GCE AS/A level

0982/01

S15-0982-01

MATHEMATICS – M3
Mechanics

A.M. MONDAY, 22 June 2015

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Take g as $9{\cdot}8\,\text{ms}^{-2}$.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle of mass 400 kg moves along a straight horizontal road under the action of a horizontal force F. The magnitude of the force F may be modelled by $500\left(\frac{x}{v+2}\right)$ N, where v ms^{-1} is the speed of the particle and x m is the distance of the particle from a point O on the road.

(a) Show that the motion of the particle satisfies the differential equation

$$4v(v+2)\frac{\mathrm{d}v}{\mathrm{d}x} = 5x. \qquad [2]$$

(b) When $x = 0$, the particle is at rest.

(i) Find an expression for x in terms of v.

(ii) Find the distance of the particle from O and the acceleration of the particle when its speed is 3 ms^{-1}. [9]

2. *(a)* An object of mass 0·5 kg is initially moving along the positive x-axis away from the origin O. The object moves under the action of a force of magnitude $6{\cdot}5x$ N which is directed towards O. The resistance to motion of the object is $2v$ N, where v ms^{-1} is the velocity of the object at time t seconds.

(i) Show that the equation of motion of the object is

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 13x = 0.$$

(ii) Find an expression for x in terms of t given that $x = 6$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 3$ when $t = 0$.

Determine the approximate value of x when t is large. [9]

(b) Find the general solution of the differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 4\frac{\mathrm{d}x}{\mathrm{d}t} + 13x = 91t + 15. \qquad [4]$$

3. A body of mass 250 kg is dropped from a hot air balloon and falls vertically downwards. During the downward motion, the body is subjected to a resistance to motion of $50v$ N, where v ms^{-1} is the speed of the body at time t seconds. The initial speed of the body may be assumed to be zero.

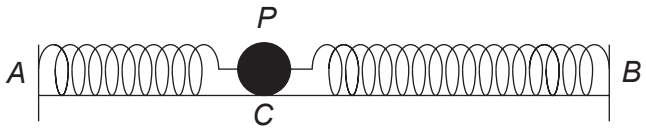
(a) Show that the motion of the body satisfies the differential equation

$$5\frac{\mathrm{d}v}{\mathrm{d}t} = 5g - v. \qquad [2]$$

(b) Find an expression for v in terms of t. Determine the speed of the body when $t = 5$. [7]

(c) Find an expression for x, the distance in metres fallen by the body in t seconds. Hence calculate the distance fallen by the body in 5 seconds. [5]

4. The diagram shows a particle *P*, of mass 7·5 kg, lying on a smooth horizontal surface. It is attached by two light springs to points *A* and *B* where *AB* is 1·4 m.
Spring *AP* has natural length 0·3 m and modulus of elasticity 15 N.
Spring *BP* has natural length 0·6 m and modulus of elasticity 20 N.



When *P* is in equilibrium, it is at the point *C*.

(a) Show that *AC* = 0·5 m. [5]

(b) The particle *P* is pulled horizontally towards *B* a distance 0·25 m from *C* and released.

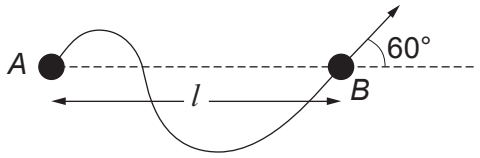
(i) Show that the subsequent motion of the particle is Simple Harmonic with period $\frac{3\pi}{5}$ seconds.

(ii) Write down the amplitude of the motion.

(iii) Determine the speed of *P* when it is 0·2 m from *C*.

(iv) Find the shortest time taken for *P* to reach a position where it is 0·2 m from *C*. [12]

5. Two particles *A* and *B*, of mass 3 kg and 5 kg respectively, are attached one to each end of a light inextensible string of length $\sqrt{3}l$ m. Initially, the particles are at rest on a smooth horizontal surface a distance *l* m apart, as shown in the diagram. Particle *B* is then projected horizontally with speed 8 ms^{-1} at an angle of 60° to the line joining the initial positions of *A* and *B* produced.



Immediately after the string becomes taut,

(a) show that the particle *A* starts to move in a direction which makes an angle of 30° with the line joining the initial positions of *A* and *B*. [2]

(b) find the speed with which each particle begins to move and determine the magnitude of the impulsive tension in the string. [9]

TURN OVER

6. A uniform ladder of mass 20 kg and length 6 m rests with its top end against a smooth vertical wall and its bottom end on rough horizontal ground. The ladder is inclined at an angle θ to the horizontal. The coefficient of friction between the ladder and the ground is 0·6. A man of mass 80 kg climbs the ladder. When he reaches $\frac{5}{6}$ of the way up, the ladder is in limiting equilibrium.

Calculate the normal reaction at the wall and the value of θ. State one modelling assumption you have made about the ladder in your solution. [9]

END OF PAPER