A-Level

Mathematics

MM05 Mechanics 5

Final Mark scheme

6360

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Q	Solution	Mark	Total	Comment
1 (a)	$\text{Period} = 2\pi\sqrt{\frac{2.45}{9.8}}$	M1		M1: Uses formula with correct length.
	$= \pi$			
	$= 3.14\text{ s}$	A1	2	A1: Correct period.
1 (b)	$\text{Average Speed} = \dfrac{2\times2\times2.45\times\frac{\pi}{10}}{\pi}$	M1 A1		M1: Correct distance found. A1: Correct expression for average speed.
	$= 0.98\text{ m s}^{-1}$	A1	3	A1: Correct average speed.
1 (c)	$\theta = \frac{\pi}{10}\cos(2t)$	M1		M1: Correct expression for θ.
	$v = -\frac{2.45\pi}{5}\sin(2t)$	A1		A1: Correct expression for velocity.
	$1.2 = \frac{2.45\pi}{5}\sin(2t)$	dM1		dM1: Forming equation to find t.
	$t = \frac{1}{2}\sin^{-1}\left(\frac{5\times1.2}{2.45\pi}\right) = 0.4469...$	A1		A1: Correct time.
	$\theta = 0.197$	A1	5	A1: Correct θ.
	OR			
	$\frac{1}{2}m\times1.2^2 = m\times9.8\times2.45\left(\cos\theta - \cos\left(\frac{\pi}{10}\right)\right)$	(M1) (A1) (A1)		M1: Energy equation with two terms correct. A1: Correct terms but allow sign errors. A1: Correct equation.
	$\cos\theta = \frac{0.72}{24.01} + \cos\left(\frac{\pi}{10}\right)$	(dM1)		dM1: Solving for θ.
	$\theta = 0.195$	(A1)	(5)	A1: Correct value of θ.
	OR			
	$1.2^2 = 2^2\left(\left(\frac{2.45\pi}{10}\right)^2 - (2.45\theta)^2\right)$	(M1) (A1) (A1)		M1: Use of $v^2 = \omega^2(a^2 - x^2)$ with consistent terms. A1: Correct terms but possible sign errors. A1: Correct terms.
	$\theta = \frac{1}{2.45}\sqrt{\left(\frac{2.45\pi}{10}\right)^2 - \left(\frac{1.2}{2}\right)^2}$	(dM1) (A1)		dM1: Solving for θ.
	$= 0.197$		(5)	A1: Correct value of θ.
	Total		**10**	

Q	Solution	Mark	Total	Comment
2 (a)	$T_1 = 0.4g + T_2$ $\frac{49}{0.5}(d-0.5) = 0.4g + \frac{49}{0.5}(2-d-0.5)$ $98d - 49 = 3.92 + 147 - 98d$ $196d = 199.92$ $d = 1.02$	M1 A1 dM1 A1	 4	M1: Three force equation with at least two terms correct. A1: Correct equation. dM1: Solving for *d*. A1: Correct *d*.
2 (b)	$0.4\frac{d^2x}{dt^2} = T_2 + 0.4g - T_1$ $= \frac{49}{0.5}(2-1.02-x-0.5) +$ $0.4\times 9.8 - \frac{49}{0.5}(x+1.02-0.5)$ $= 47.04 - 98x + 3.92 - 98x$ $- 50.96$ $= -196x$ $\frac{d^2x}{dt^2} = -490x$ $\text{Period} = \frac{2\pi}{\sqrt{490}} = \frac{\pi\sqrt{10}}{35}$	M1A1 A1 A1 A1	 5	M1: Equation of motion with at least two terms correct. A1: Correct terms but possible sign errors. A1: Correct equation. A1: Correct simplified differential equation. A1: Correct period from correct working.
2 (c) (i)	$v_{\max} = \sqrt{490}\times 0.05 = \frac{7\sqrt{10}}{20} = 1.11\,\text{m s}^{-1}$	M1A1	2	M1: Use of $a\omega$. A1: Correct max speed.
2 (c) (ii)	$v^2 = 490(0.05^2 - 0.025^2) = \frac{147}{160}$ $= 0.91875$ $v = \sqrt{0.91875} = 0.959\,\text{m s}^{-1}$	M1A1 A1	 3	M1: Use of $v^2 = \omega^2(a^2 - x^2)$ with correct ω. A1: Correct equation. A1: Correct speed.
	Total		**14**	

