Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE
In Mechanics M5 (6681/01)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

‘M’ marks
These are marks given for a correct method or an attempt at a correct method. In Mechanics they are usually awarded for the application of some mechanical principle to produce an equation.
e.g. resolving in a particular direction, taking moments about a point, applying a suvat equation, applying the conservation of momentum principle etc.
The following criteria are usually applied to the equation.

To earn the M mark, the equation
(i) should have the correct number of terms
(ii) be dimensionally correct i.e. all the terms need to be dimensionally correct
e.g. in a moments equation, every term must be a ‘force x distance’ term or ‘mass x distance’, if we allow them to cancel ‘g’ s.
For a resolution, all terms that need to be resolved (multiplied by sin or cos) must be resolved to earn the M mark.

M marks are sometimes dependent (DM) on previous M marks having been earned.
e.g. when two simultaneous equations have been set up by, for example, resolving in two directions and there is then an M mark for solving the equations to find a particular quantity – this M mark is often dependent on the two previous M marks having been earned.

‘A’ marks
These are dependent accuracy (or sometimes answer) marks and can only be awarded if the previous M mark has been earned. E.g. M0 A1 is impossible.

‘B’ marks
These are independent accuracy marks where there is no method (e.g. often given for a comment or for a graph)

A few of the A and B marks may be f.t. – follow through – marks.

3. General Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- * The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
    - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
    - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Mechanics Marking

(But note that specific mark schemes may sometimes override these general principles)

- Rules for M marks: correct no. of terms; dimensionally correct; all terms that need resolving (i.e. multiplied by cos or sin) are resolved.

- Omission or extra g in a resolution is an accuracy error not method error.

- Omission of mass from a resolution is a method error.

- Omission of a length from a moments equation is a method error.

- Omission of units or incorrect units is not (usually) counted as an accuracy error.

- DM indicates a dependent method mark i.e. one that can only be awarded if a previous specified method mark has been awarded.

- Any numerical answer which comes from use of g = 9.8 should be given to 2 or 3 SF.

- Use of g = 9.81 should be penalised once per (complete) question.

  N.B. Over-accuracy or under-accuracy of correct answers should only be penalised *once* per complete question. However, premature approximation should be penalised every time it occurs.

- Marks must be entered in the same order as they appear on the mark scheme.

- In all cases, if the candidate clearly labels their working under a particular part of a question i.e. (a) or (b) or (c),……then that working can only score marks for that part of the question.

- Accept column vectors in all cases.

- Misreads – if a misread does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, bearing in mind that after a misread, the subsequent A marks affected are treated as A ft

- Mechanics Abbreviations

  M(A) Taking moments about A.

  N2L Newton's Second Law (Equation of Motion)

  NEL Newton's Experimental Law (Newton's Law of Impact)

  HL Hooke's Law

  SHM Simple harmonic motion

  PCLM Principle of conservation of linear momentum

  RHS, LHS Right hand side, left hand side.

Question Number	Scheme	Marks
1.	**ALTERNATIVE 1**	
	Direction of line is $(\mathbf{i}+\mathbf{2j})$	B1
	$\mathbf{d}=\dfrac{\sqrt{80}}{\sqrt{1^2+2^2}}(\mathbf{i}+2\mathbf{j})=(4\mathbf{i}+8\mathbf{j})$	M1 A1
	W.D.$=\lvert(4\mathbf{i}-3\mathbf{j}).(4\mathbf{i}+8\mathbf{j})\rvert=8$ J	M1 A1 **[5]**
	ALTERNATIVE 2	
	Direction of line is $(\mathbf{i}+\mathbf{2j})$	B1
	$\cos\theta=\dfrac{(4\mathbf{i}-3\mathbf{j}).(\mathbf{i}+2\mathbf{j})}{5\sqrt{5}}=\dfrac{-2}{5\sqrt{5}}$	M1 A1
	W.D.$=\lvert(4\mathbf{i}-3\mathbf{j})\rvert\lvert\cos\theta\rvert\times\sqrt{80}=8$ J	M1 A1
	ALTERNATIVE 3	
	Direction of line is $(\mathbf{i}+\mathbf{2j})$	B1
	$\hat{\mathbf{d}}=\dfrac{(\mathbf{i}+2\mathbf{j})}{\sqrt{5}}$	M1 A1
	W.D.$=\lvert(4\mathbf{i}-3\mathbf{j}).\hat{\mathbf{d}}\rvert\times\sqrt{80}=8$ J	M1 A1
	Allow column vectors throughout	
	Notes	
	See alternatives. Answer must be positive. Ignore units. B1 For $(\mathbf{i}+\mathbf{2j})$ or multiple seen **ALTERNATIVE 1** First M1 for attempt to find the displacement vector **d** First A1 for (4**i** + 8**j**) Second M1 for WD = $(4\mathbf{i}-3\mathbf{j}).\mathbf{d}$ (**d** must be a multiple of $(\mathbf{i}+\mathbf{2j})$) Second A1 for 8 (J) **ALTERNATIVE 2** First M1 for attempt to find the angle (or cos thereof) between their **d** (must be a multiple of $(\mathbf{i}+\mathbf{2j})$) and $(4\mathbf{i}-3\mathbf{j})$ First A1 for $\cos\theta=\dfrac{-2}{5\sqrt{5}}$ oe Second M1 for W.D.$=\lvert(4\mathbf{i}-3\mathbf{j})\rvert\lvert\cos\theta\rvert\times\sqrt{80}$ Second A1 for 8 (J)	

