GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3 AS/Advanced

SUMMER 2014

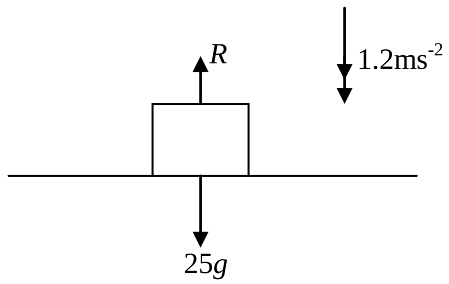
INTRODUCTION

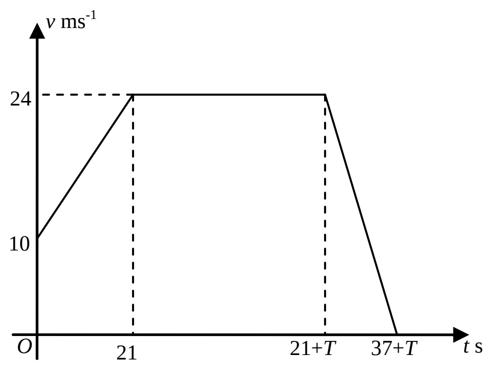
The marking schemes which follow were those used by WJEC for the Summer 2014 examination in GCE MATHEMATICS - M1-M3 & S1-S3. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

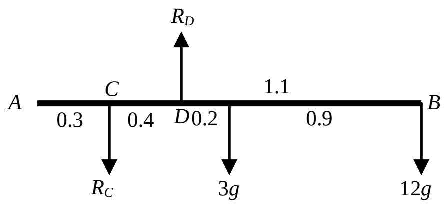
WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

M1

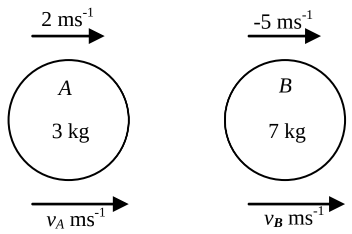
Q	Solution	Mark	Notes
1(a)			
	Apply N2L to crate	M1	R and $25g$ opposing. Dim. Correct
	$25g - R = 25 \times 1.2$	A1	correct equation Any form
	R = <u>215 (N)</u>	A1	
1(b)	$R = 25g$ = <u>245 (N)</u>	B1	

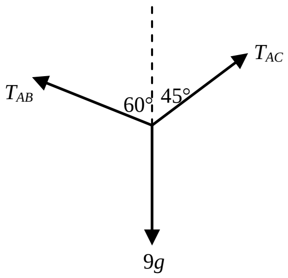
Q	Solution	Mark	Notes
2(a)	Use of $v = u + at$ with u=10, v=24, t=21	M1	oe
	$24 = 10 + 21a$	A1	
	$a = \frac{2}{3}$ (ms^{-2})	A1	accept anything derived from $\frac{2}{3}$ rounded correctly
2(b)	$s = \frac{1}{2}(u + v)t$ with v=0, u=24, t=16	M1	oe
	$s = \frac{1}{2} \times 24 \times 16$	A1	
	s = <u>192 (m)</u>	A1	
2(c)			
		B1	(0, 10) to (21, 24)
		B1	(21, 24) to (21+T, 24)
		B1	(21+T, 24) to (37+T, 0)
		B1	all labels, units and shape.
2(d)	Area under graph = 15000	M1	used
	$0.5(10+24)21 + 24T + 192 = 15000$	A1	ft (b)
		B1	$0.5(10+24)21$ or $24T$ Ft graph
	$24T = 14451$		
	T = <u>602(.125)</u>	A1	Accept 600 from correct working. Cao.

Q	Solution	Mark	Notes
3(a)	Resolve perpendicular to plane		
	$R = mg\cos\alpha$	M1	sin/cos
	$F = \mu mg\cos\alpha$	m1	correct expression
	$F = 0.6 \times 7 \times 9.8 \times \frac{4}{5}$		
	$F = \underline{32.9(28\text{ N})}$	A1	Accept rounding to 32.9.
3(b)	Apply N2L to A	M1	dim correct equation Friction opposes motion 4 terms. Accept cos.
	$T + mg\sin\alpha - F = 7a$ $T + 41.16 - 32.928 = 7a$ $T + 8.232 = 7a$	A1	ft (a)
	Apply N2L to B	M1	dim correct equation
	$3g - T = 3a$	A1	
	$3g + 8.232 = 10a$	m1	one variable eliminated Dep on both M's
	$a = \underline{3.7(632\text{ ms}^{-2})}$	A1	cao
	$T = \underline{18.1(104\text{ N})}$	A1	cao

Q	Solution	Mark	Notes
4.			
		B1	any 1 correct moment.
	Take moments about C	M1	dim correct equation. oe
	$0.4R_D = 3g\times0.6 + 12g\times1.5$ $0.4R_D = 19.8g = 194.04$	A1	correct equ any form
	$R_D = 49.5g = \underline{485.1\ (\text{N})}$	A1	cao
	Resolve vertically	M1	equation attempted. Or 2nd moment equation.
	$R_D = R_C + 15g$	A1	
	$R_C = 34.5g = \underline{338.1\ (\text{N})}$	A1	cao
	<u>Alternative solution</u>		
	Moment equation about A/centre/B	M1	
	Correct equation	B1	
	Second moment equation	M1	
	Correct equation	A1	
	Correct method for solving simultaneously	m1	Dep on both M's
	$R_C = 34.5g = \underline{338.1\ (\text{N})}$	A1	cao
	$R_D = 49.5g = \underline{485.1\ (\text{N})}$	A1	cao

Q	Solution	Mark	Notes
5(a)	Resolve perpendicular to motion	M1	equation, sin/cos
	20sin60 + Tsin30 = 28sin60	A1	
	$20\frac{\sqrt{3}}{2} + T \times \frac{1}{2} = 28\frac{\sqrt{3}}{2}$	A1	convincing
	$T = \underline{8\sqrt{3}}$		
5(b)	N2L in direction of motion	M1	dim correct all forces and No extra force
	20cos60 + Tcos30 + 28cos60 – 16 = 80a	A2	-1 each error
	$20\times\frac{1}{2} + 8\sqrt{3}\times\frac{\sqrt{3}}{2} + 28\times\frac{1}{2} - 16 = 80a$		
	a = <u>0.25 (ms^{-2})</u>	A1	cao
5(c)	N2L -16 = 80a	M1	no extra force
	a = -0.2	A1	accept +/-
	Use of $v = u + at$, v=4, u=12, a=(+/-)0.2	m1	
	4 = 12 – 0.2t	A1	ft if a<0
	t = <u>40 (s)</u>	A1	ft if a<0

Q	Solution	Mark	Notes
6(a)			
	Conservation of momentum	M1	equation required Only one sign error. Ignore common factors
	$2\times3 - 7\times5 = 3v_A + 7v_B$ $3v_A + 7v_B = -29$	A1	
	Restitution	M1	v_B, v_A opposing consistent with diagram, +/-7 with the 0.6.
	$v_B - v_A = -0.6(-5 - 2)$ $v_B - v_A = 4.2$	A1	
	$-7v_A + 7v_B = 29.4$ $3v_A + 7v_B = -29$		
	$10v_A = -58.4$	m1	one variable eliminated. Dep on both M's.
	$v_A =(\underline{-)5.84}$	A1	cao
	$v_B =(\underline{-)1.64}$	A1	cao
6(b)	Impulse = change of momentum $I = 7v_B - 7(-5)$ $I = -11.48 + 35$	M1	used
	$I = \underline{23.52\text{ (Ns)}}$	A1	ft their v_A or v_B
6(c)	$3.65 = e(5.84)$ $e = \underline{0.625}$	B1	ft v_A if > 3.65.

Q	Solution	Mark	Notes
7.			
	Resolve horizontally	M1	equation, no extra force
	T_{AB} sin 60 = T_{AC} sin45	A1	
	$\frac{\sqrt{3}}{2}\ T_{AB} = \frac{1}{\sqrt{2}}\ T_{AC}$		
	$T_{AB} = \sqrt{\frac{2}{3}}\ T_{AC}$		
	Resolve vertically	M1	equation, no extra force
	T_{AB}cos60 + T_{AC}cos45 = 9g	A1	
	$T_{AB} + \sqrt{2}\ \ T_{AC} = 18g$		
	$\sqrt{\frac{2}{3}}\ T_{AC} + \sqrt{2}\ \ T_{AC} = 18g$	m1	
	T_{AC} = <u>79.(078) (N)</u>	A1	cao allow 79
	T_{AB} = <u>64.(567) (N)</u>	A1	cao allow 65
	<u>Alternative Method</u>		
	Third angle 75º/105º	B1	
	$\frac{T_{AB}}{\sin 45} = \frac{9g}{\sin 75}$	M1	sine rule attempted
	$T_{AB} = \frac{9g \times \sin 45}{\sin 75}$	A1	si
	T_{AB} = <u>64.(567) (N)</u>	A1	cao allow 65
	$\frac{T_{AC}}{\sin 60} = \frac{9g}{\sin 75}$	M1	sine rule attempted
	$T_{AC} = \frac{9g \times \sin 60}{\sin 75}$	A1	si
	T_{AC} = <u>79.(078) (N)</u>	A1	cao allow 79

