

Please write clearly in bloc	capitals.		
Centre number		Candidate number	
Surname			 -
Forename(s)			 -
Candidate signature			 -

A-level MATHEMATICS

Unit Statistics 4

Wednesday 28 June 2017

Morning

Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer all questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question.
 If you require extra space, use an AQA supplementary answer book; do not use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

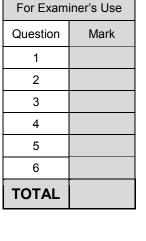
Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.







Answer all questions.

Answer each question in the space provided for that question.

1 During a study of the effect of the drug Choldrop on the level of cholesterol in blood, a randomly selected group of 11 men, all with high levels of cholesterol, was treated with the drug for three months.

> The level of low density lipoprotein, LDL, in each man's blood was measured immediately before the start of the treatment and again after the three-month course of treatment.

The results, in grams of LDL per litre of blood, are shown in the table.

Patient	Α	В	С	D	Е	F	G	Н	I	J	Κ
Before	2.98	2.70	2.61	2.93	2.56	3.05	2.94	3.41	2.22	3.07	2.88
After	2.63	2.43	2.87	2.54	2.22	2.76	2.88	2.98	2.37	2.84	2.63

Assuming that the differences in measurements of LDL are approximately normally distributed, investigate, at the 1% level of significance, whether *Choldrop* reduces the mean level of LDL in men with high levels of cholesterol.

[9 marks]

QUESTION PART REFERENCE	Answer space for question 1



QUESTION PART REFERENCE	Answer space for question 1



2	The random variable X has a geometric distribution with parameter p , mean μ variance σ^2 .	and
(a)	Given that $P(X \le 2) = 0.36$, calculate the value of p , and hence find values fo and σ^2 .	rμ
		[5 marks]
(b)) Calculate $P(\mu - 0.5 \sigma < X < \mu + 0.5 \sigma)$.	[4 marks]
QUESTION PART REFERENCE	Answer space for question 2	
REFERENCE		
	1	



QUESTION PART REFERENCE	Answer space for question 2



	newtons per s measured.	square milli	metre (N/mm ²), of r	andomiy samp	ied cubes of each	mix were	
	The results are summarised in the table.						
			Sample size (n)	Mean (\overline{x})	$\sum (x-\overline{x})^2$]	
	Concrete	M15	16 (<i>n</i>)	15.01	0.1352	_	
	mix	M15 M45	10	45.13	0.1332		
(a) (i)	variance σ_4^2 .		s mean μ_1 and vari		mix ivi45 nas mear	μ_4 and [6 mark	
						lo mark	
(ii)	Comment on	what may t	be concluded from y	our answer to	part (a)(i) .	[2 mark	
(b)	Investigate, a	t the 5% lev	vel of significance, t	he hypothesis t	that $\mu_4 - \mu_1 > 30$.	[9 mark	
	swer space for	r question	3				



Do not write outside the box

QUESTION PART	Answer space for question 3
REFERENCE	



QUESTION PART REFERENCE	Answer space for question 3



QUESTION PART REFERENCE	Answer space for question 3



4 It is suggested that the response time, *X* minutes, to an emergency request for the attendance of an ambulance, at an incident within 3 miles of an ambulance station, can be modelled by the following probability density function.

$$f(x) = \begin{cases} \frac{x^2}{18} & 0 \le x < 3\\ \frac{1}{4}(5-x) & 3 \le x \le 5\\ 0 & \text{otherwise} \end{cases}$$

To investigate this suggestion, the value of X was recorded for a sample of 324 requests, with the following results.

x	0-1	1–2	2–3	3–4	4–5	>5
Frequency	4	49	129	112	30	0

Use a χ^2 goodness of fit test and the 5% level of significance to investigate whether *X* can be modelled by the probability density function given above.

[13 marks]

QUESTION PART	Answer space for question 4
FERENCE	



QUESTION PART REFERENCE	Answer space for question 4
REFERENCE	



QUESTION PART	Answer space for question 4
REFERENCE	



QUESTION PART REFERENCE	Answer space for question 4



5 (a) The continuous random variable *X* has the cumulative distribution function F(x) where

$$F(x) = \begin{cases} 0 & x < 0\\ 1 - e^{-\lambda x} & 0 \le x < \infty \end{cases}$$

and $E(X) = \frac{1}{\lambda}$.

(i) Deduce the probability density function, f(x), of X for $0 \le x < \infty$.

[1 mark]

[3 marks]

- (ii) Use integration to find an expression for $E(X^2)$.
- (iii) Hence show that $\operatorname{Var}(X) = \frac{1}{\lambda^2}$.

[1 mark]

[2 marks]

- (b) The number of emails received by a helpdesk may be modelled by a Poisson distribution with an average of 3 emails per hour.
 - (i) Determine the probability that the helpdesk receives fewer than 15 emails during a four-hour period.
 - (ii) Calculate the probability that the time between successive emails is:
 - (A) exactly 20 minutes;
 - (B) less than 15 minutes;
 - (C) between 15 and 25 minutes.