	ALTERNATIVE 3 First M1 for attempt to find a unit vector in the direction of their direction vector First A1 for $\hat{\mathbf{d}} = \frac{(\mathbf{i}+2\mathbf{j})}{\sqrt{5}}$ oe Second M1 for W.D.$=\left\|(4\mathbf{i}-3\mathbf{j}).\hat{\mathbf{d}}\right\| \times \sqrt{80}$ ($\hat{\mathbf{d}}$ must be a multiple of $(\mathbf{i}+2\mathbf{j})$) Second A1 for 8 (J)	

Question Number	Scheme	Marks
2.	$\mathbf{R} = \sum \mathbf{F}_i = \begin{pmatrix} a \\ b \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix} = \begin{pmatrix} a-2 \\ b-2 \\ -3 \end{pmatrix}$	M1 A1
	Taking moments about O,	
	$\mathbf{G} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \times \begin{pmatrix} a \\ b \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ -1 \\ 1 \end{pmatrix} \times \begin{pmatrix} -1 \\ 1 \\ -2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix} \times \begin{pmatrix} -1 \\ -3 \\ 1 \end{pmatrix}$	M1
	$= \begin{pmatrix} -b \\ a \\ 0 \end{pmatrix} + \begin{pmatrix} 1 \\ 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 7 \\ -2 \\ 1 \end{pmatrix}$	A3
	$= \begin{pmatrix} 8-b \\ a+3 \\ 3 \end{pmatrix}$	A1 **cao**
	$8-b = \lambda(a-2)$ $a+3 = \lambda(b-2)$ Changed $3 = \lambda \times -3$	M1A1**ft**
	$a = \frac{-7}{2};\ b = \frac{5}{2}$	A1; A1 cao **[11]**
	Notes	
	First M1 for adding the 3 forces First A1 for a correct **R** Second M1 for moments about O; allow **F** x **r** Second, third, fourth A1 : **-** 1 each error or omission (A1 for each product) Allow consistent negatives (if using **F** x **r)** Fifth A1 **cao** Third M1 for using 'their **G** parallel with their **R'** to obtain three equations Sixth A1 **ft** Correct ft equations Seventh A1 -3.5 Eighth A1 2.5	

Question Number	Scheme	Marks
3.	$\dfrac{d\mathbf{r}}{dt}+\tan\left(\tfrac{1}{2}t\right)\mathbf{r}=\sin\left(\tfrac{1}{2}t\right)\cos^2\left(\tfrac{1}{2}t\right)\mathbf{i}+\mathbf{j}$	M1
	$\text{R} = e^{\int \tan\left(\frac{1}{2}t\right)dt} = \sec^2\left(\tfrac{1}{2}t\right)$	M1 A1
	$\mathbf{r}\sec^2(\dfrac{1}{2}t)=\int \sin\left(\tfrac{1}{2}t\right)\mathbf{i}+\sec^2\left(\tfrac{1}{2}t\right)\mathbf{j}\,dt$	M1 A1
	$=-2\cos\left(\tfrac{1}{2}t\right)\mathbf{i}+2\tan\left(\tfrac{1}{2}t\right)\mathbf{j}(+\mathbf{C})$	A1
	$t=0,\ \mathbf{r}=-\mathbf{i}+\mathbf{j} \Rightarrow \mathbf{C}=\mathbf{i}+\mathbf{j}$	M1
	$\mathbf{r}=\left(\cos^2\left(\tfrac{1}{2}t\right)-2\cos^3\left(\tfrac{1}{2}t\right)\right)\mathbf{i}+\left(2\sin\left(\tfrac{1}{2}t\right)\cos\left(\tfrac{1}{2}t\right)+\cos^2\left(\tfrac{1}{2}t\right)\right)\mathbf{j}$	A1 **[8]**
	Notes	
	First M1 for dividing by $\sec^2\left(\tfrac{1}{2}t\right)$ Second M1 for using a correct formula for the IF First A1 for a correct integrating factor in terms of t Third M1 for multiplying through by IF and attempting to integrate both sides Second A1 for a correct equation with LHS integrated correctly Third A1 for a fully correct equation, **C not needed** Fourth M1 for use of limits to find **C** Fourth A1 for the answer in any form **N.B.** If components used, marks can only be scored once the components have been put together	

