A-level

Mathematics

MS03 Statistics 3

Final Mark scheme

6360

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

General Notes for MS03

GN1 There is no allowance for misreads (MR) or miscopies (MC) unless specifically stated in a question

GN2 In general, a correct answer (to accuracy required) without working scores full marks but an incorrect answer (or an answer not to required accuracy) scores no marks

GN3 In general, a correct answer (to accuracy required) without units scores full marks

GN4 When applying AWFW, a slightly inaccurate numerical answer that is subsequently rounded to fall within the accepted range cannot be awarded full marks

GN5 Where percentage equivalent answers are permitted in a question, then penalise by **one accuracy mark** at the first **correct** answer but only if no indication of percentage (eg %) is shown

GN6 In questions involving probabilities, do **not** award **accuracy** marks for answers given in the form of a ratio or odds such as 13/47 given as 13:47 or 13:34

GN7 Accept decimal answers, providing that they have **at least two** leading zeros, in the form $c \times 10^{-n}$ (eg 0.00321 as 3.21×10^{-3})

GN8 Where a candidate's response to a part of a question is simply to label the part (eg (d)(i)) with nothing else (ie no attempt at a solution), then this is still treated as a response and marked as 0 rather than NR. Also, deleted work, if not replaced, should be marked and not treated as NR.

Q	Solution	Marks	Total	Comments
1	$98\% \Rightarrow z =$ **2.32 to 2.33**	B1		AWFW (2.3263)
	Require $\frac{2z\sigma}{\sqrt{n}}$ $(= \text{or} <)$ $(200 \text{ or } 0.2)$ $\Rightarrow$			
	$\begin{pmatrix}2.32 \text{ to } 2.33\\2.05 \text{ to } 2.06\end{pmatrix} \times \frac{2\times 330}{\sqrt{n}}$ $(= \text{or} <)$ 200	M1		OE; allow "no 2" Accept 0.33 and 0.2
	n $(= \text{or} >)$ $\left(\frac{(2.32 \text{ to } 2.33)\times 330}{100}\right)^2$	A1		OE; for $\sqrt{n}$ Accept 0.33 and 0.1
	n (= or >) 58.6 to 59.2 $\Rightarrow$ **60**	A1	4	CAO
		Total	**4**	

Q	Solution	Marks	Total	Comments
2	Sample is selected at **random**	B1		OE
	$\hat{p}$ = **77/440 or 7/40 or 0.175**	B1		CAO; ignore notation
	$99\% \Rightarrow z =$ **2.57 to 2.58**	B1		AWFW (2.5758)
	CI for p is			
	$0.175 \pm \begin{pmatrix}2.57 \text{ to } 2.58\\2.32 \text{ to } 2.33\end{pmatrix}\sqrt{\frac{0.175\times 0.825}{440}}$	M1		$0.175 \pm (z\text{-value within list})\times\sqrt{a}$ OE
		M1		Expression for $\sqrt{a}$
	0.175 ± 0.047	A1		CAO/AWRT (0.04666)
	or **(0.128, 0.222)**		6	AWRT
		Total	**6**	

