

General Certificate of Education (A-level) June 2013

Mathematics
MS03

## (Specification 6360)

Statistics 3

## Final

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## Key to mark scheme abbreviations

| M | mark is for method |
| :---: | :---: |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of M or m marks and is for method and accuracy |
| E | mark is for explanation |
| $\checkmark$ or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0 ) accuracy marks |
| -x EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 1(a) | $98 \% \Rightarrow z=\underline{2.32 ~ t o ~} 2.33$ | B1 |  | AWFW (2.3263) |
|  | Approximate CI for $\lambda$ : $\quad \hat{\lambda} \pm z \sqrt{\hat{\lambda}}$ | M1 |  | Used |
|  | $392 \pm 2.3263 \times \sqrt{392}$ | AF1 |  | F on $z$ |
|  | Per shift $\Rightarrow \div 12$ | M1 |  |  |
|  | Thus: $\quad \underline{32.7 \pm 3.8 \text { or }(28.8,36.5)}$ | A1 | 5 | AWRT |
| (b) | Per hour (weekday night) $\Rightarrow$ (2.05 to 2.06, 2.6 to 2.61) | BF1 |  | F on (a) |
|  | $\operatorname{Per} \text { hour }(\text { weekend })=\frac{136.8}{48}=\underline{\mathbf{2 . 8 5}}$ | B1 |  |  |
|  | Thus evidence to agree with claim | BF1 | 3 | F on comparison of value with CI Definitive conclusion $\Rightarrow \mathrm{BFO}$ |
|  | Total |  | 8 |  |



| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 3(a) | 95\% $\Rightarrow \quad z=\underline{1.96}$ | B1 |  | AWRT |
|  | $\bar{x}=\underline{1026} \quad \bar{y}=\underline{1045}$ | B1 |  | Both CAO |
|  | CI for $\mu_{\mathrm{Y}}-\mu_{\mathrm{X}}$ is $(\bar{y}-\bar{x}) \pm z \sqrt{\frac{\sigma_{\mathrm{Y}}^{2}}{n_{\mathrm{Y}}}+\frac{\sigma_{\mathrm{X}}^{2}}{n_{\mathrm{X}}}}$ | M1 m1 |  | Used <br> Accept $(\bar{x}-\bar{y})$ throughout SD term |
|  | ie |  |  |  |
|  | $(1045-1026) \pm 1.96 \sqrt{\frac{30^{2}}{8}+\frac{25^{2}}{10}}$ | AF1 |  | F on $\bar{x}, \bar{y}$ and $z$ |
|  | ie $\quad 19 \pm 25.9$ or $(-6.9,44.9)$ | A1 |  | CAO \& AWRT or AWRT |
|  | ie $\quad \underline{20 \pm 25}$ or (-5 or $-10,45$ ) | B1 | 7 | Rounding answer to nearest 5 kg |
| (b) | Fred used: machine X for sand and machine Y for gravel | B1 |  | Apparent rounding to nearest 5 kg |
|  | Use each machine for both | B1 | 2 | OE |
|  | Total |  | 9 |  |
| 4 | $\mathrm{H}_{0}: p_{\mathrm{M}}-p_{\mathrm{D}}=0.10$ | B1 |  | If B0 B0, then |
|  | $\mathrm{H}_{1}: p_{\mathrm{M}}-p_{\mathrm{D}}>0.10$ | B1 |  | award B1 for $p_{\mathrm{M}}-p_{\mathrm{D}}=0$ |
|  | $95 \% \Rightarrow z=\underline{1.64} \text { to } 1.65$ | B1 |  | AWFW (1.6449) |
|  | $z=\frac{\left(\hat{p}_{\mathrm{M}}-\hat{p}_{\mathrm{D}}\right)-0.10}{\sqrt[\hat{p}_{\mathrm{M}}\left(1-\hat{p}_{\mathrm{M}}\right)]{ }+\hat{p}_{\mathrm{D}}\left(1-\hat{p}_{\mathrm{D}}\right)}=$ | M1 |  | Used; allow pooling and/or 'no -0.10' |
|  | $\sqrt{n_{M}} n_{\text {D }}$ | m1 |  | Denominator |
|  | $\frac{(0.38-0.21)-0.10}{\sqrt{\frac{0.38 \times 0.62}{250}+\frac{0.21 \times 0.79}{100}}}=$ | A1 |  | Correct expression but allow 'no -0.10' |
|  | $\frac{0.07}{0.051}=\underline{\mathbf{1} .37}$ | A1 |  | AWRT <br> (1.3724) |
|  | No evidence, at $5 \%$ level, to suggest that the difference is more than 10 per cent | AF1 | 8 | F on CV and $z$-value <br> Definitive conclusion $\Rightarrow$ AF0 |
|  | Total |  | 8 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 5(a)(i) | $\begin{array}{rr} \hline L=X+Z & \mathrm{E}(L)=68+73 \underline{\mathbf{1 4 1}} \\ \mathrm{~V}(L)=10^{2}+15^{2}=\underline{\mathbf{3 2 5}} \end{array}$ | B1 B1 | 2 | CAO |
| (ii) | $M=X+Y$ $\mathrm{E}(M)=68+25=\underline{\mathbf{9 3}}$ | B1 |  | CAO |
|  | $\begin{aligned} \mathrm{V}(M)=10^{2}+5^{2}+2 & \times 10 \times 5 \times(-0.8) \\ = & 100+25-80=\underline{\mathbf{4 5}} \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { A1 } \end{aligned}$ | 3 | $\begin{aligned} & \text { Allow 'no 2' } \\ & \text { CAO } \end{aligned}$ |
| (b)(i) | Require: $\mathrm{P}(L<150)=$ $\mathrm{P}\left(\mathrm{Z}<\frac{150-141}{\sqrt{325}}\right)$ | M1 |  | Standardising 150 using c's $E(L) \& c^{\prime} s V(L)$ from (a)(i) |
|  | $=\mathrm{P}(\mathrm{Z}<0.5) \quad=\underline{\mathbf{0 . 6 9} \text { to } 0.692}$ | A1 | 2 | AWFW $\quad$$(0.49923)$ <br> $(0.69119)$ |
| (ii) | Require: $\mathrm{P}(X+Y>105)=\mathrm{P}(M>105)$ |  |  |  |
|  | $=\mathrm{P}\left(Z>\frac{105-93}{\sqrt{45}}\right)$ | M1 |  | Standardising 105 using c's $\mathrm{E}(M)$ \& c's $\mathrm{V}(M)$ from (a)(ii) |
|  | $=\mathrm{P}(\mathrm{Z}>1.79)=1-\mathrm{P}(\mathrm{Z}<1.79)$ | m1 |  | Correct area change <br> May be implied by a correct answer <br> or by an answer $<\mathbf{0 . 5}$ |
|  | $=\underline{0.036 ~ t o ~ 0.038 ~}$ | A1 | 3 | AWFW (0.03682) |
|  | Total |  | 10 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 6(a)(i) | $\lambda=6 \times 2.5=\underline{\mathbf{1 5}}$ | B1 |  | CAO |
|  | $\mathrm{P}(W \leq 18)=\underline{\mathbf{0 . 8 1 9} \text { to } \mathbf{0 . 8 2}}$ | B1 | 2 | AWFW (0.8195) |
| (ii) | $\mathrm{P}(W>w) \leq 0.05 \Rightarrow \mathrm{P}(W \leq w) \geq 0.95$ | M1 |  | Implied by a value of 21,22 or 23 |
|  | $w=\underline{22}$ | A1 | 2 | CAO |
| (b)(i) | $F \sim \underline{\mathbf{N}(\mathbf{3 0}, \mathbf{3 0})}$ | B1 |  | May be implied |
|  | $\begin{aligned} & \mathrm{P}(F>35)= \\ & \quad \mathrm{P}\left(Z>\frac{35.5-30}{\sqrt{30}}\right)=\mathrm{P}(Z>1.00) \end{aligned}$ | M1 B1 |  | Standardising (34.5, 35 or 35.5 ) $\text { with } \mu=\sigma^{2}$ <br> 35.5 <br> (1.00416) |
|  | $=\underline{0.157 ~ t o ~} 0.16$ | A1 | 4 | AWFW (0.15765) |
| (ii) | $\begin{aligned} & \mathrm{P}(F>f) \leq 5 \% \Rightarrow \\ & \quad \mathrm{P}\left(Z>\frac{(f+0.5)-30}{\sqrt{30}}\right) \leq 0.05 \end{aligned}$ | M1 |  | Standardising ( $f-0.5, f$ or $f+0.5$ ) with $\mu=\sigma^{2}$ |
|  | $5 \% \Rightarrow z=\underline{1.64 ~ t o ~} 1.65$ | B1 |  | AWFW (1.6449) |
|  | So $f=\underline{39}$ | Adep1 | 3 | CAO <br> Dependent on $(f+0.5)$ and on B1 |
|  | Total |  | 11 |  |


