Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE
In Mechanics M4 6680/01

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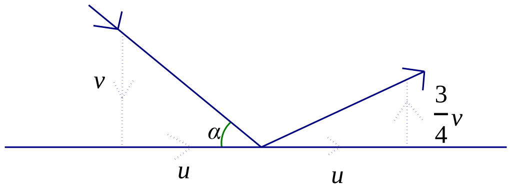
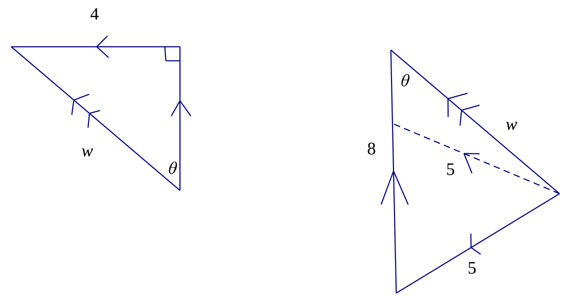
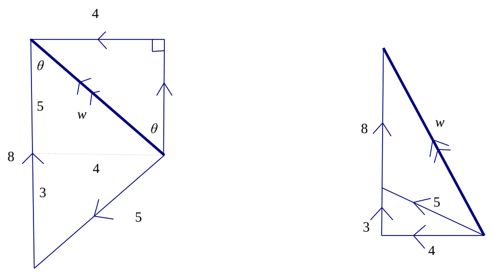
General Marking Guidance

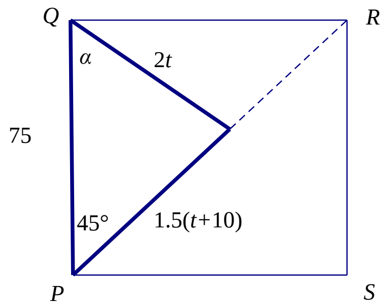
- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

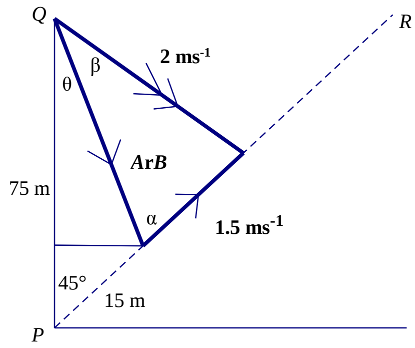
Q	Scheme	Marks
1a	For the rod: GPE $= -2mga\sin\theta$ Must be working from a fixed point	B1
	Extension in the string $= 4a\sin\theta$	B1
	GPE in the string $= \dfrac{\frac{1}{4}mgx^2}{2a}$	M1
	Total $\dfrac{mg}{8a}\times(4a\sin\theta)^2 - 2mga\sin\theta = 2mga(\sin^2\theta - \sin\theta) +$ constant **Given Answer**	A1
		(4)
1b	Differentiate: $\dfrac{dV}{d\theta} = 2mga(2\sin\theta\cos\theta - \cos\theta)$	M1A1
	Second derivative: $\dfrac{d^2V}{d\theta^2} = 2amg(2\cos^2\theta - 2\sin^2\theta + \sin\theta)$	M1A1
	Substitute $\theta = \dfrac{\pi}{6}$ in both:	M1
	$\dfrac{dV}{d\theta} = 2mga\left(2\times\dfrac{1}{2}\cos\theta - \cos\theta\right) = 0$ hence equilibrium cso Allow from working to find solutions for θ	A1
	$\dfrac{d^2V}{d\theta^2} = 2mga\left(2\times\dfrac{3}{4} - 2\times\dfrac{1}{4} + \dfrac{1}{2}\right) = 3mga > 0$ hence equilibrium stable **Given Answer**	A1
		(7)
		[11]

Q	Scheme	Marks
2		
	Velocity before & after: parallel to wall : u and u	B1
	Perpendicular to the wall : v and $\frac{3}{4}v$ Allow with ev	B1
	Kinetic energy: $\frac{1}{2}m\left(\frac{9}{16}v^2+u^2\right)=0.6\times\frac{1}{2}m\left(v^2+u^2\right)$	M1A2
	$\frac{90}{16}v^2+10u^2=6v^2+6u^2$	
	$4u^2=\frac{6}{16}v^2 \quad u^2=\frac{3}{32}v^2$	
	$\tan\alpha=\frac{v}{u}=\sqrt{\frac{32}{3}}$	M1A1
	$\alpha=73°$ (or better 72.976.....)	A1
		(8)
2 Alt		
	Velocity before & after: parallel to wall : $u\cos\alpha$ and $u\cos\alpha$	B1
	Perpendicular to the wall : $u\sin\alpha$ and $\frac{3}{4}u\sin\alpha$	B1
	Kinetic energy: $\frac{1}{2}m\left(\frac{9}{16}(u\sin\alpha)^2+(u\cos\alpha)^2\right)=0.6\times\frac{1}{2}m\left((u\sin\alpha)^2+(u\cos\alpha)^2\right)$	M1A2
	$\frac{9}{16}\sin^2\alpha+\cos^2\alpha=\frac{3}{5}=\frac{9}{16}+\frac{7}{16}\cos^2\alpha$	M1
	$\cos^2\alpha=\frac{3}{35}$, $\alpha=\cos^{-1}\sqrt{\frac{3}{35}}=73.0°$ (1.27 radians)	A1,A1
		[8]

