
A-LEVEL

Mathematics

Mechanics 4 – MM04

Mark scheme

6360
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Version1 Final Mark Scheme

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
✓ or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
a)	Σ components = 0 $2a + 8b + 11 = 0$ and $1 - 2a + 4b = 0$ Solving gives $b = -1$ and $a = -1.5$	M1 A1 A1	3	Both equations seen A1 each correct value
b)	Moments about O gives $1(1) + 3(3) - 3(1) + 8(4) + 11(2) + 4(5)$ $= 81 \text{ (Nm)}$ ALTERNATIVE Use of $\mathbf{r} \times \mathbf{F}$ three times to get $10\mathbf{k} + 29\mathbf{k} + 32\mathbf{k} = 81\mathbf{k}$ Hence magnitude = 81 (Nm)	M1 A1F A1F A1 (M1) (A1F) (A1F) (A1)	4 (4)	M1 Use of moments - at least four correct pairings A1 all signs consistent, A1 fully correct – follow through their values from part a) Magnitude correct - CAO Correct use of $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$ three times At least two determinants correctly evaluated All three fully correct - follow through their values from part a) Magnitude correct - CAO
Total			7	

Q2				
a)	Resolve vertically at R, $T_{QR} \cos 60^\circ = 250$ $T_{QR} = 500 \text{ N}$ in tension	M1 A1 E1	3	Forming a correct equation with T_{QR} Obtaining correct value of T_{QR} CAO
b)	Vertical component at hinge = 250 N Let horizontal component at hinge = X Take moments about P, $X(2 \cos 60^\circ) = 250 (4 \cos 30^\circ)$ $X = 500\sqrt{3} \text{ N}$	B1 M1A1 A1		Stated or implied by later calculation M1 – set up equation to find horizontal component of the hinge with one side correct. A1 fully correct Correct value obtained (866.025..)

<p> $\text{Magnitude} = \sqrt{(500\sqrt{3})^2 + 250^2}$ $= 901 \text{ N (3sf)}$ </p> <p>ALTERNATIVE for (b)</p> <p>Vertical component at hinge = 250 N</p> <p>For horizontal component at hinge, resolve forces horizontally</p> <p> $X = T_{\text{SR}} + T_{\text{SQ}} \cos 30^\circ$ with $T_{\text{SR}} = 250\sqrt{3}$ and $T_{\text{SQ}} = 500$ </p> <p>$X = 500\sqrt{3} \text{ N}$</p> <p> $\text{Magnitude} = \sqrt{(500\sqrt{3})^2 + 250^2}$ $= 901 \text{ N (3sf)}$ </p>	<p>A1</p> <p>(B1)</p> <p>(M1)</p> <p>(A1)</p> <p>(A1)</p> <p>(A1)</p>	<p>5</p> <p>(5)</p>	<p>Correct method for finding magnitude of reaction – CAO</p> <p>Stated or implied by later calculation</p> <p>M1 – set up a full complete and correct system of equations to find horizontal component of the hinge. A1 correct tension/compression values obtained for all required rods.</p> <p>Correct value obtained (866.025..)</p> <p>Correct method for finding magnitude of reaction – CAO</p>
Total		8	

Q3	Solution	Mark	Total	Comment
a)i)	It is a line of symmetry (and the lamina is uniform)	E1	1	Accept any equivalent statement
ii)	$\int xy dx = \int_0^a (a-x)x dx = \int_0^a ax - x^2 dx$ $= \left[\frac{ax^2}{2} - \frac{x^3}{3} \right]_0^a$ $= \frac{a^3}{6}$ <p>Area of triangle = $\frac{1}{2}a^2$</p> $\frac{\int xy dx}{\int y dx} = \frac{\frac{a^3}{6}}{\frac{a^2}{2}} = \frac{a}{3}$ <p>Coordinates of centre of mass = $(\frac{a}{3}, \frac{a}{3})$</p>	M1 A1		Use of $\int xy dx$ with evidence of correct integration seen Fully correct integration, limits and evaluation
		B1		Area of triangle seen
		m1		Dependent on first M1
		A1	5	Coordinates must be clearly stated
b)i)	<p>Moments about B</p> $P(a \sin 45^\circ) = W(\frac{a}{3})$ $P = \frac{W\sqrt{2}}{3}$	M1A1 A1	3	M1 one side correct - A1 all correct Printed answer
ii)	<p>Resolve horizontally $F = P \sin 45^\circ$ Resolve vertically $P \cos 45^\circ + R = W$ Law of friction $F = \mu R$</p> <p>Combining gives</p> $P = \frac{W\sqrt{2}\mu}{1+\mu}$ <p>Slides before toppling hence</p> $\frac{W\sqrt{2}\mu}{1+\mu} < \frac{W\sqrt{2}}{3}$ $\mu < \frac{1}{2}$	M1A1 M1A1 m1 A1		M1 Three equations seen – A1 all correct M1 Combining to obtain P - A1 if correct Inequality using P expressions formed dependent on both previous M1s Correctly solving for μ - CSO
	Total		6 15	

