## AQA

# A-LEVEL <br> Mathematics 

Statistics MS2 - MS2B
Mark scheme

6360<br>June 2015

Version/Stage 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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## Key to mark scheme abbreviations

| M | mark is for method |
| :---: | :---: |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of $M$ or m marks and is for method and accuracy |
| E | mark is for explanation |
| $\checkmark$ or ft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| -x EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns full marks, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains no marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

| Q1 | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| (a) | Use of $\operatorname{Po}(2.8)$ | M1 |  | Stated or table value ( $0.8477,0.9349$, 0.9756 or 3sf equivalents) seen |
|  | $\mathrm{P}(\leq 5)=0.935$ | A1 | 2 | AWRT |
| (b) | Use of $\operatorname{Po}(4.4)$ | B1 |  | Stated or attempt at method seen |
|  | $\mathrm{e}^{-4.4} \times 4.4{ }^{2} \div 2$ | M1 |  | Correct formula or by calculator |
|  | $=0.119$ | A1 |  | AWRT |
|  |  |  | 3 |  |
| (c) | Use of Po(15) | B1 |  | Stated or any 1 of 4 relevant values seen $0.1848,0.2676,0.7489,0.8195$ |
|  | We require $\mathrm{P}(\leq 17)$ | M1 |  | Stated or use of 0.7489 |
|  | $-\mathrm{P}(\leq 11)$ | M1 |  | Indep. Stated or use of 0.1848 |
|  | $=0.7489-0.1848=\mathbf{0 . 5 6 4}(1)$ | A1 | 4 | AWRT 0.564 |
|  |  | Total | 9 |  |

Note: (a) The mark is not awarded for simply 2.8. Some indication of Poisson is needed.
Eg. $\mathrm{Po}(2.8)$ or $\lambda=2.8$
(b) As for part (a), not simply 4.4.
(c) If $\mathrm{Po}(15)$ and $\mathrm{P}(\leq 17)-\mathrm{P}(\leq 11)$ are seen, 3 marks have been earned irrespective of later numbers.

| Q2 | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| (a) | $k=(b-a)$ | B1 | 1 | CAO |
| (b)(i) | $\begin{aligned} & \frac{1}{2}(a+b)=1 \quad \text { and } \quad \frac{1}{12}(b-a)^{2}=3 \\ & (b-a)^{2}=36 \quad \rightarrow \quad(b-a)= \pm 6 \\ & \quad b>a \text { stated giving } b-a=6 \quad \text { only } \\ & \text { or both } b-a=6 \text { and } b-a=-6 \text { used } \\ & b=4 \text { and } a=-2 \end{aligned}$ | B1 <br> M1 <br> m1 <br> A1 | 4 | For both equations (not including $k$ ) 6 or $\pm 6$ required for this mark <br> Consideration of two solutions <br> CAO not dependent on m1 |
| (ii) | $\begin{aligned} & \mathrm{P}(X<0)=\frac{\mathbf{1}}{\mathbf{3}} \\ & 4 \times p \times(1-p)^{3} \\ & \text { where } p=\text { candidate's stated } \mathrm{P}(X<0) \\ & \left.=\frac{\mathbf{3 2}}{\mathbf{8 1}} \mathbf{(}=\mathbf{0 . 3 9 5}\right) \end{aligned}$ | B1 <br> M1 <br> A1 | 3 | Stated or used (accept 0.333) $0<p<1$ <br> CAO or AWRT 0.395 |
|  |  | Total | 8 |  |



Note: (b) Integration may be used but must reach the two correct equations to earn any marks.
Many will use $b-a=6$, ignoring the $\pm$, and obtain the correct values for $a$ and $b$.
This scores B1 M1 m0 A1.