Q	Scheme	Marks
3	$\mathbf{v}_w = {}_w\mathbf{v}_m + \mathbf{v}_m$ walking: $\mathbf{v}_w = a\mathbf{j} - 4\mathbf{i}$	B1
	Running: $\mathbf{v}_w = b\mathbf{i} + (c+8)\mathbf{j}$ $\left(b^2 + c^2 = 25\right)$	B1
	Compare components and use $b^2 + c^2 = 25$:	M1
	$b = -4$	A1
	$a = c + 8,\ c^2 = 25 - 16 = 9,\ c = \pm 3$	A1
	Correct method to obtain a value of w: $w = \sqrt{4^2 + 5^2} = \sqrt{41}\,(= 6.40)$	M1
	Second value correct : $w = \sqrt{4^2 + 11^2} = \sqrt{137}\,(= 11.7)$	A1
		(7)
	Alternative:	
		
	Triangle of velocities for walking	B1
	Either form of triangle of velocities for running using their v_W	B1
	Two triangles combined using their common velocity	M1
		
	Either correct diagram seen or implied	A1
	Both possibilities shown	A1
	Correct method to obtain a value of w: $w = \sqrt{4^2 + 5^2} = \sqrt{41}\,(= 6.40)$	M1
	Second value correct : $w = \sqrt{4^2 + 11^2} = \sqrt{137}\,(= 11.7)$	A1
		(7)
		[7]

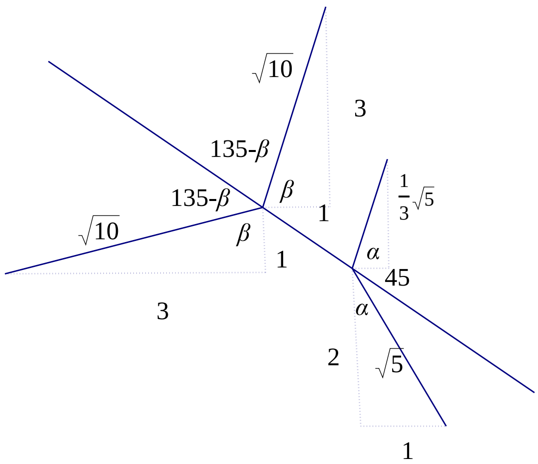
Q	Scheme	Marks
4a	Equation of motion: $\frac{1}{2}\frac{d^2x}{dt^2}\left(=-28e^{-4t}+80te^{-4t}\right)=-kx-\lambda v$	M1A2
	Differentiate: $\frac{dx}{dt}=-4\left(1.5+10t\right)e^{-4t}+10e^{-4t}=4e^{-4t}-40te^{-4t}$	M1
	$\frac{d^2x}{dt^2}=-16e^{-4t}-40e^{-4t}+160te^{-4t}=-56e^{-4t}+160te^{-4t}$	A1
	Substitute and compare coefficients: $-28e^{-4t}+80te^{-4t}=e^{-4t}\left(-1.5k-10kt-4\lambda+40\lambda t\right)$	M1
	$1.5k+4\lambda=28$ $-10k+40\lambda=80$	
	$k=8,\quad \lambda=4$	A1 A1
		(8)
Alt	Equation of motion: $\frac{1}{2}\frac{d^2x}{dt^2}\left(=-28e^{-4t}+80te^{-4t}\right)=-kx-\lambda v$	M1A2
	$\ddot{x}+2\lambda\dot{x}+2kx=0$	
	$m^2+2\lambda m+2k=0 \Rightarrow m=\frac{-2\lambda\pm\sqrt{4\lambda^2-8k}}{2}=-\lambda\pm\sqrt{\lambda^2-2k}$	M1A1
	$\Rightarrow \lambda=4$, and $\lambda^2-2k=0 \Rightarrow k=8$	M1A1 A1
		(8)
	Alternative for the last 5 marks in (a):	
	AE has a repeated root $m=-4$	M1A1
	$\Rightarrow m^2+2\lambda m+2k=m^2+8m+16$	M1
	$k=8,\quad \lambda=4$	A1A1
4b	$\frac{dx}{dt}=0$ when $t=\frac{1}{10}$	B1
	$x=2.5e^{-0.4}=1.68$	M1A1
		(3)
		[11]

