratic equation	1.1a	M1	$x^2 - \frac{13}{2}x + 9 = 0$
	Obtains correct <i>x</i> values	1.1b	A1	$x = 2 \text{ or } 4\frac{1}{2}$
	Applies distance formula to their \boldsymbol{x} and \boldsymbol{y} values	1.1a	M1	$y = 1 \text{ or } 2\frac{1}{4}$ Distance = $\sqrt{(4\frac{1}{2} - 2)^2 + (2\frac{1}{4} - 1)^2}$
	Obtains correct distance for their intersection points (non-zero values), any equivalent exact form.	1.1b	A1F	$=\frac{5\sqrt{5}}{4}$
	Total		6	

Q	Marking Instructions	AO	Marks	Typical Solution
8(a)	Recalls and applies gradient rule for e^{kx}	1.2	B1	Gradient = 4e ^{4a} Equation is
	Uses their gradient in line equation formula	1.1a	M1	$y - e^{4a} = 4e^{4a}(x - a)$
	Obtains correct equation, any form, FT their gradient.	1.1b	A1F	
	Subtotal		3	
8(b)	Substitutes $x = 0$ and $y = 0$ into their line equation	1.1a	M1	$x = 0$ and $y = 0$ gives $0 = e^{4a}(1 - 4a)$
	Finds correct value of <i>a</i> for their equation	1.1b	A1F	$a = \frac{1}{4}$
	Subtotal		2	
8(c)	Deduces correct lower limit	2.2a	B1	Any negative gradient cuts curve $0 \le m$
	Deduces correct upper limit based on their answers to (a) and (b)	2.2a	M1	With $a = \frac{1}{4}$ contact point is $(\frac{1}{4}, e)$ Gradient $(0, 0)$ to $(\frac{1}{4}, e)$ is 4e
	Obtains correct inequality	1.1b	A1	0 ≤ <i>m</i> < 4e
	Subtotal		3	
	Question Total		8	

Q	Marking Instructions	AO	Marks	Typical Solution
9	Uses 'angle in a semicircle' to justify ACB = 90°	2.4	E1	Angle ACB = 90° (Angle in a semicircle)
	Deduces that AB ² = AC ² + BC ²	2.2a	B1	$AB^2 = BC^2 + AC^2$ (Pythagoras)
	Applies area formula to one triangle	1.1a	M1	Area of ABK = $\frac{1}{2}$ AB ² sin 60° Area of BCL = $\frac{1}{2}$ BC ² sin 60°
	Applies area formula to all three triangles	1.1b	A1	Area of CAM = $\frac{1}{2}$ AC ² sin 60°
	Forms a correct equation involving BCL + CAM and completes reasoned proof.	2.1	R1	BCL + CAM = $\frac{1}{2}$ sin 60° (BC ² + AC ²) = $\frac{1}{2}$ sin 60° (AB ²) = ABK
	Total		5	

Q	Marking Instructions	AO	Marks	Typical Solution
10(a)	Applies laws of logarithms to obtain one correct term	1.1a	M1	$\ln P = \ln a + \ln C^n$
	Obtains completely correct expression	1.1b	A1	$\ln P = \ln a + n \ln C$
	Subtotal		2	
10(b)(i)	Calculates In values. Condone one slip. PI by any two correct points	1.1a	M1	In $C = -0.51$, 0.140, 0.405 In $P = 6.20$, 7.09, 7.45
	Correctly plots three points. Line not required.	1.1b	A1	(See graph below)
	Subtotal	170. 180800 174	2	
		InP ↑		
		8.0-		
		7.0	***	
		6.0-		
		5.0-		
		<u> </u>		
	-0.6 -0.4 -0.2	0	0.2	2 0.4 0.6 In <i>C</i>
10(b)(ii)	Infers significance of straight line	2.2b	E1	The three points lie on a straight line
	Subtotal		1	
10(b)(iii)	Identifies In a as the intercept. PI	3.4	M1	In a is the intercept value
	Correctly calculates <i>a</i> AWFW 960 to 1040	1.1b	A1	In <i>a</i> = 6.9 So a = 992
	Identifies <i>n</i> as the gradient. PI	3.4	M1	<i>n</i> is the gradient
	Obtains correct <i>n</i> value. AWFW 1.35 to 1.41	1.1b	A1	= 1.37
	Subtotal		4	

