A-level

Mathematics

MS04 Statistics 4

Final Mark scheme

6360

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS04

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 In general, a correct answer (to accuracy required) without units scores full marks

GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3})

GN8 Where a candidate's response to a part of a question is simply to label the part (eg (d)(i)) with nothing else (ie no attempt at a solution), then this is still treated as a response and marked as 0 rather than NR. Also, deleted work, if not replaced, should be marked and not treated as NR.

Q	Solution	Marks	Total	Comments
1				
	H_0: $\mu_D = 0$ H_1: $\mu_D > 0$ or H_1: $\mu_D < 0$	B1		Both D = Before – After or D = After – Before
	ν = **10**	B1		CAO; can be implied
	0.01 (1%) $\Rightarrow$ t = **2.76 to 2.77** or t = **–2.76 to –2.77** **or** p-value of t-calculated = **0.007 < 0.01**	B1		AWFW (2.764) AWFW (–2.764) AWRT (0.00711)
	Differences are: 0.35 0.27 –0.26 0.39 0.34 0.29 0.06 0.43 –0.15 0.23 0.25	M1		Attempt at differences Accept reversed signs
	Mean $\bar{d}$ = **±0.20**	B1		CAO; ignore sign
	Sd s_d = **0.224** **or** σ_d = **0.213** **or** Var s_d^2 = **0.0501** **or** σ_d^2 = **0.0456**	A1		AWRT (0.223875 or 0.213456) AWRT (0.050120 or 0.045564)
	$t = \dfrac{\pm 0.20 - 0}{0.224/\sqrt{11}}$	M1		OE
	= **±2.96**	A1		AWRT (2.962926)
	Reject H_0 $\Rightarrow$ **evidence**, at 1% level, that Choldrop **reduces** mean level of LDL (in men with high levels of cholesterol)	Adep1		Dep on previous 8 marks OE; but must not be definitive
			9	
Notes	**1** Inconsistent signs (eg as defined by H_1 and/or differences) $\Rightarrow$ B0 B1 B1,0 M1 B1 A1 M1 A1,0 Adep0 (max of 7 marks) **2** Unpaired t-test $\Rightarrow$ B1 B0 B0 M0 B1 A0 M0 A0 Adep0 (max of 2 marks)			
		Total	**9**	

Q	Solution	Marks	Total	Comments
2				
(a)	$P(X \leq 2) = p + (1-p)p$			
	or $2p - p^2$ **or** $1 - q^2$	M1		OE
	$2p - p^2 = 0.36$ $\quad$ $1 - q^2 = 0.36$			
	$p^2 - 2p + 0.36 = 0$ $\quad$ $q^2 = 0.64$	m1		Equating to 0.36 and simplifying
	$(p - 1.8)(p - 0.2) = 0$ $\quad$ $q = 0.8$			
	$p = \underline{\mathbf{0.2}}$ $\quad$ $p = \underline{\mathbf{0.2}}$			
	or	A1		CAO
	$q = \underline{\mathbf{0.8}}$ $\quad$ $q = \underline{\mathbf{0.8}}$			
	$\mu = \frac{1}{p} = \underline{\mathbf{5}}$	BF1		F on $0 < p < 1$
	$\sigma^2 = \frac{1-p}{p^2} = \underline{\mathbf{20}}$	A1		CAO
			5	
(b)				
	$P(\mu - 0.5\sigma < X < \mu + 0.5\sigma) =$			Use of c's values from (a)
	or	M1		
	$P(5 - \sqrt{5} < X < 5 + \sqrt{5}) =$ $P(2.8 < X < 7.2) =$			(2.7639 & 7.2361)
	$P(r < X < s)$			
	or	m1		Move to **integer** values (PI)
	$P(2 < X < 8)$ **or** $P(3 \leq X \leq 7)$			
	$= (1 - 0.8^7) - (1 - 0.8^2)$			
	$= (1 - 0.8^7) - 0.36 = 0.79 - 0.36$			
	or			
	$= 0.64 - 0.8^7 = 0.64 - 0.21$	A1		Correct expression (PI)
	or			
	$= 0.2\,(0.8^2 + 0.8^3 + 0.8^4 + 0.8^5 + 0.8^6)$			
	$= 0.2 \times 2.15$			
	$= \underline{\mathbf{0.43}}$	A1		AWRT (0.43028)
			4	
		Total	**9**	

