AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Statistics 2B

Tuesday 20 June 2017 Afternoon Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 The continuous random variable X has the cumulative distribution function

$$\mathrm{F}(x) = \begin{cases} 0 & x < 3 \\ \frac{1}{6}(x-3) & 3 \leqslant x \leqslant 9 \\ 1 & x > 9 \end{cases}$$

(a) Find the upper quartile of X.

[2 marks]

(b) Find the mean and the standard deviation of X.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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2 The total rainfall, in millimetres, during each month is recorded at a weather station. The figures recorded for the month of April for eight years are given below.

4.8 53.3 11.8 92.6 13.7 59.0 1.8 47.8

(a) State **two** assumptions that must be made about these data in order to use them to construct a confidence interval for the mean April rainfall at this weather station.

[2 marks]

(b) Assuming that these assumptions are valid, construct a 99% confidence interval for the mean April rainfall at this weather station. Give the limits to one decimal place.

[5 marks]

(c) By considering your confidence interval, comment on the likely validity of your assumptions in part **(a)**.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	**Answer space for question 2**

3 On a certain housing estate it is found that, in a lawn which has been treated with weedkiller, dandelion plants occur at an average rate of 0.1 per m^2. In a lawn which has not been treated with weedkiller, dandelion plants occur at an average rate of 0.3 per m^2.

You may assume that the number of dandelion plants in a lawn may be modelled by a Poisson distribution.

(a) Mr Brown has a rectangular lawn measuring $7\,\text{m}$ by $3\,\text{m}$. He uses weedkiller to treat his lawn.

Mrs Green has a rectangular lawn measuring $5\,\text{m}$ by $4\,\text{m}$. She does not use weedkiller on her lawn.

Find the probability that:

(i) Mr Brown's lawn has exactly 4 dandelion plants;

[2 marks]

(ii) Mrs Green's lawn has at least 6 but fewer than 10 dandelion plants;

[4 marks]

(iii) Mr Brown's lawn has at least one dandelion plant and Mrs Green's lawn has none.

[3 marks]

(b) For circular lawns of radius r metres, which have been treated with weedkiller, the standard deviation of the number of dandelion plants occurring is σ.

Show that $\sigma = kr$, where k is a constant, and find the value of k, giving your answer to three decimal places.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

QUESTION PART REFERENCE	**Answer space for question 3**

4 Students at Kentside School study one of three foreign languages, French, German or Spanish, during Years 9, 10 and 11.

Toni, the Head of Languages at Kentside, uses χ^2-tests to investigate whether, for each of the current Years 9, 10 and 11, there is a gender bias in the choice of language.

For **each** Year, the null hypothesis is that there is no association between gender and choice of language, and the test is conducted at the **10% significance level**.

(a) For **Year 9**, the observed frequencies of boys and girls studying the three languages are shown in **Table 1**.

Table 1

Observed	**French**	**German**	**Spanish**
Boys	15	8	20
Girls	13	6	28

Complete **Table 2** below to show the corresponding expected frequencies, giving each to two decimal places.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 4(a)

Table 2

Expected	**French**	**German**	**Spanish**
Boys			
Girls			

QUESTION PART REFERENCE	**Answer space for question 4(a)**

Question 4 continues on the next page

4 (b) For **Year 10**, the observed and expected frequencies of boys and girls studying the three languages are shown in **Tables 3 and 4** respectively.

Table 3

Observed	**French**	**German**	**Spanish**
Boys	18	9	15
Girls	12	6	30

Table 4

Expected	**French**	**German**	**Spanish**
Boys	14	7	21
Girls	16	8	24

Toni used these data in the χ^2-test and obtained a test statistic value of $X^2 = 6.43$, correct to three significant figures.

(i) Show the calculations necessary to obtain this value of X^2.

[2 marks]

(ii) Complete the test.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 4(b)**

QUESTION PART REFERENCE	**Answer space for question 4(b)**

Question 4 continues on the next page

4 (c) For **Year 11**, the observed and expected frequencies of boys and girls studying the three languages are shown in **Tables 5 and 6** respectively.

