AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level MATHEMATICS

Unit Mechanics 5

Friday 16 June 2017 Afternoon Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A simple pendulum consists of a light string of length 2.45 metres and a small spherical bob. The pendulum is released from rest with the string taut. When it is released, the angle between the string and the vertical is $\frac{\pi}{10}$.

(a) Find the period of the motion of the pendulum.

[2 marks]

(b) Find the average speed of the spherical bob during one complete oscillation.

[3 marks]

(c) Find the angle between the string and the vertical when the speed of the spherical bob is 1.2 m s^{-1}.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

QUESTION PART REFERENCE	**Answer space for question 1**

2 The points A and B are on a vertical line and 2 metres apart, with A above B.

Two identical elastic springs have natural length 0.5 metres and modulus of elasticity $49\ \text{N}$. One end of each spring is attached to a particle of mass $0.4\ \text{kg}$. The other ends of the springs are attached to the points A and B.

(a) Find the distance of the particle from A when it is in equilibrium.

[4 marks]

(b) The particle is then pulled down below its equilibrium position and released. Show that the period of the subsequent motion is $\frac{\pi\sqrt{10}}{35}$ seconds.

[5 marks]

(c) Given that the particle is released from rest at a point 0.05 metres below the equilibrium point:

(i) find the maximum speed of the particle;

[2 marks]

(ii) find the speed of the particle when it is midway between the equilibrium position and the point of release.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	**Answer space for question 2**

QUESTION PART REFERENCE	**Answer space for question 2**

3 A smooth wire is in the shape of the curve $y = 4 - x^2$. The wire is fixed in a vertical plane with the y-axis vertical. A small ring, of mass m, is threaded onto this wire. An elastic string has modulus of elasticity $4mg$ and natural length 1. One end of the string is fixed to the point with coordinates $(0, 0)$ and the other to the ring.

Gravitational potential energy is taken to be zero at the level of the x-axis.

(a) Show that the total potential energy, V, of this system is given by

$$V = mg\left(38 - 15x^2 + 2x^4 - 4\sqrt{16 - 7x^2 + x^4}\right)$$

where x is the x-coordinate of the position of the ring.

[5 marks]

(b) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$ and show that the system has a position of equilibrium when $x = 2$.

[4 marks]

(c) Determine the nature of this equilibrium position.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

QUESTION PART REFERENCE	**Answer space for question 3**

4 A particle moves on a curve and at time t has polar coordinates (r, θ), where $r = \sin\theta$ and $\theta = 2t$.

(a) Show that, when $r = 1$, the transverse component of the acceleration of the particle is zero.

[5 marks]

(b) Find an expression for the times when the radial component of the acceleration is zero.

[4 marks]

(c) Show that, when the radial component of the acceleration is zero, the transverse component of the velocity is also zero.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

QUESTION PART REFERENCE	**Answer space for question 4**

5 A particle, of mass m kg, is attached to one end of an elastic string which has modulus of elasticity $2.5m$ newtons. The natural length of the string is 0.5 metres. The other end of the string is attached to a fixed point O.

There is a container of fluid below O, where the surface of the fluid is 0.5 metres below O. The particle is released from rest at a point vertically below O, on the surface of the fluid. As it moves through the fluid, the particle experiences a resistance force of magnitude $2mv$ newtons, where v is the speed of the particle.

At time t seconds after the particle is released from rest, the displacement of the particle below the surface of the fluid is x metres.

Find the maximum length of the elastic string during the subsequent motion.

[15 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

QUESTION PART REFERENCE	**Answer space for question 5**

QUESTION PART REFERENCE	**Answer space for question 5**

QUESTION PART REFERENCE	**Answer space for question 5**

6 A spacecraft is descending towards the surface of a planet and fires its rockets to prepare for landing. The initial mass of the spacecraft, including fuel, is M kg and 10% of this is fuel. The fuel is expelled at a rate of λ kg s^{-1} and at a speed of U m s^{-1} relative to the spacecraft.

All the motion takes place near to the surface of the planet and the acceleration due to gravity on the planet is a constant g m s^{-2} throughout the motion. When the rockets are fired, the speed of the space craft is $\frac{U}{20}$ m s^{-1}. At time t seconds after the rockets start to fire, the velocity of the space craft is v m s^{-1}.

Assume that no air-resistance forces act on the spacecraft as it moves.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = g - \frac{\lambda U}{M - \lambda t}$$

[5 marks]

(b) Given that the spacecraft slows down at the time that the rockets are fired, show that

$$U > \frac{Mg}{\lambda}$$

[3 marks]

(c) Given that $U = \frac{3Mg}{2\lambda}$, find the speed of the spacecraft when all the fuel has been used up, giving your answer in terms of g, M and λ.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

QUESTION PART REFERENCE	**Answer space for question 6**

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED