Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MM05

Unit Mechanics 5

Friday 24 June 2011 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A simple pendulum of length L metres is set in motion. The period of the motion is 3 seconds.

(a) Find the frequency of the motion. *(1 mark)*

(b) Find the value of L. *(2 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

2 A particle moves along a straight line AB with simple harmonic motion. The point O is the mid-point of AB. When the displacement of the particle relative to O is x metres, its speed is $v\,\mathrm{m\,s^{-1}}$.

When $x = 3$, $v = 5$ and when $x = 6$, $v = 2.5$.

(a) Show that the amplitude of the motion is $3\sqrt{5}$ metres. *(4 marks)*

(b) Find the maximum speed of the particle during the motion. *(4 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

3 A railway truck, of mass m, is travelling in a straight line along a horizontal track. At time $t = 0$, the truck strikes one end of a buffer which is fixed at its other end. The buffer may be modelled as a light spring of natural length a and modulus of elasticity amn^2, where n is a positive constant. At time t, the compression of the buffer is x.

(a) In a simple model of the motion, the only force affecting the truck during this motion is the thrust from the buffer.

(i) Show that, while the truck is in contact with the buffer, the truck performs simple harmonic motion. *(2 marks)*

(ii) Find, in terms of n, the period of this motion. *(1 mark)*

(b) In a more realistic model, the motion of the truck is affected by a resistance force of magnitude mkv, where v is the speed of the truck and k is a positive constant.

(i) Show that, while the buffer is being compressed, x satisfies the equation

$$\ddot{x} + k\dot{x} + n^2 x = 0$$

(3 marks)

(ii) At time $t = 0$, the truck is travelling with speed U. Given that $k = \frac{5n}{2}$, find x in terms of n, U and t. *(6 marks)*

(iii) By means of a sketch, or otherwise, explain whether the type of damping is light, critical or heavy. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

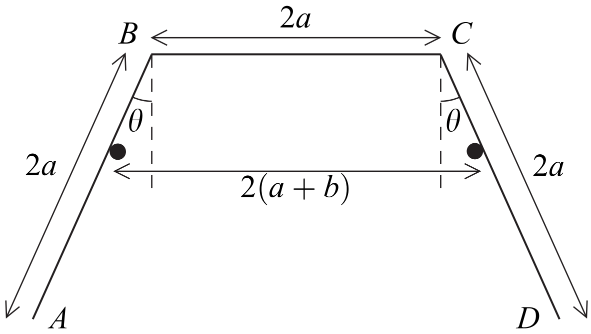
QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

4 Three uniform rods, AB, BC and CD, are each of length $2a$ and mass m. The rods are smoothly jointed at B and C and rest in equilibrium in a vertical plane. The rod BC is horizontal and the rods AB and CD rest on two small smooth fixed pegs. The pegs are at the same horizontal level and are a distance $2(a+b)$ apart. The rods AB and CD are inclined at an angle of θ to the vertical, as shown in the diagram below, where $0 < \theta < \pi$.



(a) The gravitational potential energy is taken to be zero at the level of the pegs. Show that V, the total potential energy of the system, is given by

$$V = mg(3b\cot\theta - 2a\cos\theta)$$ *(6 marks)*

(b) Hence show that any equilibrium positions of the system occur when

$$\sin^3\theta = \frac{3b}{2a}$$ *(5 marks)*

(c) It is given that $b = \dfrac{a}{3}$.

(i) Find the two values of θ for which the system is in equilibrium. *(3 marks)*

(ii) Show that, when the system is in equilibrium,

$$\frac{d^2V}{d\theta^2} = 6mga\cos\theta$$ *(3 marks)*

(iii) Hence determine, for each of the values found in part **(c)(i)**, whether the system is in stable or unstable equilibrium. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

5 A lunar module is descending in a straight line towards the surface of the moon. In order to decrease the speed of the module, the pilot fires the engines, which then eject burnt fuel vertically downwards at a constant rate of $\lambda \text{ kg s}^{-1}$ and at a constant speed of $V \text{ m s}^{-1}$ relative to the module.

When the engines have been fired for t seconds, the mass of the module and its fuel is M kg, and the speed of the module is $v \text{ m s}^{-1}$.

(a) Show that, while the engines are being fired,

$$M\frac{\mathrm{d}v}{\mathrm{d}t} = Mg_1 - \lambda V$$

where $g_1 \text{ m s}^{-2}$ is the acceleration due to gravity on the moon. *(6 marks)*

(b) (i) The module and its fuel have initial mass 1800 kg and initial speed 75 m s^{-1}. Given that $\lambda = 50$, $V = 360$ and $g_1 = 1.62$, show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = 1.62 - \frac{360}{36 - t}$$ *(4 marks)*

(ii) Hence show that the speed of the module at time t is given by

$$v = 1.62t + 360\ln\left(\frac{36 - t}{36}\right) + 75$$ *(3 marks)*

(c) When $t = 7.5$, the module is 5 metres above the surface of the moon, and the pilot stops the engines. Calculate the speed with which the module reaches the surface of the moon. *(3 marks)*

QUESTION PART REFERENCE

QUESTION PART REFERENCE

QUESTION PART REFERENCE

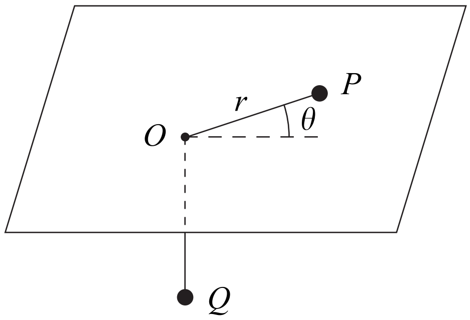
Do not write outside the box

QUESTION PART REFERENCE

Turn over ▶

Do not write outside the box

6 Two particles, P and Q, both of mass m, are attached to the ends of a light inextensible string. The string passes through a small hole, O, in a smooth horizontal table. The particle P is held in contact with the table at a distance a from O and the particle Q hangs at rest below the table. The particle P is projected horizontally with velocity $2\sqrt{ag}$ at right angles to the portion of string resting on the table. The polar coordinates of P during its subsequent motion are (r, θ) relative to O, as shown in the diagram below.



(a) Draw a diagram to show the forces acting on:

(i) the particle P; *(1 mark)*

(ii) the particle Q. *(1 mark)*

(b) By considering the forces acting on Q, explain why

$$T - mg = m\ddot{r}$$

where T is the tension in the string. *(2 marks)*

(c) Hence show that $2\ddot{r} = r\dot{\theta}^2 - g$. *(4 marks)*

(d) Hence show that $2\ddot{r} = \dfrac{4a^3g}{r^3} - g$. *(5 marks)*

(e) Deduce that P will begin to move further away from O after it is set in motion. *(2 marks)*

QUESTION PART REFERENCE

....

....

....

....

....

Do not write outside the box

QUESTION PART REFERENCE

Do not write outside the box

QUESTION PART REFERENCE

END OF QUESTIONS