[6 marks]

 QUESTION PART REFERENCE
 Answer space for question 5

 Image: Constraint of the space of the spac



QUESTION PART REFERENCE	Answer space for question 5



QUESTION PART REFERENCE	Answer space for question 5
REFERENCE	



QUESTION PART REFERENCE	Answer space for question 5
REFERENCE	



The variable \overline{X}_1 denotes the mean of a random sample of **10** observations on *X*. The variable \overline{X}_2 denotes the mean of an independent random sample of **30** observations on *X*.

(a) Two estimators proposed for μ are

6

$$Y_1 = \frac{1}{2} \left(\overline{X}_1 + \overline{X}_2 \right)$$
 and $Y_2 = \frac{1}{4} \left(\overline{X}_1 + 3\overline{X}_2 \right)$

- (i) Show that both $Y_{\!_1}$ and $Y_{\!_2}$ are unbiased estimators for μ .
- (ii) Derive simplified expressions, in terms of σ^2 , for each of $Var(Y_1)$ and $Var(Y_2)$. [3 marks]
- (iii) Calculate the efficiency of Y_1 relative to Y_2 .

(b) The variable S_1^2 denotes the unbiased estimator for σ^2 from the same sample of 10 observations on *X*. The variable S_2^2 denotes the unbiased estimator for σ^2 from the same sample of 30 observations on *X*.

Two estimators proposed for σ^2 are

$$T_1 = \frac{1}{2} \left(S_1^2 + S_2^2 \right)$$
 and $T_2 = \frac{1}{38} \left(9S_1^2 + 29S_2^2 \right)$

(i) Show that both T_1 and T_2 are unbiased estimators for σ^2 .

[2 marks]

(ii) Derive simplified expressions, in terms of σ^4 , for each of $Var(T_1)$ and $Var(T_2)$.

You may assume, for a random sample of *n* observations from the distribution $N(\mu, \sigma^2)$, that $Var(S^2) = \frac{2\sigma^4}{n-1}$.

[3 marks]

(iii) Hence state, with justification, which of T_1 and T_2 is the better unbiased estimator for σ^2 .

[2 marks]



[2 marks]

[2 marks]

QUESTION PART REFERENCE	Answer space for question 6



QUESTION PART REFERENCE	Answer space for question 6
REFERENCE	

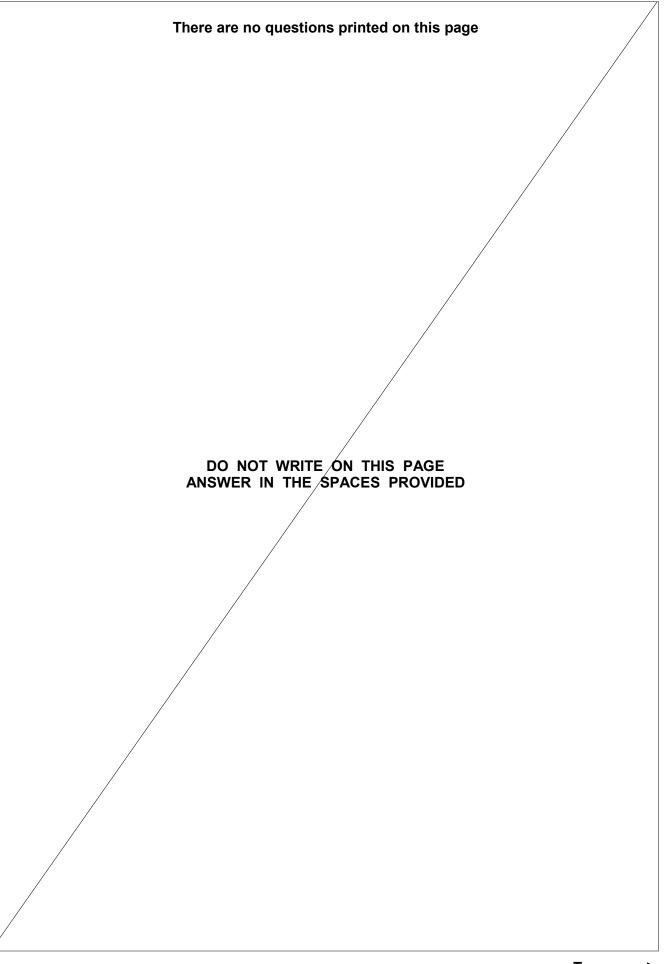


QUESTION PART	Answer space for question 6
REFERENCE	



QUESTION PART REFERENCE	Answer space for question 6
	END OF QUESTIONS









24

DO NOT WRITE ON THIS PAGE ANSWER IN THE SPACES PROVIDED

Copyright Information

For confidentiality purposes, from the November 2015 examination series, acknowledgements of third party copyright material will be published in a separate booklet rather than including them on the examination paper or support materials. This booklet is published after each examination series and is available for free download from www.aqa.org.uk after the live examination series.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2017 AQA and its licensors. All rights reserved.