Q	Solution	Marks	Total	Comments
3 (a)(i)	$s_1^2 = 0.1352/15$ = **0.009**	B1		AWRT (0.009013) Both; generous with labelling
	$s_4^2 = 0.1898/9$ = **0.021**			AWRT (0.021089)
	ν_1 (ν_4) = **9** ν_2 (ν_1) = **15**	B1		CAO both (PI); allow reversed
	95% $\Rightarrow$ F(U) = **3.123** F(L) = $(3.769)^{-1}$ = **0.265**	B1		At least F(U); allow reversed
	CI for $\frac{\sigma_4^2}{\sigma_1^2}$ is $\left(\frac{0.021/0.009}{3.123}, \frac{0.021/0.009}{3.769^{-1}}\right)$ **or** (2.333/3.123, 2.333×0.265) **or** (0.747, 8.794)	M1		Variance ratio (2.33 to 2.34) with F-values; do **not** allow reversed unless later corrected
	CI for $\frac{\sigma_4}{\sigma_1}$ is **(0.86 to 0.87, 2.96 to 2.98)**	m1 A1	**6**	Square root AWFW (0.8656, 2.9697)
(ii)	Since CI for $\frac{\sigma_4}{\sigma_1}$ or $\frac{\sigma_4^2}{\sigma_1^2}$ **includes 1**	Bdep1		Dep on CI including 1
	at 5% level, there is **no/insufficient** evidence of a difference between σ_4 and σ_1	Bdep1	**2**	Dep on Bdep1 OE; but must not be definitive
(b)	H_0: $\mu_4 - \mu_1 = 30$ H_1: $\mu_4 - \mu_1 > 30$	B1		At least H_0
	ν = 16 + 10 – 2 = **24**	B1		CAO (PI)
	$t_{24}(0.95)$ = **1.71**	B1		AWRT (1.711)
	or p-value of t-calculated = **0.009 < 0.05**			AWRT (0.008630)
	$s_p^2 = \frac{0.1352 + 0.1898}{24}$	M1		OE
	s_p^2 = **0.0135 to 0.0136**	A1		AWFW (0.013542)
	or s_p = **0.116 to 0.117**			AWFW (0.116369)
	$t = \frac{(45.13 - 15.01) - 30}{\sqrt{0.013542\left(\frac{1}{16} + \frac{1}{10}\right)}}$ = **2.56**	M1 M1 A1		Numerator; allow no"30" Denominator AWRT (2.558106)
	Reject H_0 $\Rightarrow$ **evidence**, at 5% level, that $\mu_4 - \mu_1 > 30$	Adep1	**9**	Dep on previous 8 marks OE; but must not be definitive
		Total	**17**	