Q	Solution				Mark	Notes
8(a)		mass	AD	AB		
	$ABCD$	72	6	3	B1	
	XYZ	12	6	2	B1	
	E	24	3	4		
	F	36	9	4	B1	both E and F correct
	Jewel	120	x	y	B1	masses in correct proportions.
8(a)(i)	Moments about AD				M1	masses and moments consistent.
	$120x + 12\times6 = 72\times6 + 24\times3 + 36\times9$				A1	ft table if triangle subt.
	$120x = 756$					
	$x = \frac{63}{10} = \underline{6.3\text{(cm)}}$				A1	cao
8(a)(ii)	Moments about AB				M1	masses & moments consistent
	$120y + 12\times2 = 72\times3 + 24\times4 + 36\times4$				A1	ft table if triangle subt.
	$120y = 432$					
	$y = \frac{18}{5} = \underline{3.6\text{ (cm)}}$				A1	cao
8(b)	$PC = 12 - x$					
	$PC = \underline{5.7\text{ (cm)}}$				B1	ft their x if < 12.

M2

Q	Solution	Mark	Notes
1(a)	EE = $\frac{1}{2}\times\frac{\lambda x^2}{l}$, λ=625, x=(+/-)0.1, l=0.2	M1	
	EE = $\frac{1}{2}\times\frac{625\times 0\cdot 1^2}{0\cdot 2}$		
	EE = <u>15.625 (J)</u>	A1	
1(b)	KE = $\frac{1}{2}\times 0\cdot 8v^2$ (= $0.4v^2$)	B1	
	WD by resistance = 46 × 0.1 (= 4.6)	B1	
	Work-energy Principle	M1	3 terms, no PE.
	$\frac{1}{2}0.8v^2 + 46\times 0.1 = 15.625$	A1	FT their EE
	$0.4v^2 = 15.625 - 4.6$		
	$0.4v^2 = 11.025$		
	$v = \sqrt{\frac{11\cdot 025}{0\cdot 4}}$		
	v = <u>5.25 (ms^{-1})</u>	A1	cao

Q	Solution	Mark	Notes
2(a)	$F - R = ma$	M1	used, F and R opposing.
	$30t^{-2} - 150 = 5a$		
	$6t^{-2} - 30 = a$	A1	
	$\frac{dv}{dt} = 6t^{-2} - 30$		Answer given
(b)	$24 = \frac{6}{t^2} - 30$	M1	Ft (a) if same form
	$\frac{6}{t^2} = 54$		
	$t = \frac{1}{3}$	A1	cao, accept 0.3.
2(c)	Integrate w.r.t. t	M1	Increase in powers
	$v = -6t^{-1} - 30t\ (+ C)$	A1	
	$t = \frac{1}{3}$, $v = 18$	m1	
	$18 = -18 - 10 + C$		
	$C = 46$		
	$v = -6t^{-1} - 30t + 46$		
	When $v = 10$	m1	
	$10 = -\frac{6}{t} - 30t + 46$		
	$5t^2 - 6t + 1 = 0$	m1	recognition of quadratic Some attempt to solve.
	$(5t - 1)(t - 1) = 0$		
	$t = \frac{1}{5}, 1$	A1	cao

Q	Solution	Mark	Notes
3(a)	$T = \dfrac{P}{v}$, $P = 90\times1000$, $v = 4.8$	M1	si
	$T = \dfrac{90\times1000}{4\cdot8}$	A1	si
	$T = 18750$		
	N2L	M1	dim correct, all forces T, R opposing.
	$T - mg\sin\alpha - R = ma$	A1	
	$18750 - 4000\times9.8\times\dfrac{2}{49} - R = 4000\times1.2$	A1	
	$R = 18750 - 1600 - 4800$		
	$R = \underline{12350\ (\text{N})}$	A1	cao
3(b)	N2L with $a = 0$	M1	all forces.
	$T = \dfrac{90\times1000}{v}$	B1	si
	$T - 1600 - 12800 = 0$	A1	
	$v = \underline{6.25\ \text{ms}^{-1}}$	A1	

