



GCE MARKING SCHEME

SUMMER 2017

**MATHEMATICS - M3
0982-01**

INTRODUCTION

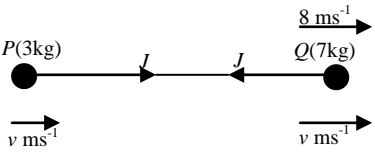
This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

Mathematics M3 (June 2017)
Markscheme

Q	Solution	Mark	Notes
1(a)	$\frac{dx}{dt} = 2 - x$ $\int \frac{dx}{2-x} = \int dt$ $-\ln 2-x = t + (C)$ <p>When $t = 0, x = 0$ $C = -\ln 2$</p> $t = \ln \left \frac{2}{2-x} \right $ <p>When $x = 1$ $t = \ln 2 = (0.693)$</p> $e^{-t} = \frac{2-x}{2}$ $x = 2(1 - e^{-t})$	<p>M1</p> <p>A1</p> <p>m1</p> <p>A1</p> <p>A1</p> <p>m1</p> <p>A1</p>	<p>sep variables, (2-x) required</p> <p>correct integration ft x-2</p> <p>use of initial conditions ft if ln present.</p> <p>cao</p> <p>correct method inversion</p> <p>any correct exp. cao</p>
1(b)	$\frac{d^2x}{dt^2} = -\frac{dx}{dt}$ $\frac{d^2x}{dt^2} = -(2-x) = x-2$ $\frac{d^2x}{dt^2} = 2(1 - e^{-t}) - 2$ $\frac{d^2x}{dt^2} = -2e^{-t}$ <p><u>Alternative</u></p> $x = 2(1 - e^{-t})$ $\frac{dx}{dt} = 2e^{-t}$ $\frac{d^2x}{dt^2} = -2e^{-t}$	<p>M1</p> <p>m1</p> <p>A1</p> <p>(M1)(A1)</p> <p>(A1)</p>	<p>substitute for x</p> <p>ft similar expressions</p> <p>ft $\frac{dx}{dt} = -2e^{-t}$ only.</p>

Q	Solution	Mark	Notes
2	 <p>Impulse = change in momentum Applied to Q $J = 7 \times 8 - 7v$</p> <p>Applied to P $J = 3v$</p> <p>$3v = 56 - 7v$ $v = \underline{5.6 \text{ (ms}^{-1}\text{)}}$</p> <p>$J = \underline{16.8 \text{ (Ns)}}$</p>	M1 A1 B1 m1 A1 A1	allow +/- J cao cao

Q	Solution	Mark	Notes
3(a)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 5x = 0$ <p>Auxilliary equation $m^2 - 6m + 5 = 0$ $(m - 1)(m - 5) = 0, m = 1, 5$ G.S. is $x = Ae^t + Be^{5t}$</p> <p>When $t = 0, x = 8$ and $\frac{dx}{dt} = 16$</p> $A + B = 8$ $\frac{dx}{dt} = Ae^t + 5Be^{5t}$ $A + 5B = 16$ <p>Solving, $A = 6, B = 2$ $x = 6e^t + 2e^{5t}$</p>	M1 A1 m1 B1 A1	ft 2 real roots used both ft similar expressions both values cao
3(b)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} + 10x = 0$ <p>Auxilliary equation $m^2 - 6m + 10 = 0$ $m = 3 \pm i$ C.F. is $x = e^{3t}(Asint + Bcost)$ Using initial conditions $B = 8$</p> $\frac{dx}{dt} = 3e^{3t}(Asint + Bcost) + e^{3t}(Acost - Bsint)$ $16 = 24 + A, A = -8$ $x = 8e^{3t}(-sint + cost)$	M1 A1 m1 B1 A1	ft complex roots used both ft similar expression both values cao
3(c)	$\frac{d^2x}{dt^2} - 6\frac{dx}{dt} = (12t - 26),$ <p>Auxilliary equation $m^2 - 6m = 0$ $m = 0, 6$ C.F. is $x = A + Be^{6t}$</p> <p>For P.I. try $x = at^2 + bt$ $2a - 6(2at + b) = 12t - 26$ $a = -1$ $2a - 6b = -26, b = 4$ $x = A + Be^{6t} - t^2 + 4t$</p> $8 = A + B$ $\frac{dx}{dt} = 6Be^{6t} - 2t + 4$ $16 = 6B + 4$ $B = 2, A = 6$ $x = 2e^{6t} - t^2 + 4t + 6$	M1 A1 m1 A1 B1 A1	ft 0, another real root allow $at+b$ correct LHS comparing coefficients both values cao ft similar CF+PI both values cao

Q	Solution	Mark	Notes
4(a)	N2L applied to P $-3v^2 = 0.5 \frac{dv}{dt}$ $\frac{dv}{dt} = -6v^2$	M1 A1	Dimensionally correct All forces convincing
4(b)	$-\int \frac{dv}{v^2} = 6 \int dt$ $\frac{1}{v} = 6t + (C)$ When $t=0, v=2$ $C = \frac{1}{2}$ $\frac{1}{v} = 6t + \frac{1}{2}$ $v = \frac{2}{12t+1}$	M1 A1 m1 A1	separating variables correct integration use of initial conditions cao, any correct exp.
4(c)	$v \frac{dv}{dx} = -6v^2$ $\frac{dv}{dx} = -6v$ $\int \frac{dv}{v} = -6 \int dx$ $\ln v = -6x + (C)$ when $x = 0, v = 2$ $C = \ln 2$ $-6x = \ln v - \ln 2$ $v = 2e^{-6x}$	M1 m1 A1 m1 A1	separating variables correct integration use of initial conditions cao, any correct exp.
4(d)	Rate of work = $F.v$ Rate of work = $3v^2 \times v$ Rate of work = $3(2e^{-6x})^3$ Rate of work = $24e^{-18x}$	M1 A1 A1	used cao, any correct exp.