Q	Solution	Marks	Total	Comments
3 (a)	H_0: (λ or μ) = 8 H_1: (λ or μ) < 8	B1		Both; accept $\lambda/p = 0.008$
	P(FE $\leq$ 5 | λ = 8) = 0.1912 or 0.3134 or 0.0996	M1		Any one
	= 0.191	A1		AWRT
	< 0.05	m1		Correct comparison (PI)
	Accept H_0 $\Rightarrow$ **no evidence**, at 5% level, that upgrade **has reduced** average number of faulty envelopes per pack	Adep1	5	Dep on previous 4 marks OE; but must not be definitive
(b)	H_0: (λ or μ) = (8 or 400) H_1: (λ or μ) < (8 or 400)	(B1)		Both; iff not scored in (a)
	1% (0.01) $\Rightarrow$ z = **<u>–2.32 to –2.33</u>** **or** p-value of z-calculated = **<u>0.005 < 0.01</u>**	B1		AWFW (–2.3263) AWRT (0.0046 to 0.0051)
	$z = \frac{348-400}{\sqrt{400}}$ **or** $\frac{6.96-8}{\sqrt{8/50}}$ = **<u>–2.6</u>** **or** $z = \frac{348.5-400}{\sqrt{400}}$ = **<u>–2.57 to –2.58</u>**	M1 A1 A1		$(x-\lambda)\big/\sqrt{\lambda/(n)}$ or $\left(\left(x(\pm 0.5)\right)-\lambda\right)\big/\sqrt{\lambda/(n)}$ A **correct** expression CAO (–2.6) AWFW (–2.575)
	Reject H_0 $\Rightarrow$ **evidence**, at 1% level, that refurbishment **has reduced** average number of faulty envelopes per pack	Adep1	5	Dep on previous 4 (or 5) marks OE; but must not be definitive
Note	**1** Use of Poisson and P($X \leq 348$ | $\lambda = 400$) = 0.0043 (AWRT) < 0.01 $\Rightarrow$ (B1) B1 (0.01) M1 (Po(400)) A2 (0.0043) A1dep1			
		Total	**10**	

Q	Solution	Marks	Total	Comments
4 (a)	$P(C1) = 0.45 \times 0.80 + 0.25 \times 0.10 =$ **<u>0.385</u>**	B1	(1)	CAO; (77/200)
(b)	$P(M \mid C1) = \dfrac{0.45 \times 0.80}{0.385}$ **or** $\dfrac{0.36}{\textbf{(a)}}$	M1		
	= **<u>0.935</u>**	A1	(2)	AWRT; (72/77) (0.93506)
(c)	$P(E \mid C2) =$			
	$\dfrac{0.25 \times 0.85}{(0.45 \times 0.05) + (0.25 \times 0.85) + (0.30 \times 0.25)}$	M1		Numerator
	or			
	$\dfrac{0.2125}{0.0225 + 0.2125 + 0.0750}$ **or** $\dfrac{0.2125}{0.3100}$	M1		Denominator
	= **<u>0.685 to 0.686</u>**	A1	(3)	AWFW; (85/124) (0.68548)
(d)	$P(W' \mid C3') = \dfrac{P(M \cap C3') + P(E \cap C3')}{P(C3')} =$			
	$\dfrac{(0.45 \times 0.85) + (0.25 \times 0.95)}{(0.45 \times 0.85) + (0.25 \times 0.95) + (0.30 \times 0.25)}$	M1 M1		Numerator
	or	M1		Denominator; accept [Num + (0.30 × 0.25)] or [(a) + (Den(c)]
	$\dfrac{0.3825 + 0.2375}{0.3825 + 0.2375 + 0.0750}$ **or** $\dfrac{0.6200}{0.6950}$			
	or			
	$1 - \dfrac{0.3 \times 0.25}{0.695} = 1 - 0.108$	(M1) (M1) (M1)		1 – Numerator Denominator
	= **<u>0.892</u>**	A1	(4)	AWRT; (124/139) (0.89209)
		Total	**10**	