Q	Scheme	Marks
5	 Using distances	
	Distance travelled by Ali: $1.5(t+10)$	M1A1
	Distance travelled by Beth: $2t$ and correct triangle seen or implied	B1
	Cosine rule: $(2t)^2 = 75^2 + 1.5^2(t+10)^2 - 2\times 75\times 1.5(t+10)\cos 45$	M1A1
	$1.75t^2 + 114.1t - 4259 = 0$	M1
	$t = \dfrac{-114.1 \pm \sqrt{114.1^2 + 4\times 1.75\times 4259}}{3.5} = 26.5$	A1
	Sine rule: $\dfrac{\sin\alpha}{1.5(t+10)} = \dfrac{\sin 45}{2t}$	M1A1
	$\dfrac{\sin\alpha}{1.5\times 36.5} = \dfrac{\sin 45}{2\times 26.5} \Rightarrow \alpha = 46.9°$ to side PQ or equivalent	M1A1
		(11)
5 Alt	Position vector of A: $\begin{pmatrix} \frac{1.5}{2}(t+10) \\ \frac{1.5}{2}(t+10) \end{pmatrix}$ or with t	B1
	Position vector of B: $\begin{pmatrix} 2t\sin\alpha \\ 75 - 2t\cos\alpha \end{pmatrix}$ value for time consistent	M1A1
	Equate components:	M1A1
	Form equation in t: $4t^2 = \frac{9}{4}\times\frac{1}{2}(t+10)^2 + \left(75 - \frac{3}{2\sqrt{2}}(t-10)\right)^2$	M1A1
	Simplify and solve: $14t^2 + 912.8t - 34072 = 0$	M1
	$t = 26.5$	A1
	Substitute t and solve for α	M1
	$\Rightarrow \alpha = 46.9°$ to side PQ	A1
		(11)

5 Alt	 Using triangle of velocities	
	Using distances: $\tan\theta = \dfrac{15/\sqrt{2}}{75 - 15/\sqrt{2}}$ $\theta = 9.35°$	M1A1
	$\alpha = 45° + \theta = 54.35°$	
	Distance to travel at relative velocity: $\sqrt{\left(15/\sqrt{2}\right)^2 + \left(75 - 15/\sqrt{2}\right)^2} = \sqrt{10.61^2 + 64.39^2} = 65.3 \text{ (m)}$	B1
	Using relative velocities: $\dfrac{\sin\alpha}{2} = \dfrac{\sin\beta}{1.5}$ their α, β	M1A1
	$\beta = 37.5°$	
	$\Rightarrow$ Beth should travel at $\theta + \beta = 46.9°$ to side PQ or equivalent	M1A1
	Relative velocity: $\dfrac{v}{\sin(180 - \alpha - \beta)} = \dfrac{2}{\sin\alpha}$	M1A1
	$v = 2.46 \text{ (ms}^{-1}\text{)}$	
	Time to intercept $= \dfrac{65.3}{2.46} = 26.5 \text{ (s)}$	M1A1
		[11]

Q	Scheme	Marks
6a	$v^2 = kg\left(5e^{-\frac{x}{2k}} - 4\right) \quad \Rightarrow \frac{v^2}{2} = \frac{kg}{2}\left(5e^{-\frac{x}{2k}} - 4\right)$	
	$\Rightarrow \frac{d}{dx}\left(\frac{v^2}{2}\right) = -\frac{1}{2k}\frac{kg}{2}\left(5e^{-\frac{x}{2k}}\right) = -\frac{5g}{4}e^{-\frac{x}{2k}}$	M1
	From v^2: $5ge^{-\frac{x}{2k}} = \frac{v^2}{k} + 4g \quad \Rightarrow \frac{d}{dx}\left(\frac{v^2}{2}\right) = -\left(\frac{v^2}{4k} + g\right)$	M1
	$\Rightarrow ma = -\left(\frac{mv^2}{4k} + mg\right)$	A1
	So resistance is $\frac{mv^2}{4k}$ (Given answer)	A1
		(4)
6b	At max height $v = 0$	M1
	$\Rightarrow \left(5e^{-\frac{x}{2k}} - 4\right) = 0$, $e^{-\frac{x}{2k}} = \frac{4}{5}$, $x = 2k\ln\left(\frac{5}{4}\right)$	M1A1
		(3)
6c	$x = 0, \quad v = \sqrt{kg}$	B1
	Differential equation in v and t: $\frac{dv}{dt} = -\left(g + \frac{v^2}{4k}\right)$	B1
	Separate variables: $-\int \frac{1}{4k}dt = \int \frac{1}{4kg + v^2}dv$	M1
	Integrate: $-\frac{T}{4k} = \left[\frac{1}{\sqrt{4kg}}\tan^{-1}\left(\frac{v}{\sqrt{4kg}}\right)\right]_{\sqrt{kg}}^{0}$	M1A1
	Use limits: $T = \frac{4k}{\sqrt{4kg}}\left(\tan^{-1}\frac{1}{2} - \tan^{-1}0\right) = \sqrt{\frac{4k}{g}}\arctan\left(\frac{1}{2}\right)$ **Given answer**	M1A1
		(7)
		[14]

