Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
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5	
6	
TOTAL	

AQA

General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MM05

Unit Mechanics 5

Monday 28 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the space provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A simple pendulum of length 0.4 metres is set in motion.

(a) Find the period of motion of the pendulum. *(2 marks)*

(b) The length of the pendulum is to be shortened so that it completes 50 oscillations per minute. Determine the amount by which the pendulum is to be shortened. *(4 marks)*

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2 A particle moves along a straight line between the points A and B with simple harmonic motion. The point O is the mid-point of AB. At time t seconds, the particle is x metres from O and moving with speed $v\,\mathrm{m\,s^{-1}}$.

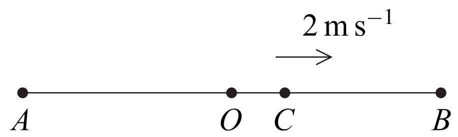
The motion of the particle satisfies the equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + \omega^2 x = 0$$

(a) The particle completes one oscillation in 7.5 seconds.

Show that $\omega = \frac{4\pi}{15}$. *(2 marks)*

(b) When the particle passes through the point C, as shown in the diagram, $x = 1$ and $v = 2$.



(i) Show that the amplitude of the motion is 2.59 metres, correct to three significant figures. *(2 marks)*

(ii) When the particle first passes through C, it is heading away from O towards B.

Find the time that it takes to move from C to B and back to C, giving your answer to two significant figures. *(5 marks)*

(iii) Find the maximum speed of the particle during the oscillations. *(2 marks)*

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3 A particle P moves in a plane so that, at time t, its polar coordinates (r, θ) with respect to a fixed origin O are such that

$$\dot{r} = -3r \quad \text{and} \quad r\dot{\theta} = 5\theta$$

It is given that $r = 2$ when $\theta = 1$.

(a) (i) By differentiating the expression for $r\dot{\theta}$ with respect to time, show that, when $\theta = 1$,

$$\ddot{\theta} = \frac{55}{4}$$

(4 marks)

(ii) Find the transverse component of the acceleration of P when $\theta = 1$. *(2 marks)*

(b) (i) Show that

$$\frac{dr}{d\theta} = \frac{-3r^2}{5\theta}$$

(2 marks)

(ii) Hence show that

$$\frac{1}{r} = \frac{1}{2} + \frac{3}{5}\ln\theta$$

(4 marks)

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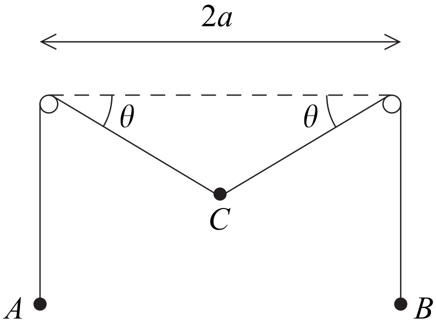
4 Three particles, each of mass m, are attached to a light inextensible string of length $12a$. Particle A is attached to one end of the string, particle B is attached to the other end of the string, and particle C is attached to the mid-point of the string. The string is placed over two smooth pegs, which are at the same horizontal level and a distance $2a$ apart, as shown in the diagram.



The sections of the string attached to C make an angle θ with the horizontal, where $0 < \theta < \dfrac{\pi}{2}$.

The gravitational potential energy is taken to be zero at the level of the pegs.

(a) Show that the distance of the particles A and B below the level of the pegs is

$$a(6 - \sec\theta)$$ *(2 marks)*

(b) Hence show that V, the total potential energy of the system, is given by

$$V = -mga(12 - 2\sec\theta + \tan\theta)$$ *(4 marks)*

(c) Hence show that there is only one position where the system is in equilibrium, and find the value of θ in this position. *(5 marks)*

(d) Determine whether, in this position, the system is in stable or in unstable equilibrium. *(3 marks)*

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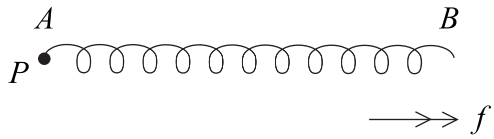
5 A light spring AB lies at rest and unstretched on a smooth horizontal surface. A particle, P, of mass m is attached to the end A of the spring.

The spring has stiffness $2mn^2$, where n is a positive constant. The end B of the spring is set in motion and moves with constant acceleration of magnitude f in the direction AB, as shown in the diagram. The particle P is consequently forced into motion.



At time t after the motion begins, P is moving with speed v and experiences a resistant force of magnitude $3mnv$. The extension of the spring is x and the displacement of P from its initial position in the direction AB is y.

(a) Show that

$$y = \frac{1}{2}ft^2 - x$$ *(3 marks)*

(b) Hence show that P has velocity

$$ft - \frac{dx}{dt}$$ *(1 mark)*

(c) Hence show that

$$\frac{d^2x}{dt^2} + 3n\frac{dx}{dt} + 2n^2x = f + 3nft$$ *(5 marks)*

(d) Given that $n = 1$, and that a particular integral for this differential equation is

$$x = \frac{f}{4}(6t - 7)$$

find an expression for x, in terms of f and t. *(7 marks)*

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6 A spherical hailstone falls under gravity through still air. As it falls, it acquires moisture from the air, causing it to increase in mass.

You may assume that the hailstone remains spherical throughout the motion and that no external forces other than gravity act to affect the motion.

At time t seconds, the radius of the hailstone is r, its mass is m, and the radius, r, is increasing at a rate λr, where λ is a positive constant. The density of the hailstone is ρ, a positive constant.

(a) Show that, at time t,

$$\frac{dm}{dt} = 4\pi\lambda\rho r^3$$

(3 marks)

(b) At time t, the velocity of the hailstone is v.

Show that

$$v\frac{dm}{dt} + m\frac{dv}{dt} = mg$$

(3 marks)

(c) (i) Given that, when $t = 0$, $v = u$, show that

$$v = \frac{g}{3\lambda} + \left(u - \frac{g}{3\lambda}\right)e^{-3\lambda t}$$

(8 marks)

(ii) Hence show that, as the hailstone falls, v approaches a constant value, and state that value. *(2 marks)*

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END OF QUESTIONS