| Q3 | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| (a)(i) | Mean of sample is $\mathbf{9 0 9 . 2}$ <br> Use of 1.96 $909.2 \pm 1.96 \times \frac{2.2}{\sqrt{8}}$ <br> 907.7, 910.7 <br> Notes: 1 Seen use of $s \Rightarrow$ B1 B1 M0 A0 max <br> 2 Seen use of $t \Rightarrow \mathrm{~B} 1$ В0 м 0 A 0 max <br> 3 Seen use of $t$ and $s \Rightarrow$ B1 B0 M0 A0 max | B1 <br> B1 <br> M1 <br> A1 | 4 | If wrong here, the B1 here may be earned for a correct value seen in (ii) <br> AWRT <br> Allow for M1 if AWFW 1.64 to 1.65 used instead of 1.96 <br> For both. AWRT |
| (ii) | $\begin{aligned} & t_{7}=2.365 \\ & s=2.39 \text { or } 2.24 \text { (or } s^{2}=5.72 \text { or } 5.00(5) \text { ) } \\ & 909.2 \pm(2.36 \text { to } 2.37) \times s e \\ & \text { where } s e=2.39 / \sqrt{ } 8 \text { or } 2.24 / \sqrt{ } 7 \\ & \mathbf{9 0 7 . 2}, \mathbf{9 1 1 . 2} \end{aligned}$ <br> Notes: 1 Seen use of $2.2 \Rightarrow$ В1 в0 м 0 A0 max 2 Seen use of $z \Rightarrow$ B0 B1 M0 A0 max <br> 3 Seen use of $z$ and $2.2 \Rightarrow$ Во В0 м0 А0 | B1 <br> B1 <br> M1 <br> A1 | 4 | AWFW 2.36 to 2.37 <br> AWRT <br> Allow for M1 if AWFW 1.89 to 1.90 used instead of (2.36 to 2.37) <br> OE in terms of $s^{2}$ <br> For both. AWRT |
| (b) | Both confidence intervals are above 907 so mean/average weight is probably acceptable <br> One of data values (or 905.6) is below 907 (or underweight) | Edep1 <br> E1 | 2 | OE Dependent on A1 in (i) and A1 in (ii). Must specify both, 907 and mean/average. |
|  |  | Total | 10 |  |

Note: In both (a)(i) and (ii), where working is shown, condone accuracy to more than 4 s.f.
Where working is not shown, if accurate to 4 s.f. allow B4. If not accurate to 4 s.f., award B1 for AWRT 908-911 in (i) and another B1 for AWRT 907-911 in (ii).

| Q4 | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| (a) | (The 100 vehicles can be regarded as a) random (sample). | B1 | 1 | Must say random and be about the sample. Do not penalise "and independent", but any mention of "normal" anywhere in (a) scores B0 |
| (b) | So test statistic in critical region. (Reject $\mathrm{H}_{0}$ ), significant evidence that mean speed has reduced. | B1 <br> B1 <br> B1 <br> M1 <br> m1 <br> A1 <br> B1 <br> Adep1 | 8 | Both. Must be "Population mean", $\mu_{x}$ or $\mu$. <br> CAO <br> AWFW 3.055 to 3.060. <br> AWFW 3.040 to 3.045 <br> Denominator is division of candidate's sd by $\sqrt{ } 100$ or $\sqrt{ } 99$ <br> Numerator is $\pm(\bar{x}-44.1$ or 40$)$ <br> AWFW -2.695 to -2.735 <br> AWFW -2.32 to -2.33 <br> AWFW -2.36 to -2.37 <br> Dep on preceding A1 and B1, but not on B1 for hypotheses. Must have context and mean (or average). |
| (c) (i) <br> (ii) | Concluding that the mean speed has reduced (or changed) when in fact it has not <br> Concluding that the mean speed is still 44.1 when in fact it has reduced (or changed) | E1 E1 | 2 | Must be in context. Must refer to mean speed ( $\mu$ ) <br> Must be in context. Must refer to mean speed ( $\mu$ ) |
|  |  | Total | 11 |  |

Note: (a) "It is random" is sufficient for B1.
"It is random and normally distributed" scores B0.
"The vehicles arrive at random" scores B0
The final A mark is not awarded for the negative statement "There is no significant evidence that the mean speed is 44.1 " or equivalent. There is significant evidence of a reduction in the mean. A definite statement "the mean speed has reduced" is accepted for A1.

