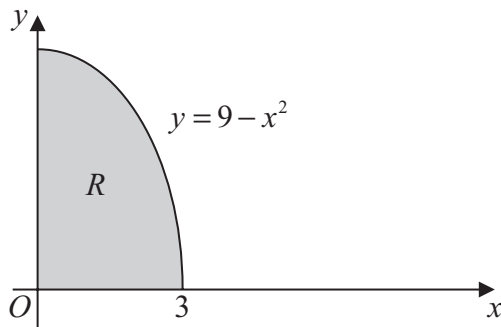






2.

**Figure 1**

The shaded region  $R$  is bounded by the curve with equation  $y = 9 - x^2$ , the positive  $x$ -axis and the positive  $y$ -axis, as shown in Figure 1. A uniform solid  $S$  is formed by rotating  $R$  through  $360^\circ$  about the  $x$ -axis.

Find the  $x$ -coordinate of the centre of mass of  $S$ .

**(9)**

3.

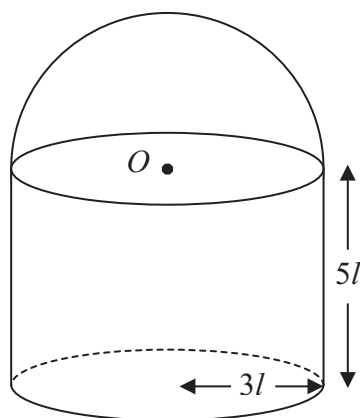


Figure 2

A solid consists of a uniform solid right cylinder of height  $5l$  and radius  $3l$  joined to a uniform solid hemisphere of radius  $3l$ . The plane face of the hemisphere coincides with a circular end of the cylinder and has centre  $O$ , as shown in Figure 2.

The density of the hemisphere is **twice** the density of the cylinder.

(a) Find the distance of the centre of mass of the solid from  $O$ .

(5)

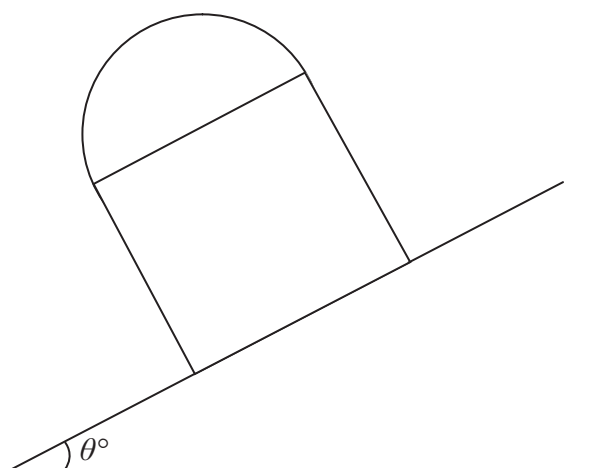


Figure 3

The solid is now placed with its circular face on a plane inclined at an angle  $\theta^\circ$  to the horizontal, as shown in Figure 3. The plane is sufficiently rough to prevent the solid slipping. The solid is on the point of toppling.

(b) Find the value of  $\theta$ .

(4)

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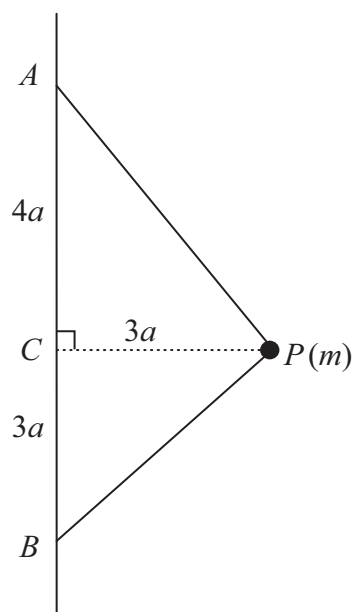
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4.