Q	Solution	Mark	Total	Comment
3 (a)	$EPE=\frac{4mg}{2\times 1}\left(\sqrt{x^2+y^2}-1\right)^2$	M1		M1: EPE in terms of *x* and *y*.
	$=2mg\left(\sqrt{x^2+16-8x^2+x^4}-1\right)^2$	A1		A1: EPE in terms of *x*.
	$=2mg\left(\sqrt{16-7x^2+x^4}-1\right)^2$	A1		A1: EPE expanded correctly.
	$=2mg\left(17-7x^2+x^4-2\sqrt{16-7x^2}\right.$ …			
	$GPE=mgy=mg(4-x^2)$	B1		B1: Correct GPE.
	$V=mg\left(38-15x^2+2x^4-4\sqrt{16-7x^2+x^4}\right)$	A1	5	A1: Correct final result from correct working.
3 (b)	$\frac{dV}{dx}=mg\left(-30x+8x^3-\frac{8x^3-28x}{\sqrt{16-7x^2+x^4}}\right)$	M1A1		M1: Differentiates with no more than one error. A1: Correct derivative.
	$x=2, \frac{dV}{dx}=0$	dM1		dM1: Substitutes $x=2$.
	So there is a position of equilibrium when x = 2.	A1	4	A1: Obtains correct conclusion.
3 (c)	At x =1.9 $\frac{dV}{dx}=-2.99mg$	M1		M1: Substitutes a value of *x* just less than 2.
	At x =2.1 $\frac{dV}{dx}=3.94mg$	M1		M1: Substitutes a value of *x* just greater than 2.
	As increasing this corresponds to a minimum value of *V* and hence is a position of stable equilibrium.	A1	3	A1: Uses values to reach the correct conclusion.
	OR			
	$\frac{d^2V}{dx^2}=mg\left(24x^2-30x+\frac{(8x^3-28x^2)(2x^3-7)}{\left(\sqrt{16-7x^2+x^4}\right)^{1.5}}+\frac{(28\text{-}24x^2)}{\sqrt{16-7x^2+x^4}}\right)$	(M1) (A1)		M1: Second derivative of the correct format. A1: Correct second derivative.
	At $x=2,\ \frac{d^2V}{dx^2}=34mg$ This corresponds to a minimum value of *V* and hence is a position of stable equilibrium.	(A1)	(3)	A1: Correct value at $x=2$ and correct conclusion.
	Total		**12**	

Q	Solution	Mark	Total	Comment
4 (a)	$1 = \sin 2t$	M1		M1: Using $r = 1$ to form an equation.
	$t = \frac{\pi}{4} + n\pi$	A1		A1: Finding *t*.
	$\dot{r} = 2\cos 2t$	B1		B1: Correct $\dot{\theta}$.
	$\dot{\theta} = 2$			
	$\ddot{\theta} = 0$			
	$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8\cos 2t$	M1		M1: Expression transverse component.
	$\cos\left(\frac{\pi}{2} + 2n\pi\right) = 0$	A1	5	A1: Obtaining zero from correct working.
	OR			
	$1 = \sin\theta$			
	$\cos\theta = 0$	(M1) (A1)		M1: Using $r = 1$ to form an equation. A1: Finding $\cos\theta$.
	$\dot{r} = 2\cos\theta$			
	$\dot{\theta} = 2$	(B1)		B1: Correct $\dot{\theta}$.
	$\ddot{\theta} = 0$			
	$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 8\cos\theta = 0$	(M1) (A1)	(5)	M1: Expression transverse component. A1: Obtaining zero from correct working.
4 (b)	$\ddot{r} = -4\sin 2t$	M1		M1: Finding radial component.
	$\ddot{r} - r\dot{\theta}^2 = -4\sin 2t - 4\sin 2t$	A1		A1: Correct radial component.
	$= -8\sin 2t$			
	$0 = -8\sin 2t$	M1		M1: Forming equation to find *t*.
	$t = 0 + \frac{n\pi}{2}$	A1	4	A1: Correct time(s).
4 (c)	$r\dot{\theta} = 2\sin 2t$	M1		M1: Finding transverse component of the velocity.
	$\sin(n\pi) = 0$			
	$r\dot{\theta} = 0$	A1	2	A1: Correct conclusion from correct working.
	Total		**11**	