Q	Solution	Marks	Total	Comments
4	$\mathrm{P}(a \le X < b) = \int_a^b \frac{x^2}{18}\,\mathrm{d}x = \left[\frac{x^3}{54}\right]_a^b$	M1		Used for $0 \le X < 3$; PI
	or $\mathrm{F}(x) = \frac{x^3}{54}$			
	$\mathrm{P}(0 \le X < 1) =$ **<u>1/54 or 0.019</u>** $\mathrm{P}(1 \le X < 2) =$ **<u>7/54 or 0.130</u>** $\mathrm{P}(2 \le X < 3) =$ **<u>19/54 or 0.352</u>**	A2 (–1 ee)		CAO/AWRT (0.01852) (0.12963) (0.35185)
	$\mathrm{P}(c \le X < d) = \int_c^d \frac{1}{4}(5-x)\,\mathrm{d}x = \left[\frac{5x}{4} - \frac{x^2}{8}\right]_c^d$	M1		Used for $3 \le X \le 5$; PI
	or $\mathrm{F}(x) = \frac{5x}{4} - \frac{x^2}{8} - \frac{17}{8}$			
	$\mathrm{P}(3 < X \le 4) =$ **<u>3/8 or 0.375</u>** $\mathrm{P}(4 < X \le 5) =$ **<u>1/8 or (0.124 to 0.125)</u>**	A1 A1	**(6)**	CAO CAO/AWFW
	H_0: pdf given is appropriate H_1: pdf given is not appropriate	B1		OE At least H_0
	$\nu = 5 - 1 =$ **<u>4</u>**	B1		CAO
	$\chi^2(0.95) =$ **<u>9.49</u>**			AWRT (9.488)
	or p-value of χ^2-calculated = **<u>(0.12 to 0.13) < 0.05</u>**	B1		AWFW (0.12219)
		M1		$E = 324 \times p$
		M1		Attempt at $\sum(O-E)^2/E$
	χ^2(calc) = **<u>7.15 to 7.35</u>**	A1		AWFW (7.27204)
	Accept H_0 $\Rightarrow$ **no evidence**, at 5% level, that pdf given **is not appropriate** model	Adep1	**(7)**	Dep on previous 12 marks OE; but must not be definitive
		Total	**13**	

x	O	E	$O-E$	$(O-E)/E^2$
0–1	4	6	–2	0.6667
1–2	49	42	7	1.1667
2–3	129	114	15	1.9737
3–4	112	121.5	–9.5	0.7428
4–5	30	40.5	–10.5	2.7222
T	**324**	**324.0**	**0.0**	**7.2721**

Q	Solution	Marks	Total	Comments
5(a) (i)	$f(x) = \frac{d}{dx}F(x) = \underline{\boldsymbol{\lambda e^{-\lambda x}}}$	B1	**1**	CAO
(ii)	$E(X^2) = \int_0^\infty x^2(\lambda e^{-\lambda x})dx =$	M1		Ignore limits
	$\left[x^2(-e^{-\lambda x})\right]_0^\infty - \int_0^\infty 2x(-e^{-\lambda x})\,dx =$	A1		**Correct** integration by parts Ignore limits
	$\frac{2}{\lambda}\int_0^\infty x(\lambda e^{-\lambda x})\,dx$ $= \frac{2}{\lambda}E(X) = \underline{\frac{\mathbf{2}}{\boldsymbol{\lambda^2}}}$	A1	**3**	Fully **complete & correct** derivation to this point (OE) plus (OE) **correct** answer
(iii)	$\text{Var}(X) = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$	Adep1	**1**	Dep on previous A1 AG
(b) (i)	$P(E < 15 \mid \text{Po}(12)) = 0.7720$ **or** 0.8444	M1		
	$= \underline{\mathbf{0.772}}$	A1	**2**	AWRT (0.7720)
(ii) (A)	$P(T = 20) =$ **0 or zero or nought or nothing or nil**	B1	**(1)**	
(B)	$\underline{\boldsymbol{T \sim \text{Exp}(\lambda = 1/20)}}$	B1		OE (PI); accept $\text{Exp}(\lambda_h = 3)$
	$P(T < 15) = 1 - e^{-15/20}$ **or** $1 - e^{-3/4}$ **or** $1 - 0.47237$	M1		
	$= \underline{\mathbf{0.527 \text{ to } 0.528}}$	A1	**(3)**	AWFW (0.52763)
(C)	$P(15 < T < 25) =$ $(1 - e^{-25/20}) - (1 - e^{-15/20})$	M1		$e^{-25/20} = e^{-3\times(5/12)}$ $e^{-15/20} = e^{-3\times(3/12)}$
	or $[(1 - e^{-25/20}) - (B)]$ **or** $(e^{-15/20} - e^{-25/20})$ $= 0.71350 - 0.52763 = \underline{\mathbf{0.185 \text{ to } 0.186}}$ **or** $= 0.47237 - 0.28650 = \underline{\mathbf{0.185 \text{ to } 0.186}}$	A1	**(2)**	AWFW (0.18586)
			6	
		Total	**13**	