| Q | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| 7(a) | $\begin{aligned} & \mathrm{H}_{0}: p=0.50 \\ & \mathrm{H}_{1}: p>0.50 \end{aligned}$ | $\begin{aligned} & \text { B1 } \\ & \text { B1 } \end{aligned}$ |  | Here or in (b)(i) |
|  | $\begin{aligned} \mathrm{P}(X \geq 29 \mid \mathrm{B}(50,0.50)= \\ 1-(\mathbf{0 . 8 3 8 9} \text { or } \mathbf{0 . 8 9 8 7}) \end{aligned}$ | $\begin{aligned} & \text { M1 } \\ & \text { M1 } \end{aligned}$ |  | Use of $\mathrm{B}(50,0.50)$; may be implied |
|  | $=\underline{0.16 ~ t o ~} 0.165$ | A1 |  | AWFW (0.16112) |
|  | No evidence to support the claim | AF1 | 6 | F on $10 \%$ and ( $p$-value $>0.10$ ) Definitive conclusion $\Rightarrow$ AF0 |
| (b)(i) | $10 \% \Rightarrow \quad z=\underline{\mathbf{1} .28}$ | B1 |  | AWRT (1.2816) |
|  | $z=\frac{\frac{271}{500}-0.5}{\sqrt{\frac{0.5 \times 0.5}{500}}}=\underline{\mathbf{1 . 8 7} \text { to } \mathbf{1 . 8 9}}$ | M1 A1 |  | Accept use of $\hat{p}$ in denominator giving $z=1.88511$ <br> AWFW <br> (1.87830) |
|  | Evidence to support the claim | AF1 | 4 | F on CV and $z$-value <br> Definitive conclusion $\Rightarrow$ AF0 |
| (ii) | $\begin{aligned} \text { Power } & =1-\mathrm{P}(\text { Type II error }) \\ & =1-\mathrm{P}\left(\text { accept } \mathrm{H}_{0} \mid \mathrm{H}_{0} \text { false }\right) \\ & \text { or } \mathrm{P}\left(\text { reject } \mathrm{H}_{0} \mid \mathrm{H}_{0} \text { false }\right) \\ & \text { or } \mathrm{P}\left(\text { accept } \mathrm{H}_{1} \mid \mathrm{H}_{1} \text { true }\right) \end{aligned}$ | B1 |  | Any one stated or used |
|  | $\mathrm{P}(\hat{P}>0.529 \mid \mathrm{B}(500,0.55))=$ | M1 |  | Use of $\mathrm{B}(500,0.55)$ <br> M0 for use of 0.529 or 0.5 |
|  | $\mathrm{P}\left(Z>\frac{0.529-0.55}{\sqrt{0.55 \times 0.45}}\right)=\mathrm{P}(Z>-$ | M1 |  | Accept use of 0.529 in denominator giving $z=0.94075$ but not use of 0.5 Ignore inequality and sign |
|  | $\underline{0.94})$ | A1 |  | AWRT <br> (0.94388) |
|  | $=\underline{0.82 ~ t o ~ 0.83 ~}$ | A1 | 5 | AWFW (0.82738) |
|  | Total |  | 15 |  |
|  | TOTAL |  | 75 |  |