Q	Solution	Mark	Total	Comment
5	$m\dfrac{d^2x}{dt^2} = mg - \dfrac{2.5mx}{0.5} - 2m\dfrac{dx}{dt}$	M1A1		M1: Forming equation of motion, with at least two terms correct. A1: Correct differential equation.
	$\dfrac{d^2x}{dt^2} + 2\dfrac{dx}{dt} + 5x = g$			
	$PI \quad x = \dfrac{g}{5}$	B1		B1: Correct PI.
	$CF \quad \lambda^2 + 2\lambda + 5 = 0$			
	$\lambda = \dfrac{-2 \pm \sqrt{2^2 - 4\times1\times5}}{2\times1} = -1 \pm 2i$	M1 A1		M1: Solving auxiliary equation. A1: Correct roots.
	$x = e^{-t}(A\sin 2t + B\cos 2t) + \dfrac{g}{5}$	A1		A1: Correct general solution.
	$t = 0, x = 0$			
	$0 = B + \dfrac{g}{5}$	M1		M1: Finding one constant.
	$B = -g/5$	A1		A1: Correct constant.
	$\dot{x} = -e^{-t}(A\sin 2t + B\cos 2t)$ $+ e^{-t}(2A\cos 2t - 2B\sin 2t)$	M1		M1: Finding derivative.
	$\dot{x} = e^{-t}((-A - 2B)\sin 2t$ $+ (-B + 2A)\cos 2t)$	A1		A1: Correct derivative.
	$t = 0, \dot{x} = 0$			
	$0 = -B + 2A$			
	$A = -\dfrac{g}{10}$	A1		A1: Second constant correct.
	$x = ge^{-t}\left(-\dfrac{\sin 2t}{10} - \dfrac{\cos 2t}{5}\right) + \dfrac{g}{5}$			
	$\dot{x} = ge^{-t}\left(\dfrac{5\sin 2t}{8}\right)$			
	$\dot{x} = 0 \Rightarrow t = \dfrac{\pi}{2}$	M1 A1		M1: Finding time for zero speed. A1: Correct time.
	$x = ge^{-\frac{\pi}{2}}\left(-\dfrac{\sin\pi}{10} - \dfrac{\cos\pi}{5}\right) + \dfrac{g}{5}$	M1		M1: Using time to find max length.
	Max Length of String $= \dfrac{g}{5}\left(1 + e^{-\frac{\pi}{2}}\right) + 0.5 = 2.87 \text{ m}$	A1		A1: Correct maximum length.
			15	
	Total		**15**	

Q	Solution	Mark	Total	Comment
6 (a)	$mg\delta t = (m+\delta m)(v+\delta v) + (-\delta m)(v+\delta v+U) - mv$	M1 A1		M1: Use of momentum-impulse equation. (Must have $v+U$) A1: Correct equation.
	$mg\delta t = m\delta v - U\delta m$			
	$mg = m\frac{dv}{dt} - U\frac{dm}{dt}$	dM1		dM1: Forming differential equation.
	$\frac{dm}{dt} = -\lambda, m = M - \lambda t$	M1		M1: Expression for M in terms of t.
	$mg = m\frac{dv}{dt} + \lambda U$			
	$\frac{dv}{dt} = g - \frac{\lambda U}{M - \lambda t}$	A1	5	A1: Correct result from correct working.
6 (b)	$t = 0, m = M, \frac{dv}{dt} < 0$	M1		M1: Statement that acceleration is less than zero.
	$g - \frac{\lambda U}{M} < 0$	dM1		dM1: Use of time as zero.
	$U > \frac{Mg}{\lambda}$	A1	3	A1: Correct result from correct working.
6 (c)	$\int 1 dv = \int g - \frac{\lambda U}{M - \lambda t} dt$			
	$v = gt + U\ln(M - \lambda t) + c$	M1A1		M1: Integrating to obtain linear and ln terms. A1: Correct integral.
	$t = 0, v = \frac{U}{20} \Rightarrow c = \frac{U}{20} - U\ln M$	A1		A1: Correct constant.
	$v = gt + U\ln\left(\frac{M - \lambda t}{M}\right) + \frac{U}{20}$			
	$t = \frac{M}{10\lambda}$	M1		M1: Correct time.
	$v = \frac{gM}{10\lambda} + \frac{3gM}{2\lambda}\ln\left(\frac{9}{10}\right) + \frac{3gM}{40\lambda}$			
	$= \frac{3gM}{2\lambda}\ln\left(\frac{9}{10}\right) + \frac{7gM}{40\lambda}$	A1	5	A1: Correct velocity.
	Total		**13**	