Q	Scheme	Marks
7a	Impulse on *A*: $I = 2(\mathbf{i}+3\mathbf{j}-3\mathbf{i}-\mathbf{j})$	M1A1
	$= -4\mathbf{i}+4\mathbf{j} = 4(-\mathbf{i}+\mathbf{j})$	A1
	Impulse parallel to l.o.c., hence l.o.c. parallel to $-\mathbf{i}+\mathbf{j}$ (Given answer)	A1
		(4)
7b	Impulse equal and opposite: $4\mathbf{i}-4\mathbf{j} = 3(\mathbf{v}+\mathbf{i}-2\mathbf{j})$	M1A1
	$3\mathbf{v} = \mathbf{i}+2\mathbf{j}$, $\mathbf{v} = \frac{1}{3}(\mathbf{i}+2\mathbf{j})$	A1
		(3)
	Alt using CLM: $2(3\mathbf{i}+\mathbf{j})+3(-\mathbf{i}+2\mathbf{j}) = 2(\mathbf{i}+3\mathbf{j})+3\mathbf{v}$ M1A1	
	$3\mathbf{v} = \mathbf{i}+2\mathbf{j}$, $\mathbf{v} = \frac{1}{3}(\mathbf{i}+2\mathbf{j})$ A1	
7c	Components of velocities parallel to $-\mathbf{i}+\mathbf{j}$: *A* before : $(3\mathbf{i}+\mathbf{j})\cdot\frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j}) = \frac{-2}{\sqrt{2}}$ *A* after : $(\mathbf{i}+3\mathbf{j})\cdot\frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j}) = \frac{2}{\sqrt{2}}$ *B* before : $(-\mathbf{i}+2\mathbf{j})\cdot\frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j}) = \frac{3}{\sqrt{2}}$ *B* after : $\frac{1}{3}(\mathbf{i}+2\mathbf{j})\cdot\frac{1}{\sqrt{2}}(-\mathbf{i}+\mathbf{j}) = \frac{1}{3\sqrt{2}}$ follow through from 7(b) NB: the marks are all available if the unit vector $(\sqrt{2})$ is not used.	M1A3
	Coefficient of restitution: $\frac{1}{\sqrt{2}}\left(2-\frac{1}{3}\right) = \frac{e}{\sqrt{2}}(3+2)$	M1
	$e = \frac{1}{3}$	A1
		(6)

	Alternative (non-vector form)	
		
	Components parallel to the line of centres:	M1A3
	A before: $-\sqrt{10}\cos(135-\beta)=-\sqrt{10}\left(-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{10}}+\frac{1}{\sqrt{2}}.\frac{3}{\sqrt{10}}\right)=-\frac{2}{\sqrt{2}}$	
	A after: $\sqrt{10}\cos(135-\beta)=\sqrt{10}\left(-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{10}}+\frac{1}{\sqrt{2}}.\frac{3}{\sqrt{10}}\right)=\frac{2}{\sqrt{2}}$	
	B before: $\sqrt{5}\cos(45-\alpha)=\sqrt{5}\left(\frac{1}{\sqrt{2}}.\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{5}}\right)=\frac{3}{\sqrt{2}}$	
	B after: $\frac{\sqrt{5}}{3}\cos(45+\alpha)=\frac{\sqrt{5}}{3}\left(\frac{1}{\sqrt{2}}.\frac{2}{\sqrt{5}}-\frac{1}{\sqrt{2}}.\frac{1}{\sqrt{5}}\right)=\frac{1}{3\sqrt{2}}$ follow through from 7(b)	
	Coefficient of restitution: $\frac{1}{\sqrt{2}}\left(2-\frac{1}{3}\right)=\frac{e}{\sqrt{2}}(3+2)$	M1
	$e=\frac{1}{3}$	A1
		[13]

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