**Figure 4**

A light inextensible string has its ends attached to two fixed points  $A$  and  $B$ . The point  $A$  is vertically above  $B$  and  $AB = 7a$ . A particle  $P$  of mass  $m$  is fixed to the string and moves in a horizontal circle of radius  $3a$  with angular speed  $\omega$ . The centre of the circle is  $C$  where  $C$  lies on  $AB$  and  $AC = 4a$ , as shown in Figure 4. Both parts of the string are taut.

(a) Show that the tension in  $AP$  is  $\frac{5}{7}m(3a\omega^2 + g)$ . (8)

(b) Find the tension in  $BP$ . (2)

(c) Deduce that  $\omega \geq \frac{1}{2}\sqrt{\left(\frac{g}{a}\right)}$ . (2)

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5. A particle  $P$  of mass  $m$  is attached to one end of a light elastic string of natural length  $l$  and modulus of elasticity  $3mg$ . The other end of the string is attached to a fixed point  $O$  on a rough horizontal table. The particle lies at rest at the point  $A$  on the table, where  $OA = \frac{7}{6}l$ . The coefficient of friction between  $P$  and the table is  $\mu$ .

(a) Show that  $\mu \geq \frac{1}{2}$ .

(4)

The particle is now moved along the table to the point  $B$ , where  $OB = \frac{3}{2}l$ , and released from rest. Given that  $\mu = \frac{1}{2}$ , find

- (b) the speed of  $P$  at the instant when the string becomes slack,

(5)

- (c) the total distance moved by  $P$  before it comes to rest again.

(3)

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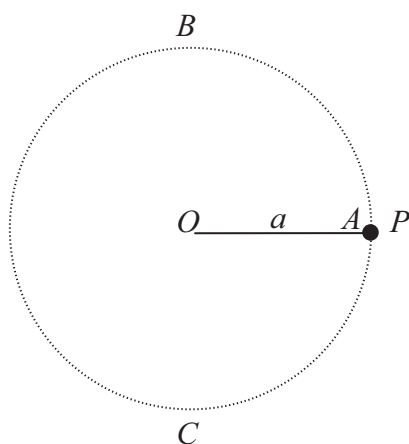
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6.



**Figure 5**

A particle  $P$  is attached to one end of a light inextensible string of length  $a$ . The other end of the string is attached to a fixed point  $O$ . The particle is held at the point  $A$ , where  $OA = a$  and  $OA$  is horizontal. The point  $B$  is vertically above  $O$  and the point  $C$  is vertically below  $O$ , with  $OB = OC = a$ , as shown in Figure 5. The particle is projected vertically upwards with speed  $3\sqrt{ag}$ .

(a) Show that  $P$  will pass through  $B$ . (6)

(b) Find the speed of  $P$  as it reaches  $C$ . (2)

As  $P$  passes through  $C$  it receives an impulse. Immediately after this, the speed of  $P$  is  $\frac{5}{12}\sqrt{11ag}$  and the direction of motion of  $P$  is unchanged.

(c) Find the angle between the string and the downward vertical when  $P$  comes to instantaneous rest. (4)

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7. A particle  $P$  of mass  $0.5$  kg is attached to the mid-point of a light elastic string of natural length  $1.4$  m and modulus of elasticity  $2$  N. The ends of the string are attached to the points  $A$  and  $B$  on a smooth horizontal table, where  $AB = 2$  m. The mid-point of  $AB$  is  $O$  and the point  $C$  is on the table between  $O$  and  $B$  where  $OC = 0.2$  m. At time  $t = 0$  the particle is released from rest at  $C$ . At time  $t$  seconds the length of the string  $AP$  is  $(1 + x)$  m.

(a) Show that the tension in  $BP$  is  $\frac{2}{7}(3 - 10x)$  N. (2)

(b) Find, in terms of  $x$ , the tension in  $AP$ . (1)

(c) Show that  $P$  performs simple harmonic motion with period  $2\pi\sqrt{\left(\frac{7}{80}\right)}$  s. (6)

(d) Find the greatest speed of  $P$  during the motion. (2)

The point  $D$  lies between  $O$  and  $A$ , where  $OD = 0.1$  m.

(e) Find the time taken by  $P$  to move directly from  $C$  to  $D$ . (4)

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