

6. A random variable X has probability density function given by

$$f(x) = \begin{cases} \frac{1}{2} & 0 \leq x < 1 \\ x - \frac{1}{2} & 1 \leq x \leq k \\ 0 & \text{otherwise} \end{cases}$$

where k is a positive constant.

- (a) Sketch the graph of $f(x)$. (2)
- (b) Show that $k = \frac{1}{2}(1 + \sqrt{5})$. (4)
- (c) Define fully the cumulative distribution function $F(x)$. (6)
- (d) Find $P(0.5 < X < 1.5)$. (2)
- (e) Write down the median of X and the mode of X . (2)
- (f) Describe the skewness of the distribution of X . Give a reason for your answer. (2)