Q4	Solution	Mark	Total	Comment
a)	Use of $\mathbf{r} \times \mathbf{F}$	M1		Use of $\mathbf{r} \times \mathbf{F}$ or $\mathbf{F} \times \mathbf{r}$ three times
	$\begin{vmatrix} \mathbf{i} & -1 & 3 \\ \mathbf{j} & 1 & -2 \\ \mathbf{k} & 0 & 0 \end{vmatrix} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$	A1		CAO
	$\begin{vmatrix} \mathbf{i} & 2 & 4 \\ \mathbf{j} & 0 & -1 \\ \mathbf{k} & 5 & 2 \end{vmatrix} = \begin{bmatrix} 5 \\ 16 \\ -2 \end{bmatrix}$	A1		CAO
	$\begin{vmatrix} \mathbf{i} & -6 & 0 \\ \mathbf{j} & 2 & 3 \\ \mathbf{k} & 1 & -4 \end{vmatrix} = \begin{bmatrix} -11 \\ -24 \\ -18 \end{bmatrix}$	A1		CAO
	$\text{Total} = \begin{bmatrix} -6 \\ -8 \\ -21 \end{bmatrix}$	A1F	5	Sum of their three $\mathbf{r} \times \mathbf{F}$
b)i)	$\begin{bmatrix} 7 \\ 0 \\ -2 \end{bmatrix}$	B1	1	Sum of three given forces
(b)(ii)	$\begin{vmatrix} \mathbf{i} & x & 7 \\ \mathbf{j} & y & 0 \\ \mathbf{k} & z & -2 \end{vmatrix} = \begin{pmatrix} -6 \\ -8 \\ -21 \end{pmatrix}$	M1		Setting up equation to find point
	$\begin{pmatrix} -2y \\ 2x+7z \\ -7y \end{pmatrix} = \begin{pmatrix} -6 \\ -8 \\ -21 \end{pmatrix}$	A1		Evaluation of determinant – LHS
	so $y = 3$ and $x = -4$ $z = 0$	M1		Equating components and finding correct y value
		A1		Any correct valid combination of x and z seen
	$\mathbf{r} = \begin{pmatrix} -4 \\ 3 \\ 0 \end{pmatrix} + t \begin{pmatrix} 7 \\ 0 \\ -2 \end{pmatrix}$	M1		M1 Correct structure of a straight line with their a and b used
		A1		A1 Fully correct - CSO
	Total		6	
			12	

Q5	Solution	Mark	Total	Comment
a)	$MI = \frac{4}{3}(2m)(l\sqrt{2})^2 = \frac{16}{3}ml^2$	M1A1	2	M1 Correct structure for MI of rod A1 correct length – can be unsimplified
b)	$MI = \frac{1}{3}ml^2 + m(\sqrt{5}l)^2 = \frac{16}{3}ml^2$	M1 A1	2	Must use $I_G + md^2$ Correct MI obtained – fully simplified CSO
c)	$MI = 2(\frac{4}{3}ml^2) + 3(\frac{16}{3}ml^2)$ $= \frac{56ml^2}{3}$	M1A1F A1	3	Five MI combined for M1 A1 fully correct – follow through part b) above Printed answer – CSO – must have part b) fully correct
d)	Gain in KE = $\frac{1}{2}I\omega^2 = \frac{1}{2}(\frac{56}{3}ml^2)\omega^2 = \frac{28}{3}ml^2\omega^2$ Loss in PE: For rods AB and AD = $mg l$ For rods BC and CD = $3mg l$ For rod AC = $4mg l$ Using conservation of energy $\frac{28}{3}ml^2\omega^2 = 12mg l$ Hence $\omega = \sqrt{\frac{9g}{7l}}$	B1 M1 A1 m1 A1	5	Correct gain in KE – can be unsimplified Considering change in PE for five rods – at least three correct All correct or correct total ($12mg l$) seen Use of KE gained = PE lost – dependent on first M1 Any equivalent form
e)	Angular momentum immediately before = $\left(\frac{56ml^2}{3}\right)\left(\sqrt{\frac{9g}{7l}}\right)$ MI of framework and particle = $\frac{56}{3}ml^2 + (3m)\left(\frac{l}{3}\right)^2 = 19ml^2$ Conservation of angular momentum $\left(\frac{56ml^2}{3}\right)\left(\sqrt{\frac{9g}{7l}}\right) = 19ml^2\omega'$ Hence $\omega' = \left(\frac{56}{57}\right)\left(\sqrt{\frac{9g}{7l}}\right) = \frac{8}{19}\sqrt{\frac{7g}{l}}$	B1F M1A1 m1 A1	5	FT their ω M1 Finding new MI or use of moment of momentum – A1 fully correct Forming an equation using momentum – dependent on M1 above Any equivalent form - CSO
	Total		17	