Alternative method for (b) using critical value for $\bar{x}$

| Q4 | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| (b) | $\mathrm{H}_{0}: \mu_{x}=44.1$ |  |  | Both. Must be "Population mean", $\mu_{x}$ |
|  | $\mathrm{H}_{1}: \mu_{\mathrm{x}}<44.1$ | B1 |  | or $\mu$. |
|  | ( $\bar{x}=$ ) 43.27 | B1 |  | CAO |
|  | sd = $3.0579 \quad(\mathrm{var}=9.35$ AWRT) |  |  | AWFW 3.055 to 3.060. |
|  | $\begin{aligned} & \text { or } \\ & \text { sd }=3.0425(\mathrm{var}=9.26 \text { AWRT }) \end{aligned}$ | B1 |  | AWFW 3.040 to 3.04 |
|  | $C V: z=-2.32(63)$ |  |  | AWFW -2.32 to -2.33 |
|  | or $t=-2.36(46)$ | B1 |  | AWFW -2.36 to -2.37 |
|  | $\bar{x}_{\mathrm{cv}}=44.1-\mathrm{CV} \times \frac{3.0579}{\sqrt{ } 100} \text { or } \times \frac{3.0425}{\sqrt{ } 99}$ | M1 |  | Division of candidate's sd by $\sqrt{ } 100$ or $\sqrt{ } 99$ |
|  |  | m1 |  | Rest of formula |
|  | $=43.37$ to 43.395 | A1 |  | AWFW 43.37 to 43.395 |
|  | $43.27<43.37 \text { or } 43.395$ |  |  | Dep on preceding A1 and B1 but not |
|  | (Reject $\mathrm{H}_{0}$ ), significant evidence that mean speed has reduced. | Adep1 |  | on B1 for hypotheses. Must have context and mean (or average). |

Alternative method for (b) using confidence interval for $\bar{x}$



Note: (a) Final A mark is not awarded for the double negative statement "No significant evidence that there is no association ....". There is significant evidence of an association. A definite conclusion "there is an association between age at leaving education and rate ..." is accepted for A1 "Association" is the expected word. Use of independent must say "tax rate is not independent of age ...". No other words are accepted.

| Q6 | Solution | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: |
| (a) | $\begin{aligned} & F(0.4)=\frac{0.4}{2}-\frac{0.16}{16}=0.2-0.01=\mathbf{0 . 1 9} \\ & F(0.8)=\frac{0.8}{2}-\frac{0.64}{16}=0.4-0.04=\mathbf{0 . 3 6} \\ & P(0.4<X<0.8)=0.36-0.19=\mathbf{0 . 1 7} \end{aligned}$ | M1 A1 | 2 | For either, can be implied by correct answer. <br> CAO |
| (b) | Clear correct use of differentiation of $\mathrm{F}(x)$. | B1 | 1 | AG Sight of $\mathrm{F}^{\prime}(x), \frac{d}{d x}, \frac{d y}{d x}$ etc. $=$ correct answer |
| (c)(i) | $\begin{aligned} \mathrm{E}(X) & =\int_{0}^{4}\left(\frac{1}{2} x-\frac{1}{8} x^{2}\right) \mathrm{d} x \\ & =\left[\frac{1}{4} x^{2}-\frac{1}{24} x^{3}\right]_{0}^{4} \\ & =4-\frac{8}{3}=\frac{4}{3} \end{aligned}$ | M1 <br> A1 <br> A1 | 3 | Attempt at integrating $x f(x)$ (condone omission of limits and $\mathrm{d} x$ ) <br> Integration completed correctly with limits <br> OE exact form |
| (ii) | $\begin{aligned} \mathrm{E}\left(X^{2}\right) & =\int_{0}^{4}\left(\frac{1}{2} x^{2}-\frac{1}{8} x^{3}\right) \mathrm{d} x \\ & =\left[\frac{1}{6} x^{3}-\frac{1}{32} x^{4}\right]_{0}^{4} \\ & =\frac{32}{3}-8=\frac{8}{3} \\ \operatorname{Var}(X) & =\mathrm{E}\left(X^{2}\right)-\mathrm{E}(X)^{2}=\frac{8}{3}-\left(\frac{4}{3}\right)^{2} \quad\left(=\frac{8}{9}\right) \end{aligned}$ | M1 <br> A1 <br> A1 <br> A1 | 4 | Attempt at integrating $x^{2} \mathrm{f}(x)$ (condone omission of limits and $\mathrm{d} x$ ) <br> Integration completed correctly with limits <br> OE exact form <br> AG |
| (d) | $\begin{aligned} & \mathrm{E}(Y)=3 \mathrm{E}(X)-2=3 \times \frac{4}{3}-2=\mathbf{2} \\ & \operatorname{Var}(Y)=3^{2} \times \operatorname{Var}(X)=9 \times \frac{8}{9}=\mathbf{8} \end{aligned}$ | $\begin{gathered} \text { B1F } \\ \text { B1 } \end{gathered}$ | 2 | FT their (c)(i) provided $0<\mathrm{E}(X)<4$ CAO |
|  |  | Total | 12 |  |