Q	Solution	Mark	Notes
5(a)	$v^2 = -4x^2 + 8x + 21$ $2v \frac{dv}{dx} = -8x + 8$ $v \frac{dv}{dx} = -4(x - 1)$ $\frac{d^2x}{dt^2} = -4(x - 1)$ <p>Let $y = x - 1$, $\frac{dy}{dt} = \frac{dx}{dt}$, $\frac{d^2y}{dt^2} = \frac{d^2x}{dt^2}$,</p> $\frac{d^2y}{dt^2} = -4y = -2^2y$ <p>Hence motion is simple harmonic</p> <p>Centre of motion is $x = 1$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>A1</p> <p>B1</p> <p>B1</p>	<p>attempt to differentiate</p> <p>or $dv/dx =$</p> <p>convincing</p>
5(b)	$\omega = 2$ <p>Period = $\frac{2\pi}{2} = \pi$</p> <p>Amplitude is given by $x - 1$ when $v = 0$</p> $-4x^2 + 8x + 21 = -4(x - 1)^2 + 25 = 0$ $(x - 1) = \pm 2.5$ <p>Amplitude = $a = 2.5$</p> <p><u>Alternative solution</u></p> $v^2 = \omega^2[a^2 - y^2]$ $v^2 = 2^2[2.5^2 - (x - 1)^2]$ <p>Hence $\omega = 2$</p> <p>Period = $\frac{2\pi}{2} = \pi$</p> <p>Amplitude = $a = 2.5$</p> <p><u>Alternative solution</u></p> <p>Amplitude is given when $v = 0$</p> $-4x^2 + 8x + 21 = 0$ $(2x + 3)(2x - 7) = 0$ $x = -1.5, 3.5$ <p>amplitude = $3.5 - 1 = 2.5$</p>	<p>B1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>(M1)</p> <p>(B1)</p> <p>(B1)</p> <p>(A1)</p> <p>(M1)</p> <p>(A1)</p>	<p>convincing</p> <p>$v = 0$</p> <p>cao</p> <p>attempt to write equation in correct form</p> <p>cao</p> <p>used</p> <p>cao</p>

5(c) $(x - 1) = 2.5 \sin(2t)$
 $x = 2.5 \sin(2t) + 1$

M1

$$3 - 1 = 2.5 \sin(2t)$$
$$2t = \sin^{-1}\left(\frac{2}{2.5}\right)$$

m1 use of 3-centre

m1 inversion ft a, ω , centre

$$2t = 0.927295$$
$$t = \underline{0.4636 \text{ (s)}}$$

A1 cao

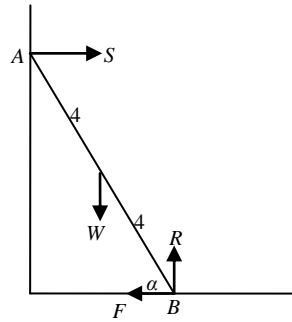
Q

Solution

Mark

Notes

6(a)



Resolve vertically

$$R = W$$

B1

Resolve horizontally

$$S = F = \mu R = \mu W$$

B1

Moments about B

$$W \times 4 \cos \alpha = S \times 8 \sin \alpha$$

M1

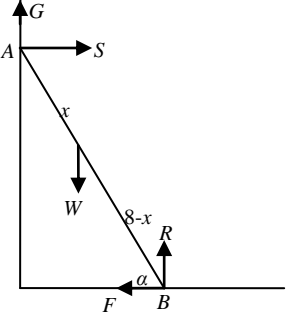
dim correct, all forces
no extra except friction A

$$16W = \mu W \times 8 \times 3$$

$$\mu = \frac{2}{3}$$

A1

cao

Q	Solution	Mark	Notes
6(b)	 <p> $F = 0.6R$ $G = 0.6S$ </p> <p>Resolve vertically</p> <p> $G + R = W$ $0.6S + R = W$ </p> <p>Resolve horizontally</p> <p> $S = F$ $S = 0.6R$ </p> <p> $0.6 \times 0.6R + R = W$ $1.36R = W$ </p> <p>Moments about A</p> <p> $Wx \cos \alpha + 0.6R \times 8 \sin \alpha = R \times 8 \cos \alpha$ $1.36Rx \frac{4}{5} + 4.8R \times \frac{3}{5} = 8R \times \frac{4}{5}$ $5.44x + 14.4 = 32$ $5.44x = 17.6$ $x = \frac{55}{17} = \underline{3.2353 \text{ (m)}}$ </p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>M1</p> <p>A2</p> <p>m1</p> <p>A1</p>	<p>both</p> <p>dimensionally correct All forces, no extra</p> <p>dimensionally correct All forces, no extra</p> <p>dimensionally correct All forces, no extra -1 each error</p> <p>substitute to obtain one common factor force</p> <p>cao</p>