Q	Solution	Marks	Total	Comments
5(a) (i)	E(X) = $(0 \times 0.15) + (1 \times 0.4) + (2 \times 0.3) + (3 \times 0.15)$			
	$= 0 + 0.4 + 0.6 + 0.45 =$ **<u>1.45</u>**	B1		CAO; (29/50)
	$E(X^2) = (1^2 \times 0.4) + (2^2 \times 0.3) + (3^2 \times 0.15)$ $= 0.4 + 1.2 + 1.35 =$ <u>2.95 or 59/20</u>	M1		PI
	$\text{Var}(X) = 2.95 - 1.45^2$	m1		Use of $\{E(X^2) - (E(X))^2\}$ iff > 0
	= **<u>0.847 to 0.848</u>**	A1	4	AWFW; (339/400) (0.8475)
(ii)	$\text{Var}(Y) = 1.95 - 0.85^2$ = **<u>1.22 to 1.23</u>**	B1		AWFW; (491/400) (1.2275)
	$\text{Cov}(X, Y) = 0.90 - 1.45 \times 0.85$	M1		FT only on E(X) from (a)(i)
	= **<u>–0.333 to –0.332</u>**	A1	3	AWFW; (–133/400) (–0.3325)
(iii)	Cor(X, Y) = $\dfrac{-0.3325}{\sqrt{0.8475 \times 1.2275}}$ **or** $\dfrac{-133}{\sqrt{339 \times 491}}$	M1		FT from (a)(i) & (ii)
	= **<u>–0.326</u>**	A1	2	AWRT (–0.32560)
(b) (i)	$E(T) = 1.45 + 0.85$ = **<u>2.3</u>**	B1		CAO
	$\text{Var}(T) = 0.8475 + 1.2275 + 2 \times -0.3325$	M1		PI; use of $\sigma_X^2 + \sigma_Y^2 \pm 2 \times \text{Cov}(X, Y)$ or $\sigma_X^2 + \sigma_Y^2 \pm 2 \times \sigma_X \times \sigma_Y \times \text{Cor}(X, Y)$; here or in (b)(ii)
	= **<u>1.41</u>**	A1	(3)	AWRT
(ii)	$E(D) = 1.45 - 0.85$ = **<u>0.6</u>**	B1		CAO
	$\text{Var}(D) = 0.8475 + 1.2275 - 2 \times -0.3325$			
	= **<u>2.74</u>**	A1	(2)	AWRT
			5	
		Total	**14**	

Q	Solution	Marks	Total	Comments
6 (a)	$\mathrm{E}(X) = \sum_{x=0}^{\infty} x \binom{n}{x} p^x (1-p)^{n-x} =$	M1		Used; ignore limits until A1
	$np \sum_{x=1}^{\infty} \frac{(n-1)!}{(x-1)!(n-x)!} p^{x-1} (1-p)^{n-x} =$	M1		Factor of np plus $n!$ to $(n-1)!$ and $x!$ to $(x-1)!$
	Using $u = x-1$ and $m = n-1$ gives $np \sum_{u=0}^{m} \frac{m!}{u!(m-u)!} p^u (1-p)^{m-u} = np \times 1 = np$	A1	3	Fully **complete & correct** derivation but allow minor slips in limits AG
Note	**1** Other valid derivations are possible and acceptable			
(b)	$\mathrm{E}(Y(Y-1)) = \sum_{y=0}^{\infty} y(y-1) \frac{e^{-\lambda}\lambda^y}{y!} =$	M1		Used; ignore limits until A1
	Using $v = x-2$ gives $\lambda^2 \sum_{y=2}^{\infty} \frac{e^{-\lambda}\lambda^{y-2}}{(y-2)!} = \lambda^2 \sum_{v=2}^{\infty} \frac{e^{-\lambda}\lambda^v}{v!}$ $= \lambda^2 \times 1 = \lambda^2$	A1		Factor of λ^2 plus $x!$ to $(x-2)!$ and fully **complete & correct** derivation but allow minor slips in limits
	Var(Y) = E(Y(Y – 1)) + E(Y) – $(\mathrm{E(Y)})^2$ **or** $(\mathrm{E(Y)})^2 = \lambda^2 + \lambda$ so Var(Y) = = $\lambda^2 + \lambda - \lambda^2 = \lambda$	B1	3	Fully **correct** deduction AG
Note	**1** Other valid derivations are possible and acceptable			
(c)(i)	A ~ B(50, 0.005) P(FS = 1) = $\binom{50}{1}(0.005)(0.995)^{49}$	M1		PI
	= **<u>0.195 to 0.196</u>**	A1	2	AWFW (0.19556)
SC	**1** Use of $0.25e^{-0.25}$ = 0.19470 ⇒ B1			
(ii)	B ~ B(250, 0.005) ⇒ **<u>Po(1.25)</u>**	B1		PI
	P(FS < 2) = $e^{-1.25}(1+1.25)$	M1		PI; a **correct** Poisson expression
	= **<u>0.644 to 0.645</u>**	A1	3	AWFW (0.64464)
(iii)	C ~ B(50000, 0.005) ⇒ **<u>N(250, 248.75)</u>**	B1 B1		PI: normal and 250 CAO 248.75 CAO
	P(FS > 240) = $\mathrm{P}\left(Z > \frac{240.5-250}{\sqrt{248.75}}\right) =$	M1 A1		Allow 240 or 239.5 and/or $\sqrt{250}$ **Fully correct** expression
	P(Z > –0.60234) = **<u>0.725 to 0.727</u>**	Adep1	5	Dependent on A1 (0.72653)
SC	**1** B1 (Po(250)) M1 (0.745 or 0.724 or 0.702 (AWRT)) A1 (0.724 (AWRT)) ⇒ max of 3 marks			
		Total	**16**	