Q	Solution	Mark	Notes
4(a)	$\mathbf{r} = \mathbf{p} + t\mathbf{v}$	M1	used
	$\mathbf{r}_A = (3 - t)\mathbf{i} + (5 + 2t)\mathbf{j} + (20 + t)\mathbf{k}$	A1	
	$\mathbf{r}_B = (-2 + 3t)\mathbf{i} + (x - 4t)\mathbf{j} + (15 + 2t)\mathbf{k}$	A1	
4(b)	$\mathbf{r}_B - \mathbf{r}_A =$	M1	
	$(-5 + 4t)\mathbf{i} + (x - 5 - 6t)\mathbf{j} + (-5 + t)\mathbf{k}$	A1	ft (a) similar expressions.
	$AB^2 = x^2 + y^2 + z^2$	M1	
	$AB^2 = (-5 + 4t)^2 + (x - 5 - 6t)^2 + (-5 + t)^2$	A1	cao
4(c)	Differentiate	M1	powers reduced
	$\frac{dAB^2}{dt} = 2(-5 + 4t)(4) + 2(x - 5 - 6t)(-6)$		
	$+ 2(-5 + t)(1)$		
	$-40 + 32t - 12x + 60 + 72t - 10 + 2t = 0$	m1	equating to 0.
	$106t + 10 = 12x$		
	When $t = 5$		
	$x = \underline{45}$	A1	cao

Q	Solution	Mark	Notes
5(a)	$u_H = \dfrac{42}{2\cdot5}$ = $\underline{16.8\ (\text{ms}^{-1})}$	B1	
	$s = u_Vt + 0.5at^2$, $s = 3$, $t = 2.5$, a=(±)9.8	M1	
	$3 = 2.5u_V - 4.9\times2.5^2$	A1	
	u_V = $\underline{13.45\ (\text{ms}^{-1})}$	A1	cao, accept 13.4, 13.5.
5(b)	$v_V = u_V + at$, $u_V = 13.45$, a = (±)9.8, t=2.5	M1	
	$v_V = 13.45 - 9.8 \times 2.5$	A1	ft from (a)
	$v_V = -11.05$		
	magnitude of vel = $\sqrt{u_H{}^2 + v_V{}^2}$	m1	
	= $\underline{20.11\ (\text{ms}^{-1})}$	A1	cao
	$\theta = \tan^{-1}\left(\dfrac{11\cdot05}{16\cdot8}\right)$	m1	
	θ = $\underline{33.33°}$ (below horizontal)	A1	cao
5(c)	$s = ut + 0.5at^2$, $s = 0$, u=13.45, a=(±)9.8	M1	
	$0 = 13.45t - 4.9t^2$		
	t =2.7449		
	Distance = 2.7449 × 16.8	m1	
	Distance = 46.11		
	Required distance = 46.11 – 42 = $\underline{4.11\ (\text{m})}$	A1	cao

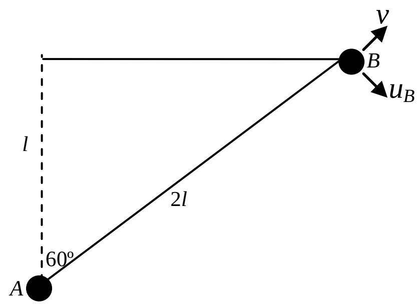
Q	Solution	Mark	Notes
6(a)	$\mathbf{a} = \dfrac{dv}{dt}$	M1	differentiation attempted.
			Vectors required.
	$\mathbf{a} = 8\cos 2t\,\mathbf{i} - 75\sin 5t\,\mathbf{j}$	A1	
	At $t = \dfrac{3\pi}{2}$, ($\mathbf{a} = -8\mathbf{i} + 75\mathbf{j}$)	m1	substitution of *t*.
	Magnitude of force $= 3\times\sqrt{8^2 + 75^2}$	M1	or $\mathbf{F} = 3(-8\mathbf{i} + 75\mathbf{j})$
	= <u>226.28 (N)</u>	A1	cao
6(b)	$\mathbf{r} = \int 4\sin 2t\ \mathbf{i} + 15\cos 5t\,\mathbf{j}\ dt$	M1	integration attempted
	$\mathbf{r} = -2\cos 2t\,\mathbf{i} + 3\sin 5t\,\mathbf{j}\ (+\ \mathbf{c})$	A1	
	At $t = 0$,		
	$-2\mathbf{i} + 3\mathbf{j} = -2\mathbf{i} + \mathbf{c}$	m1	
	$\mathbf{c} = 3\mathbf{j}$	A1	
	$\mathbf{r} = -2\cos 2t\,\mathbf{i} + 3\sin 5t\,\mathbf{j} + 3\mathbf{j}$		
6(c)	Particle crosses the *y*-axis when		
	$-2\cos 2t = 0$	M1	
	$2t = \dfrac{\pi}{2}$		
	$t = \dfrac{\pi}{4}$	A1	cao
	Distance from origin $= 3\sin(5\times\dfrac{\pi}{4}) + 3$	m1	substitute *t* into **r**
	= <u>0.88 (m)</u>	A1	cao