| Q7 | Solution |  |  |  |  |  | Marks | Total | Comments |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| (a) | (I) $a$ requires the " $=3$ " value using $\operatorname{Po}(2)$ $=\left(\mathrm{e}^{-2} \times 2^{3}\right) \div 3$ ! or $0.8571-0.6767$ or $0.1804 \ldots$ from calculator $=\mathbf{0 . 1 8 0}$ <br> (II) $b=1-\mathrm{P}($ demand $\leq 3)=1-0.8571=\mathbf{0 . 1 4 3}$ <br> (III) $b=1-(0.135+0.271+0.271+0.180)=\mathbf{0 . 1 4 3}$ <br> (IV) $a=1-(0.135+0.271+0.271+0.143)=\mathbf{0 . 1 8 0}$ <br> (I) \& (II) or (I) \& (III) or (II) \& (IV) <br> SC If M0 can award B1 for $a+b=0.323$ derived from sum of probabilities $=1$ |  |  |  |  |  | M1 <br> m1 <br> A1 |  | One M1 for correct use of correct Poisson for either $a$ or $b$. <br> A dependent m 1 for use of Poisson again for $b$ or $a$ or for subsequent use of probability sum = 1 <br> A1 for both correct calculations AG |
| (b) | $\begin{aligned} & \mathrm{E}(X)= \\ & 1 \times 0.135+2 \times 0.271+3 \times 0.271+4 \times 0.180+5 \times 0.143 \\ & (=0.135+0.542+0.813+0.72+0.715) \\ & =2.925 \\ & \mathrm{E}\left(X^{2}\right)=1^{2} \times 0.135+2^{2} \times 0.271+3^{2} \times 0.271+ \\ & 4^{2} \times 0.180+5^{2} \times 0.143 \\ & (=0.135+1.084+2.439+2.88+3.575) \\ & =\mathbf{1 0 . 1 1 3} \\ & \\ & \text { S.D. }=\sqrt{ }\left(10.113-2.925^{2}\right)=\mathbf{1 . 2 5} \end{aligned}$ |  |  |  |  |  | M1 <br> A1 <br> M1 <br> A1 <br> B1 | 5 | Evidence of at least two of the five products added <br> OE AWFW 2.92 to 2.93 <br> Evidence of at least two of the five products added <br> AWRT 10.1 <br> AWRT |
| (c) | $1 \times \mathrm{E}(X)$ $=£ 1.89$ or profit// <br> Profit <br> $\mathrm{P}(X=x)$ <br> E(Profit) <br> $0.715=£$ | $0.5 \times$ ss tab -1 0.135 -0.135 89 | $5-\mathrm{E}$ <br> 0.5 0.271 $+0.13$ | ) <br> 2 0.271 $5+0 .$ | $\begin{gathered} \hline \frac{3.5}{0.180} \\ \hline 42+0 \end{gathered}$ | $\begin{array}{\|c\|} \hline 5 \\ \hline 0.143 \\ \hline \end{array}$ $630+$ | M1 <br> A1 <br> (M1) <br> (A1) | 2 | Candidate's $\mathrm{E}(X)$ AWRT Condone omission of ' $£$ ' <br> AWRT Condone omission of ' $£$ ' |

Note: (a) One of the three methods of getting 0.180 - formula, subtraction of two figures from tables, or direct calculation showing fourth decimal place (4) - must be seen before the M1 for use of Poisson is awarded. Similarly for $0.143(0.1429)$ done using Poisson.

If value of $\mathrm{E}(X)$ calculated in part (b) is used retrospectively in part (a) to calculate $a$ and $b$, then only the SC B1 can be earned.


