AQA

A-LEVEL
Mathematics

MM05 – Mechanics 5
Mark scheme

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1. (a)	$\frac{4\pi}{7} = 2\pi\sqrt{\frac{x}{9.8}}$	**M1**		M1: Equation with no more than one error.
		A1		A1: Correct equation.
				A1: Correct distance.
	$x = \frac{4\times 9.8}{49} = 0.8$	**A1**		
			3	
1. (b)	$\theta = \frac{\pi}{20}\cos\left(\frac{7}{2}t\right)$	**B1**		B1: Uses $\omega = 3.5$
				M1: Seeing $\pm\pi/40$ used.
				A1: Correct equation.
	$-\frac{\pi}{40} = \frac{\pi}{20}\cos\left(\frac{7}{2}t\right)$	**M1**		
		A1		
	$\cos\left(\frac{7}{2}t\right) = -\frac{1}{2}$			
		dM1		dM1: Solving equation for t.
	$\frac{7}{2}t = \frac{2\pi}{3}$			
		A1		A1: Correct time.
	$t = \frac{4\pi}{21} = 0.598$ s		**5**	
	Total		**8**	

Q	Solution	Mark	Total	Comment
2. (a)	$T_A = \frac{4g}{0.5}(d - 0.5)$	**B1**		B1: One correct tension.
	$T_B = \frac{3g}{0.4}(2 - d - 0.4)$	**B1**		B1: Other correct tension.
	$2.6g + T_B = T_A$	**M1A1**		M1:Equation with correct terms.
				A1: Correct equation.
	$2.6g + 12g - 7.5gd = 8gd - 4g$	**A1**		
	$d = \frac{18.6}{15.5} = 1.2$		**5**	A1: Correct distance.
2. (b) (i)	$T_A = 8g(1.2 + x - 0.5)$	**B1F**		B1: One correct tension in terms of x.
	$T_B = 7.5g(0.8 - x - 0.4)$	**B1F**		B1: Other correct tension in terms of x..
	$2.6\ddot{x} = 2.6g + T_B - T_A$	**M1**		M1: equation with correct terms.
	$= 2.6g + 3g - 7.5gx - 5.6g - 8gx$	**A1**		A1: Correct equation.
	$= -15.5gx$			
	$\ddot{x} = -\frac{15.5g}{2.6}x$	**A1**		M1: Correct expression.
			5	
2. (b) (ii)	Period $= 2\pi\sqrt{\frac{2.6}{15.5g}} = 0.822$ s	**M1A1**		M1: Using their ω to find period.
			2	A1: Correct period.
2. (b) (iii)	$v_{\max} = 0.2\sqrt{\frac{15.5g}{2.6}} = 1.53\ \text{m s}^{-1}$	**M1A1**		M1: Using their ω to find max speed.
			2	A1: Correct max speed.
	Total		**14**	

Q	Solution	Mark	Total	Comment
3. (a)	$V = -mg \times a\tan\theta + 2 \times \frac{2mg}{2 \times a}\left(\frac{a}{\cos\theta} - a\right)^2$	**B1**		B1: Correct GPE
	$= mga\left(\frac{2}{\cos^2\theta} - \frac{4}{\cos\theta} + 2 - \tan\theta\right)$	**M1**		M1: Correct EPE
	$= mga(2(\sec\theta - 1)^2 - \tan\theta)$	**A1**	**3**	A1: Correct total ACF
3. (b)	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = mga\left(\frac{4\sin\theta}{\cos^3\theta} - \frac{4\sin\theta}{\cos^2\theta} - \frac{1}{\cos^2\theta}\right)$	**M1A1**		M1: Attempts to differentiate. A1: Correct derivative.
	$\theta = 0.7$ $\frac{\mathrm{d}V}{\mathrm{d}\theta} = -0.355mga$	**dM1**		dM1: Substitution of 0.7.
	$\theta = 0.8$	**dM1**		dM1: Substitution of 0.8.
	$\frac{\mathrm{d}V}{\mathrm{d}\theta} = 0.513mga$	**A1**		A1: Correct values obtained.
	As there is a sign change there must be a zero between 0.7 and 0.8. Hence there is a position of equilibrium.	**A1F**	**6**	A1F: Correct conclusion based on their values.
3. (c)	Stable, as the energy is a minimum	**B1**		B1: Stable
	because $\frac{dV}{d\theta}$ changes from negative to positive.	**B1**	**2**	B1: Correct justification.
	Total		**11**	

