AQA

Please write clearly in block capitals.

Centre number		Candidate number	

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level MATHEMATICS

Unit Mechanics 3

Wednesday 7 June 2017 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:

- The blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working, otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$, unless stated otherwise.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
TOTAL	

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 Bernoulli's equation for fluid flow at any point in a streamline is

$$\frac{1}{2}\rho v^2 + \rho g h + P = C$$

where ρ $\mathrm{kg\,m^{-3}}$ is the density of the fluid;
v $\mathrm{m\,s^{-1}}$ is the speed of the fluid flow at a point on the streamline;
g is the acceleration due to gravity;
h metres is the height of the point above a fixed plane of reference;
P $\mathrm{N\,m^{-2}}$ is the pressure at the chosen point;
and C $\mathrm{N\,m^{-2}}$ is a constant.

By using dimensional analysis, show that the equation is dimensionally consistent.

[5 marks]

QUESTION PART REFERENCE	**Answer space for question 1**

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QUESTION PART REFERENCE	Answer space for question 1

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2 A projectile is fired from a point O with an initial velocity of $70\ \mathrm{m\ s^{-1}}$ at an angle θ above the horizontal. The projectile travels in a vertical plane through the point O. During the motion, the horizontal and upward vertical displacements of the projectile from O are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x\tan\theta - \frac{x^2\left(1+\tan^2\theta\right)}{1000}$$

[5 marks]

(b) (i) The projectile hits a small target which is 500 metres horizontally from O and 25 metres vertically below O.
Find the two possible values of θ.

[5 marks]

(ii) Find the shortest possible time of the flight of the projectile from O to the target.

[2 marks]

QUESTION PART REFERENCE	Answer space for question 2

QUESTION PART REFERENCE	**Answer space for question 2**

3 A particle of mass $2\ \text{kg}$ is moving in a straight line on a horizontal surface under the action of a single variable force. At time t seconds the force has magnitude $(10-2t)$ newtons.

(a) Find the magnitude of the impulse given to the particle by the force between the times $t=0$ and $t=4$.

[3 marks]

(b) When $t=0$ the particle has velocity $3\ \text{m s}^{-1}$.

Find the velocity of the particle when $t=4$.

[2 marks]

(c) Find the times when the particle has velocity $11\ \text{m s}^{-1}$.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 3**

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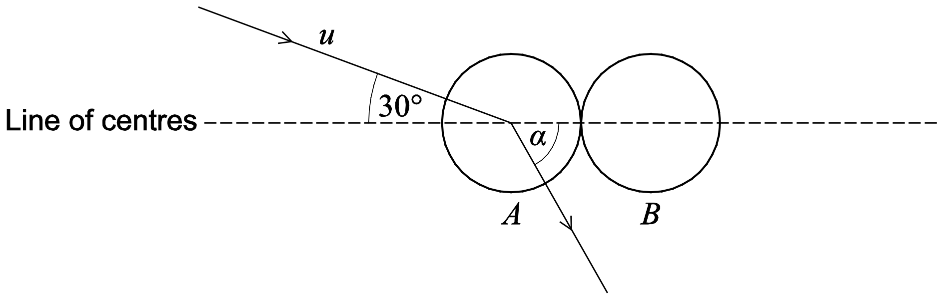
QUESTION PART REFERENCE	Answer space for question 3

4 Two smooth and uniform spheres, A and B, of equal radii have masses $3m$ and $2m$ respectively. The sphere A is moving with speed u on a smooth horizontal surface and collides with the sphere B, which is stationary on the same surface. Just before the collision, the direction of motion of A makes an angle of $30°$ with the line of centres of the spheres. Immediately after the collision, the direction of motion of A makes an angle α with the line of centres of the spheres, as shown in the diagram.



The coefficient of restitution between A and B is $\frac{2}{3}$.

(a) Find the value of α.

[9 marks]

(b) Find, in terms of m and u, the magnitude of the impulse exerted by A on B.

[2 marks]

QUESTION PART REFERENCE	**Answer space for question 4**

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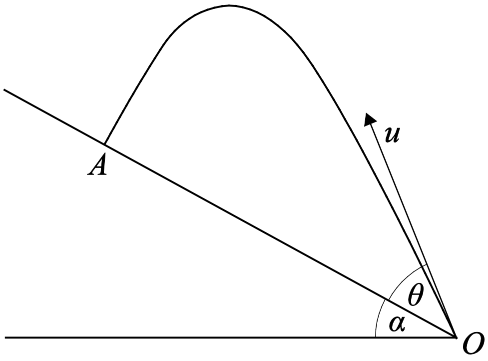
5 A projectile is fired from a point O on a plane which is inclined at an angle α to the horizontal, where $\tan\alpha = \frac{2}{3}$. The projectile is fired up the plane with velocity u at an angle θ above the plane. The motion of the projectile is in a vertical plane containing a line of greatest slope of the inclined plane. The projectile strikes the plane for the first time at **right angles** to the plane at a point A.



(a) Show that $\tan\theta = \frac{3}{4}$.

[8 marks]

(b) Given that OA = 20 m, show that the time taken by the projectile to travel from O to A is 2.71 seconds, correct to three significant figures.

[4 marks]

(c) Find the value of u .

[3 marks]

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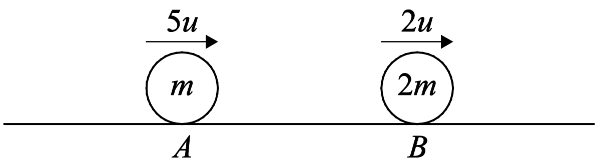
QUESTION PART REFERENCE	Answer space for question 5

6 Two smooth uniform spheres, A and B, of equal radii have masses m and $2m$ respectively. The spheres are moving in a straight line and in the same direction on a smooth horizontal surface with constant speeds $5u$ and $2u$ respectively.



The sphere A collides directly with the sphere B. After the collision, A and B move in the same direction.

The coefficient of restitution between A and B is $\frac{2}{3}$.

(a) Show that, in terms of u, the speeds of A and B immediately after the collision are $\frac{5u}{3}$ and $\frac{11u}{3}$ respectively.

[6 marks]

(b) At a moment after the collision, B is moving in a straight line directly towards a point O and is at a distance s from O. At this moment, a sphere C is moving on the same surface in a straight line **perpendicular** to the direction of motion of B and passes through O with constant speed $\frac{11u}{4}$.

Model the spheres as particles.

Find, in terms of s, the shortest distance between B and C in the subsequent motion.

[5 marks]

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7 Two ships, A and B, are sailing with constant velocities. At noon, A and B are $48\ \text{km}$ apart, with B on a bearing of $045°$ from A. Two hours later, B is $20\ \text{km}$ due north of A.

(a) Find the velocity of B relative to A, giving your answer as a speed together with a bearing.

[6 marks]

(b) It is given that A is sailing with a speed of $12\ \text{km h}^{-1}$ and B is sailing on a bearing of $210°$.

(i) Find the bearings of the two possible directions in which A could be sailing. Give your answers correct to the nearest degree.

[4 marks]

(ii) Hence find the greater of the two possible speeds of B.

[2 marks]

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END OF QUESTIONS

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