Q6	Solution	Mark	Total	Comment
a)	$\rho = \frac{m}{8a}$ <p>MI of elemental piece = $(\rho dx)x^2$</p> $\text{MI of rod} = \int_{-2a}^{6a} x^2 p dx = \int_{-2a}^{6a} x^2 \left(\frac{m}{8a}\right) dx$ $= \left[\frac{mx^3}{24a} \right]_{-2a}^{6a}$ $= \frac{m(6a)^3}{24a} - \frac{m(-2a)^3}{24a}$ $= \frac{28ma^2}{3}$	<p>B1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p></p> <p></p> <p>4</p>	<p>Connecting ρ and m</p> <p>Use of correct elemental piece and evidence of integration</p> <p>Correct integration</p> <p>Correct limits to obtain printed answer</p>
b)i)	$mg(2a \sin \theta) = \frac{28}{3} ma^2 \ddot{\theta}$ $\ddot{\theta} = \frac{3g \sin \theta}{14a}$ <p>ALTERNATIVE</p> $\frac{1}{2} \left(\frac{28}{3} ma^2 \right) \dot{\theta}^2 = 2mga(\cos 60^\circ - \cos \theta)$ $2mga \sin \theta \dot{\theta} = \frac{28}{3} ma^2 \dot{\theta} \ddot{\theta}$ $\ddot{\theta} = \frac{3g \sin \theta}{14a}$	<p>M1A1</p> <p>A1</p> <p>(M1A1)</p> <p>(A1)</p>	<p>3</p> <p>(3)</p>	<p>Use of $C = I \ddot{\theta}$ M1 one side correct - A1 fully correct CAO</p> <p>Use of conservation of energy and differentiation M1 one side correct - A1 fully correct CAO</p>
ii)	<p>Gain in KE = $\frac{1}{2} I \dot{\theta}^2 = \frac{1}{2} \left(\frac{28}{3} ma^2 \right) \dot{\theta}^2$</p> <p>Change in PE = $2mga(\cos 60^\circ - \cos \theta)$</p> $\frac{1}{2} \left(\frac{28}{3} ma^2 \right) \dot{\theta}^2 = 2mga(\cos 60^\circ - \cos \theta)$ $\frac{14}{3} a \dot{\theta}^2 = 2g \left(\frac{1}{2} - \cos \theta \right)$ <p>Hence $\dot{\theta} = \sqrt{\frac{3g(1-2\cos \theta)}{14a}}$</p>	<p>B1</p> <p>M1A1</p> <p>M1</p> <p>A1</p> <p>A1</p>	<p></p> <p></p> <p>6</p>	<p>Correct KE seen</p> <p>M1 either potential energy term seen – A1 fully correct</p> <p>Use of conservation of energy with their PE and KE terms</p> <p>Evidence of correct substitution, simplification and cancelling</p> <p>Fully correct rearrangement - CSO</p>

iii)	<p>Using $F = ma$ along the rod</p> $mg \cos \theta - X = 2am\dot{\theta}^2$ $mg \cos \theta - X = (2am)\frac{3g}{14a}(1 - 2\cos \theta)$ <p>Hence $X = \frac{mg}{7}(13\cos \theta - 3)$</p>	<p>M1</p> <p>A1F</p> <p>A1</p>	<p>3</p> <p>16</p> <p>75</p>	<p>Correct structure of $F = ma$</p> <p>Substitution of their expression</p> <p>CAO – can be unsimplified</p>
	Total		16	
	TOTAL		75	