Q	Solution	Mark	Total	Comment
4. (a)	No transverse acceleration			
	$\frac{1}{r}\frac{d}{dt}(r^2\dot{\theta}) = 0$	**M1**		M1: Uses transverse component equal to zero.
	$r^2\dot{\theta} = c$			
	$\dot{\theta} = \frac{c}{r^2}$	**A1**		A1: Correct conclusion from correct working.
	Where c is a constant.		**2**	
4. (b)	$m(\ddot{r} - r\dot{\theta}^2) = -\frac{km}{r^2}$	**M1** **A1**		M1: Uses Newton's second law with radial component. A1: Correct equation
	$\ddot{r} - r\left(\frac{c}{r^2}\right)^2 = -\frac{k}{r^2}$	**M1**		M1: Substitutes for $\dot{\theta}^2$.
	$\ddot{r} = \frac{c^2}{r^3} - \frac{k}{r^2}$	**A1**		A1: Correct result from substitution.
	$\ddot{r} = 0, r = 2a \Rightarrow c^2 = 2ak$			
	$\ddot{r} = \frac{2ak}{r^3} - \frac{k}{r^2}$	**A1**	**5**	A1: Correct expression for $\ddot{r}$.
4. (c)	$\dot{r}\ddot{r} = \frac{2ak}{r^3}\dot{r} - \frac{k}{r^2}\dot{r}$	**M1**		M1: Multiplying by $\dot{r}$.
	$\frac{1}{2}\dot{r}^2 = -\frac{ak}{r^2} + \frac{k}{r} + d$	**dM1**		dM1: Integrating.
	$\dot{r} = U, r = a$			
	$\frac{1}{2}U^2 = -\frac{k}{a} + \frac{k}{a} + d$			
	$d = \frac{1}{2}U^2$	**A1**		A1: Correct constant.
	$r = 2a$			
	$\frac{1}{2}\dot{r}^2 = -\frac{k}{4a} + \frac{k}{2a} + \frac{1}{2}U^2$	**dM1**		dM1: Substituting and simplifying correctly.
	$\dot{r} = \sqrt{\frac{k}{2a} + U^2}$	**A1**	**5**	A1: Correct final answer.
	Total		**12**	

Q	Solution	Mark	Total	Comment
5. (a)	$m\ddot{x} = mg - cm\dot{x} - 2 \times 2mx$	**M1**		M1: Equation of motion with correct terms.
	$\ddot{x} + c\dot{x} + 4x = g$	**A1**		A1: Correct equation.
	For critical damping			
	$c^2 - 16 = 0$	**dM1**		dM1: Equation to find c from discriminant.
	$c = 4$	**A1**	**4**	A1: Obtaining $c = 4$ from correct working.
				AG
5. (b)	**CF**			
	$\lambda^2 + 4\lambda + 4 = 0$			
	$\lambda = -2$	**M1**		M1: Finding λ
	$x = e^{-2t}(A + Bt)$	**A1**		A1: Correct form for CF.
	PI			
	$x = \frac{g}{4}$	**B1**		B1: Correct PI.
	General Solution			
	$x = e^{-2t}(A + Bt) + \frac{g}{4}$			
	$x = 0, t = 0$			
	$A = -\frac{g}{4}$	**M1**		M1: Finding A.
		A1		A1: A correct.
	$\dot{x} = -2e^{-2t}(A + Bt) + Be^{-2t}$			
	$= e^{-2t}(B - 2A + Bt)$			
	$\dot{x} = 0, t = 0$			
	$B = 2A$	**M1**		M1: Finding B.
		A1		A1: B correct.
	$B = -\frac{g}{2}$			
	$x = \frac{g}{4} - \frac{ge^{-2t}}{4}(1 + 2t)$	**A1**	**8**	A1: Correct final answer
5. (c)	$\frac{g}{4}$	**B1**	**1**	B1: CAO
5. (d)	$\dot{x} = -\frac{g}{4}(e^{-2t} \times 2 - 2e^{-2t}(1 + 2t))$			
	$= gte^{-2t}$	**B1**		B1: Correct $\dot{x}$.
	$\ddot{x} = g(e^{-2t} - 2te^{-2t})$	**M1**		M1: Obtains second derivative.
	$\ddot{x} = 0 \Rightarrow t = \frac{1}{2}$	**A1**		A1: Correct time.
	$\dot{x}_{\max} = \frac{g}{2e}$	**A1**	**4**	A1: Correct final answer.
	Total		**17**	

Q	Solution	Mark	Total	Comment
6. (a)	$(m+\delta m)(v+\delta v)-mv=-mg\delta t$	**M1A1**		M1: Equation with correct terms. A1: Correct equation.
	$m\delta v+v\delta m=-mg\delta t$			
	$m\dfrac{\mathrm{d}v}{\mathrm{d}t}+v\dfrac{\mathrm{d}m}{\mathrm{d}t}=-mg$	**M1**		M1: Correct equation with derivatives.
	$m\dfrac{\mathrm{d}v}{\mathrm{d}t}+v\lambda m=-mg$			
	$\dfrac{\mathrm{d}v}{\mathrm{d}t}=-\lambda v-g$	**A1**		A1: Correct DE from correct working.
			4	
6. (b)	$\int\dfrac{1}{\lambda v+g}\mathrm{d}v=\int-\mathrm{d}t$	**M1**		M1: Separation of variables.
	$\dfrac{1}{\lambda}\ln(\lambda v+g)=-t+c$	**A1**		A1: Correct integration.
	$t=0, v=U$	**M1**		M1: Finding constant.
	$c=\dfrac{1}{\lambda}\ln(\lambda U+g)$	**A1**		A1: Correct constant.
	$v=0$			M1: Use of $v = 0$
	$\dfrac{1}{\lambda}\ln(g)=-t+\dfrac{1}{\lambda}\ln(\lambda U+g)$	**M1**		
	$t=\dfrac{1}{\lambda}\ln\left(\dfrac{\lambda U}{g}+1\right)$	**A1**		A1: Correct time.
			6	
6. (c)	$\dfrac{dm}{dt}=\lambda m$	**M1**		M1: Use of DE to find m.
	$m=Me^{\lambda t}$	**A1**		A1: Correct expression for m.
	$m=M\left(\dfrac{\lambda U}{g}+1\right)$	**A1**		A1: Correct mass.
			3	
	Total		**13**	