Q	Solution	Marks	Total	Comments
6(a) (i)	$\mathrm{E}(Y_1)=\frac{1}{2}(\mu+\mu)$ = $\underline{\boldsymbol{\mu}}$ $\mathrm{E}(Y_2)=\frac{1}{4}(\mu+3\mu)$ = $\underline{\boldsymbol{\mu}}$	M1 A1	**2**	One **correct** application of $\mathrm{E}(\bar{X})$ **Two correct** derivations
(ii)	$\mathrm{V}(Y_1)=\frac{1}{2^2}\left(\frac{\sigma^2}{10}+\frac{\sigma^2}{30}\right)$ = $\underline{\boldsymbol{\frac{\sigma^2}{30}}}$ $\mathrm{V}(Y_2)=\frac{1}{4^2}\left(\frac{\sigma^2}{10}+\frac{3^3\sigma^2}{30}\right)$ = $\underline{\boldsymbol{\frac{\sigma^2}{40}}}$	M1 m1 A1	**3**	**Correct** use of V on (c **or** $\bar{X}$) **Correct** use of V on (c **and** $\bar{X}$) **Two correct** derivations
(iii)	RE (Y_1 to Y_2) = $\frac{1/\mathrm{Var}(Y_1)}{1/\mathrm{Var}(Y_2)}$ = $\left(30/\sigma^2\right)/\left(40/\sigma^2\right)$ = $\underline{\textbf{3/4 or 0.75}}$	M1 A1	**2**	**Correct** use of values from (ii) CAO
(b) (i)	$\mathrm{E}(T_1)=\frac{1}{2}\left(\sigma^2+\sigma^2\right)$ = $\underline{\boldsymbol{\sigma^2}}$ $\mathrm{E}(T_2)=\frac{1}{38}\left(9\sigma^2+29\sigma^2\right)$ = $\underline{\boldsymbol{\sigma^2}}$	M1 A1	**2**	**One correct** application of E(S^2) **Two correct** derivations
(ii)	$\mathrm{V}(T_1)=\frac{1}{2^2}\left(\frac{2\sigma^4}{9}+\frac{2\sigma^4}{29}\right)$ = $\underline{\boldsymbol{19\sigma^4/261 \text{ or } 0.073\sigma^4}}$ $\mathrm{V}(T_2)=\frac{1}{38^2}\left(\frac{9^2\times 2\sigma^4}{9}+\frac{29^2\times 2\sigma^4}{29}\right)$ = $\underline{\boldsymbol{\sigma^4/19 \text{ or } 0.053\sigma^4}}$	M1 A1 A1	**3**	**One correct** use of $2\sigma^4/(n-1)$ CAO/AWRT (0.072797) CAO/AWRT (0.052632)
(iii)	$\left\{\mathrm{V}(T_2)=\sigma^4/19\right\} < \left\{19\sigma^4/261=\mathrm{V}(T_1)\right\}$ $\Rightarrow$ T_2 is the better **or** RE (T_1 to T_2) = $\underline{\textbf{(261/361 or 0.723) < 1}}$ **or** RE (T_2 to T_1) = $\underline{\textbf{(361/261 or 1.383) > 1}}$ $\Rightarrow$ T_2 is the better	Bdep1 Bdep1 (Bdep1) (Bdep1)	**2**	Dep on A1 A1; (19/261) > (1/19) Dep on Bdep1 CAO/AWRT (0.722992) Dep on A1 A1 CAO/AWRT (1.383142) Dep on Bdep1
		Total	**14**	