



GCE AS/A Level

0982/01



MATHEMATICS – M3
Mechanics

FRIDAY, 23 JUNE 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as 9.8 ms^{-2} .

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle moves along the x -axis such that its displacement x metres at time t seconds satisfies the differential equation

$$\frac{dx}{dt} + x = 2.$$

The particle passes through the origin when $t = 0$.

- (a) Find the time when the particle reaches the point $x = 1$, and determine an expression for x at time t . [7]
- (b) Hence find an expression for the acceleration of the particle at time t . [3]
2. Two particles P and Q , of mass 3 kg and 7 kg respectively, are attached one to each end of a light inextensible string. Initially, the string is slack and the particles are at rest on a smooth horizontal surface. The particle Q is then projected across the surface with speed 8 ms^{-1} away from P along the straight line passing through the initial positions of P and Q . Find the speed with which the particles begin to move immediately after the jerk and determine the impulsive tension in the string during the jerk. [6]

3. The function x satisfies the differential equation

$$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + (10 - k)x = \frac{1}{50}k(k - 5)(12t - 26),$$

where k is a constant. When $t = 0$, $x = 8$ and $\frac{dx}{dt} = 16$. Find x in each of the following cases.

- (a) $k = 5$. [5]
- (b) $k = 0$. [5]
- (c) $k = 10$. [8]
4. An object P , of mass 0.5 kg , moves along a horizontal straight line. The object experiences a resistive force of magnitude $3v^2 \text{ N}$, where $v \text{ ms}^{-1}$ is the speed of P at time t seconds. When $t = 0$, P is at a point O and moving with speed 2 ms^{-1} .
- (a) Show that v satisfies the differential equation
- $$\frac{dv}{dt} = -6v^2. \quad [2]$$
- (b) Find an expression for v in terms of t . [4]
- (c) Obtain an expression for v in terms of x , where x metres is the distance of P from O at time t seconds. [5]
- (d) Determine, in terms of x , the rate at which work is being done against the resistance when P is at a distance x metres from O . [3]

5. The speed $v \text{ ms}^{-1}$ of a particle moving along the x -axis is given by

$$v^2 = -4x^2 + 8x + 21.$$

- (a) Show that the motion is simple harmonic and write down the centre of the motion. [5]
- (b) Show that the period of the motion is π seconds and determine the amplitude. [4]
- (c) Given that when $t = 0$, the particle is at the centre of the motion and moving with positive velocity, write down an expression for x in terms of t and calculate the time taken for the particle to reach $x = 3$ for the first time. [4]
6. A ladder AB , of length 8 m and weight WN , rests with one end A against a vertical wall and the other end B on horizontal ground. The ladder makes an angle α with the horizontal where $\tan \alpha = \frac{3}{4}$. The coefficient of friction between the ladder and the wall is λ and the coefficient of friction between the ladder and the ground is μ .
- (a) Consider the case when the ladder is **uniform**. Given that $\lambda = 0$ and the ladder is on the point of slipping, determine the value of μ in this case. [4]
- (b) Consider the case when the ladder is **non-uniform** and its centre of mass is x m from A . Given that $\lambda = \mu = 0.6$ and the ladder is on the point of slipping, calculate the value of x in this case. [10]

END OF PAPER