Question Number	Scheme	Marks
4.	$\int_0^2 x^2 \mathrm{d}x = \frac{8}{3}$	M1
	$\rho = \frac{3M}{8}$	A1
	$\delta m = y\rho\delta x$	
	$= \frac{3Mx^2\delta x}{8}$	M1
	$\delta I = \frac{1}{3}\delta m y^2$	
	$= \frac{M}{8}x^6\delta x$	M1
	$I = \frac{M}{8}\int_0^2 x^6 \mathrm{d}x = \frac{16M}{7}$ kgm^2	M1 A1
		[6]
	Notes	
	First M1 for finding the area of the lamina First A1 for a correct mass per unit area (ρ) Second M1 for an expression for the mass of the strip in terms of *M* and *x* only. Third M1 for a CORRECT expression for the MI of the strip in terms of *M* and *x* only. Fourth M1 for summing these MI's from $x = 0$ to $x = 2$ Second A1 for the answer	

Question Number	Scheme	Marks
5(a)	$(m+\delta m)(v+\delta v)+(-\delta m)(v-U)-mv=-mg\delta t$	M1 A2
	$mv+v\delta m+m\delta v+U\delta m-v\delta m-mv=-mg\delta t$	
	$m\frac{\mathrm{d}v}{\mathrm{d}t}+U\frac{\mathrm{d}m}{\mathrm{d}t}=-mg$ GIVEN ANSWER	A1 (4)
(b)	$mg+U\frac{\mathrm{d}m}{\mathrm{d}t}=-mg$	
	$U\frac{\mathrm{d}m}{\mathrm{d}t}=-2mg$	M1
	$\int_M^m \frac{\mathrm{d}m}{m}=\frac{-2g}{U}\int_0^t \mathrm{d}t$	M1
	$\left[\ln m\right]_M^m=\frac{-2gt}{U}$	A1
	$m=Me^{\frac{-2gt}{U}}$ GIVEN ANSWER	A1 (4)
(c)	$(1-\lambda)M=Me^{\frac{-2gT}{U}}$	M1
	$T=\frac{U}{2g}\ln(\frac{1}{1-\lambda})$	A1
	$v=gT=\frac{U}{2}\ln(\frac{1}{1-\lambda})$	M1 A1
	$\text{KE}=\frac{1}{2}(1-\lambda)M\left[\frac{U}{2}\ln(\frac{1}{1-\lambda})\right]^2=\frac{1}{8}MU^2(1-\lambda)\left[\ln(\frac{1}{1-\lambda})\right]^2$	M1 A1 (6)
		[14]
	Notes	
	5(a) First M1 for use of Impulse-Momentum principle, dimensionally correct, with correct no. of terms, with usual rules. First and second A1 for a correct equation Third A1 for the correct given answer correctly obtained.	
	5(b) First M1 for putting $\frac{\mathrm{d}v}{\mathrm{d}t}=g$ into the DE and collecting terms Second M1 for separating the variables and attempting to integrate both sides First A1 for a correct integral on both sides with correct limits oe Second A1 for the GIVEN ANSWER correctly obtained.	

	5(c) First M1 for putting *m* = $(1-\lambda)M$ and solving for *T* First A1 for a correct expression for *T* Second M1 for using *v* = *u* + *at* oe with *u* = 0, *a* = *g* and *t* = their *T* to find *v* Second A1 for a correct *v* Third M1 for use of KE formula (M0 if they use *M* for mass) Third A1 for a correct expression in any form.	