Q	Solution	Marks	Total	Comments
7 (a)	H$_0$: $\mu_X - \mu_Y = 1.5$ H$_1$: $\mu_X - \mu_Y >$ **1.5**	B1 B1		Award for $\mu_X - \mu_Y = 0$ Allow any valid notation
	5% (0.05) $\Rightarrow$ z = **1.64 to 1.65** **or** p-value of z-calculated = **0.08 > 0.05**	B1		AWFW (1.6449) AWRT (0.0810)
	$\bar{x}$ = **4.53** and $\bar{y}$ = **2.88**	B1		CAO: both
	s_X^2 = **0.300** and s_Y^2 = **0.120** **or** s_X = **0.548** and s_Y = **0.346**	B1		AWRT; both (0.30006 and 0.12002) AWRT; both (0.54777 and 0.34644)
	$z = \dfrac{(4.53-2.88)-1.5}{\sqrt{\dfrac{0.300}{40}+\dfrac{0.120}{30}}} = \dfrac{0.15}{\sqrt{0.0115}} = \mathbf{1.40}$	M1 M1 A1		Numerator; allow (4.53 – 2.88) Denominator; OE AWRT (1.39863)
	Accept H$_0$ $\Rightarrow$ **no evidence**, at 5% level, that difference in mean lengths is **more than** 1.5 metres	Adep1	9	Dep on previous 8 marks OE; but must not be definitive
Notes	**1** Invalid pooling of variances $\Rightarrow$ z =1.31 $\Rightarrow$ B1 B1 B1 B1 B1 M1 M0 A0 Adep 0 (max = 6) **2** Use of σ^2/σ (0.293/0.541 and 0.116/0.341) $\Rightarrow$ B1 B1 B1 B1 B1 M1 M0 A0 Adep 0 (max = 6) unless ($\div(n-1)$) in z **3** Omission of 1.5 throughout $\Rightarrow$ z = 15.4 $\Rightarrow$ B1 B0 B1 B1 B1 M1 M0 A0 Adep0 (max = 5)			
(b)	CV is given by: $\dfrac{(\bar{X}-\bar{Y})-1.5}{\sqrt{\dfrac{0.300}{40}+\dfrac{0.120}{30}}}$ = 1.64 to 1.65	M1		Fully **correct** equality
	$(\bar{X}-\bar{Y})$ = (0.107 × 1.64 to 1.65) + 1.5 = (0.175 to 0.177) + 1.5 = **1.68**	A1	2	AWRT; AG (1.67641)
(c)	Power = P(reject H$_0$ | H$_0$ false) = $\text{P}\left((\bar{X}-\bar{Y})>1.68 \mid (\mu_X-\mu_Y)=1.85\right)$ =			
	$\text{P}\left(Z>\dfrac{1.68-1.85}{\sqrt{\dfrac{0.300}{40}+\dfrac{0.12}{30}}}\right)$ or $\text{P}\left(Z>\dfrac{-0.17}{\sqrt{0.0115}}\right)$	M1 A1		Correct use of 1.68 (OE) Ignore sign of inequality Correct numerical expression
	= P(Z > **–1.58 to –1.63**)	A1		AWFW (PI); ignore sign (–1.58511)
	= **0.94 to 0.95**	A1	4	AWFW (0.94353)
		Total	**15**	