10(c)	Explains significance of a	2.4	E1	a is the price for a 1 carat diamond
	Subtotal		1	
10(d)	Substitutes their values into <i>P</i> equation with <i>C</i> = 2 Or Uses the graph to read off a value for ln P	3.4	M1	992 × 2 ^{1.37}
	Calculates correct value of P for their values. AWFW 2440 to 2770 FT provided > 2000 and < 3000 must include units	3.2a	A1F	= £2560
	Subtotal		2	
	Question Total		12	

Q	Marking Instructions	AO	Marks	Typical Solution
11	Circles correct answer	1.1b	B1	3 N
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
12	Ticks correct box	1.1b	B1	$\sqrt{13^2 + 10^2}$
	Total		1	

Q	Marking Instructions	AO	Marks	Typical Solution
13	Uses definition of acceleration	1.2	B1	Gradient of v–t graph represents acceleration
	Uses appropriate gradient formula OE	1.1a	M1	$a = \frac{v - u}{t}$
	Substitutes u , v and t into gradient formula and rearranges to find given equation \mathbf{AG}	2.1	R1	$\Rightarrow v = u + at$
	Total		3	

Q	Marking Instructions	AO	Marks	Typical Solution
14	Uses appropriate constant acceleration equation to find the acceleration	3.4	M1	$v^2 = u^2 + 2as$ Using $s = 3.2, u = 0$ and $v = 4$
	Obtains correct value for a	1.1b	A1	a = 2.5
	Uses their <i>a</i> value with Newton's second Law to determine magnitude of force	3.4	M1	$F = ma \Longrightarrow F = 0.25$
	Calculates the magnitude of the given vector	1.1b	B1	$\sqrt{7^2 + 24^2} = 25$
	Deduces, using given information, correct vector form of the force	2.2a	A1	$\mathbf{F} = \frac{1}{100} (7\mathbf{i} + 24\mathbf{j})$
	Total		5	

Q	Marking Instructions	AO	Marks	Typical Solution
15(a)	Integrates a to find v with at least one term correct	3.4	M1	$v = \int a dt$
	Finds fully correct final expression Condone presence of + c	1.1b	A1	$v = 4t - t^3$
	Subtotal		2	
15(b)	Integrates v to find s with at least one term correct	3.1b	M1	$s = \int v dt$
	Integrates their answer to (a) correctly including constant of integration	1.1b	A1F	$s = 2t^2 - \frac{1}{4}t^4 + k$
	Uses given conditions to find constant	3.4	M1	$39 = 8 - 4 + k \Longrightarrow k = 35$
	Equates their expression for s to zero and finds a value for t	1.1a	M1	$0 = t^4 - 8t^2 - 140$
	Obtains correct value of t to required accuracy	1.1b	A1	t = 4.06 seconds
	Subtotal		5	
	Question Total		7	

Q	Marking Instructions	AO	Marks	Typical Solution
16(a)	Models the motion of the container and load with at least one side of the equation correct.	3.3	M1	T - (m+2)g = (m+2)a
	Forms fully correct equation	1.1b	A1	
	Forms fully correct equation for particle	3.3	B1	5g - T = 5a
	Completes a rigorous argument by eliminating T and rearranging to express a in terms of m . AG	2.1	R1	5g - (m+2)g = (5+2+m)a $(3-m)g = (7+m)a$
	·			$\therefore a = \left(\frac{3-m}{m+7}\right)g$
	Subtotal		4	
16(b)	Deduces correct limits Condone $0 \le m < 3$	2.2a	B1	0 < m < 3
	Subtotal		1	
16(c)	Uses appropriate constant acceleration equation to find the acceleration	3.4	M1	$s = ut + \frac{1}{2}at^2$
	Calculates correct value for a	1.1b	A1	Using $s = 2$, $u = 0$ and $t = 1$ $a = 4$
	Forms equation for a in terms of m using their a value	3.4	M1	$4 = \left(\frac{3-m}{m+7}\right) g$
	Solves to find <i>m</i> . AWRT 0.10 Condone 0.1	3.2a	A1	$m = \frac{3g - 28}{4 + g} = 0.10 \text{ kg}$
	Subtotal		4	
16(d)	Describes any valid assumption not related to those assumptions already stated in the question. Eg The particle is at least 2m above the ground Eg The particle does not collide with the load	3.5b	E1	I assumed that the top of the container does not reach the pulley
	Subtotal		1	
	Question Total		10	