Question Number	Scheme	Marks
6(a)	$\text{MI} = 2 \times \dfrac{4ma^2}{3}, + \left(\dfrac{ma^2}{3} + m(2a\cos 30^{o})^2\right)$ $= 6ma^2$ GIVEN ANSWER	M1 A1, A1 A1 (4)
(b)	$X - 3mg\cos 60^{o} = 0$ since $\dot{\theta}=0$ $X = \dfrac{3mg}{2}$ $3mg\sin 60^{o} \pm Y = 3m.\dfrac{2}{3}2a\cos 30^{o}\ddot{\theta}$ $M(A)$, $3mga = 6ma^2\ddot{\theta}$ OR $6a\ddot{\theta} = 2\sqrt{3}g\sin 60^{o}$ Eliminating $\ddot{\theta}$, $Y = \dfrac{mg\sqrt{3}}{2}$ $R = \sqrt{X^2 + Y^2} = \dfrac{mg}{2}\sqrt{3+3^2} = mg\sqrt{3}$	M1 A1 M1 A1A1 M1 A1 **DM**1 A1 M1A1 (11) [**15]**
	Notes	
	6(a) M1 for attempt at MI, dimensionally correct, with correct no. of terms First A1 for first two terms Second A1 for third term unsimplified Third A1 for correct GIVEN ANSWER	
	6(b) First M1 for equation of motion inwards with usual rules and RHS = 0 First A1 for a correct value of *X* Second M1 for equation of motion tangentially with usual rules Second and Third A1 for a correct equation. A1A0 if one error. Third M1 for equation of rotational motion about *A* to give equation in $\ddot{\theta}$ only Fourth A1 for a correct equation **N.B. If *m* is used consistently in all 3 equations, treat as only one error and penalise in first equation.**	

	Equation of rotational motion could be obtained by differentiating an energy equation and putting $\theta = 0$: $3mg.\frac{2}{3}2a\cos 30^o.(\sin(\theta + 30^o) - \sin 30^o)) = \frac{1}{2}6ma^2\dot{\theta}^2$ $2mga\sqrt{3}(\sin(\theta + 30^o) - \sin 30^o)) = 3ma^2\dot{\theta}^2$ Differentiating, $2mga\sqrt{3}\cos(\theta + 30^o) = 6ma^2\ddot{\theta}$ When $\theta = 0$, $2mga\sqrt{3}\cos 30^o = 6ma^2\ddot{\theta}$ $6a\ddot{\theta} = 2\sqrt{3}g\cos 30^o)$ Fourth **DM**1, dependent on previous 2 M's, for eliminating to give *Y* in terms of *mg* Fifth M, independent, for finding magnitude using their *X* and *Y*, <u>in terms of *mg* only</u> Fifth A1 for correct answer	

Question Number	Scheme	Marks
7(a)	$3mg.2l\sin\theta + 4mg.4l\sin\theta = -2m(a^2+40l^2)\ddot{\theta}$	M1 A1
	For small θ, $\sin\theta \simeq \theta$	
	$-\dfrac{11gl}{(a^2+40l^2)}\theta = \ddot{\theta}$	M1
	SHM; $T = 2\pi\sqrt{\dfrac{(a^2+40l^2)}{11gl}}$	M1A1 (5)
7(b)	$4mg.4l + 3mg.2l = \dfrac{1}{2}2m(a^2+40l^2)\omega^2$	M1 A1
	$\omega^2 = \dfrac{22gl}{(a^2+40l^2)}$ GIVEN ANSWER	A1 (3)
(c)	$J.2l = 2m(a^2+40l^2)(\dfrac{1}{2}\omega - -\omega)$	M1 A2
	$J = \dfrac{3m}{2}\sqrt{\dfrac{22g(a^2+40l^2)}{l}}$	A1 (4)
(d)	$\dfrac{1}{2}2m(a^2+40l^2)(\dfrac{1}{2}\omega)^2 = 3mg.2l\sin\theta + 4mg.4l\sin\theta$	M1 A1
	$\dfrac{1}{8}2m(a^2+40l^2)\dfrac{22gl}{(a^2+40l^2)} = 22mgl\sin\theta$	**DM**1
	$\sin\theta = \dfrac{1}{4} \Rightarrow \theta = \sin^{-1}\dfrac{1}{4}$ GIVEN ANSWER	A1 (4)
		[16]
	Notes	
	7(a) First M1 for equation of motion about *A* with usual rules First A1 for a correct equation Second M1 for using small angle approx. and putting into appropriate form (with – sign) Third M1 for SHM and use of correct formula for period Second A1 for correct answer	
	7(b) M1 for conservation of energy equation with usual rules First A1 for a correct equation Second A1 for correct GIVEN ANSWER	

	7(c) M1 for impulse-momentum equation with usual rules (*I* and ω do not need to be substituted) First A1 for a correct equation without *I* and ω substituted Second A1 for a correct equation with *I* and ω substituted Third A1 for correct answer (must be positive)	
	7(d) First M1 for conservation of energy equation with usual rules First A1 for a correct equation without ω substituted Second DM1, dependent, for substituting for ω Second A1 for correctly obtaining GIVEN ANSWER	

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