Table 5

Observed	**French**	**German**	**Spanish**
Boys	13	6	27
Girls	4	5	25

Table 6

Expected	**French**	**German**	**Spanish**
Boys	9.775	6.325	29.900
Girls	7.225	4.675	22.100

(i) Before conducting the χ^2-test, Toni combined the data in the French and German columns. Explain why this was necessary.

[1 mark]

(ii) When the French and German columns have been combined, state the **formula** that should be used to calculate the value of the test statistic for the χ^2-test.

[1 mark]

(iii) State the critical value that must then be used.

[1 mark]

QUESTION PART REFERENCE	**Answer space for question 4(c)**

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5 The discrete random variable X is defined by

$$\mathrm{P}(X = x) = \begin{cases} c(8-x) & x = 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of c.

[1 mark]

(b) Three independent values of X are taken. Find the probability that at least one of these is greater than 4.

[3 marks]

(c) Find $\mathrm{E}(X)$ and show that $\mathrm{Var}(X)$ has the same value.

[5 marks]

(d) Variable Y is related to X by the equation

$$Y = \frac{42}{X}$$

Find $\mathrm{E}(Y)$.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

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6 Helen is a conservationist who monitors the health of fish in a river. She is investigating whether a new water-treatment plant has affected the weight of adult trout in the river. Previously, the mean weight of adult trout in the river has been 1.25 kg.

Helen catches 10 adult trout and measures the weight, X kilograms, of each fish. Her summarised data are

$$\sum x = 16.6 \qquad \sum (x - \bar{x})^2 = 4.858$$

You may assume that these data constitute a random sample from a normal distribution.

(a) Carry out a hypothesis test, at the 10% level of significance, to investigate whether there has been a change in the mean weight of adult trout in the river.

[8 marks]

(b) At the end of the fishing season, data from several hundred anglers show that the mean weight of trout caught along the river this season is 1.35 kg.

State, with a reason, whether, in your test in part **(a)**, you made a Type I error, a Type II error or no error.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

7 (a) The continuous random variable X has probability density function defined by

$$f(x) = \begin{cases} k(x+a)^2(x-a)^2 & -a \leqslant x \leqslant a \\ 0 & \text{otherwise} \end{cases}$$

(i) Sketch the graph of f on the axes below.

[3 marks]

(ii) Find the value of k. You may assume that $(x+a)^2(x-a)^2 = x^4 - 2a^2x^2 + a^4$.

[4 marks]

(iii) State the value of $\mathrm{E}(X)$.

[1 mark]

(iv) Find the value of $\mathrm{Var}(X)$.

[4 marks]

(b) The continuous random variable Y is related to the variable X by the formula

$$Y = 7(X+a)$$

(i) State the value of $\mathrm{E}(Y)$.

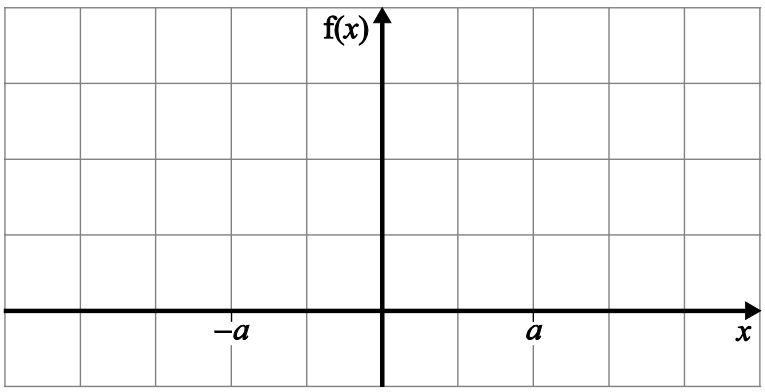
[1 mark]

(ii) State the value of $\mathrm{Var}(Y)$.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 7



QUESTION PART REFERENCE	**Answer space for question 7**

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QUESTION PART REFERENCE	**Answer space for question 7**

END OF QUESTIONS

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