Q	Solution	Mark	Notes
7(a)	Conservation of energy	M1	
	$0.5m(4u)^2 = mg(2l) + 0.5mu^2$	A1	
	$16u^2 = 4gl + u^2$		
	$u^2 = \frac{4}{15}gl$	A1	convincing
7(b)(i)	Conservation of energy	M1	
	$0.5m(4u)^2 = 0.5mv^2 + mgl(1 - \cos\theta)$	A1	
	$v^2 = 16u^2 – 2gl + 2gl\cos\theta$		
	$v^2 = \frac{34}{15}gl + 2gl\cos\theta$	A1	
	N2L towards centre of circle	M1	
	$T - mg\cos\theta = \frac{mv^2}{l}$	A1	
	$T = \frac{34}{15}mg + 3mg\cos\theta$	m1	If M1s gained, substitute for v^2.
	$T = \frac{mg}{15}(34+ 45\cos\theta)$	A1	any correct form
7(b)(ii)	when $T = 0$, $\cos\theta = -\frac{34}{45}$	M1	putting $T = 0$ in acos ± b
	$\theta = 139.1°$	A1	Ft cos = a, a<0.

Q	Solution	Mark	Notes
1(a)	N2L $500 - 100v = 1200\frac{dv}{dt}$	M1	
	$\frac{dv}{dt} = \frac{500-100v}{1200} = \frac{5-v}{12}$	A1	convincing
1(b)	$\int 12\frac{dv}{5-v} = \int dt$	M1	sep. var. (5-v) together.
	$-12\ln(5 - v) = t + (C)$	A1	correct integration
	When $t = 0$, $v = 0$, $C = -12\ln 5$	m1	allow +/-, oe
	$t = 12\ln\left(\frac{5}{5-v}\right)$		
	$\frac{5}{5-v} = e^{\frac{t}{12}}$	m1	inversion ft similar exp.
	$v = 5(1 - e^{-t/12})$	A1	cao
	limiting speed = 5 (ms^{-1})	B1	Ft similar expression
1(c)	When $v = 4$, $t = 12\ln\left(\frac{5}{5-4}\right)$	M1	
	$t = 12\ln 5$ (= 19.31s)	A1	cao

Q	Solution	Mark	Notes
2(a)	Period = $\dfrac{2\pi}{\omega} = 2$	M1	
	$k = \omega = \pi$	A1	
2(b)	$x = 0.52\cos\pi t$	B1	for amp=0.52
	When $t = \dfrac{1}{3}$, $x = 0.52\cos\dfrac{\pi}{3}$	M1	allow asin/acos, c’s a
	$x = 0.26$	A1	cao
2(c)	$0.4 = 0.52\cos\pi t$	M1	allow sin/cos
	$\cos\pi t = \dfrac{0.4}{0.52}$		
	$t = 0.22$	A1	cao
	$t = 1.78$	A1	FT t,ie 2-first t.
2(d)	$v^2 = \omega^2(0.52^2 - x^2)$	M1	used. oe
	$v^2 = \pi^2(0.52^2 – 0.2^2)$	m1	sub $x = 0.2$
	$v = \pi(0.48)$ (= 1.508 ms^{-1})	A1	cao
2(e)	max $v = a\omega$	M1	used
	$=0.52\pi$ (= 1.634 ms^{-1})	A1	cao

Q	Solution	Mark	Notes
3			

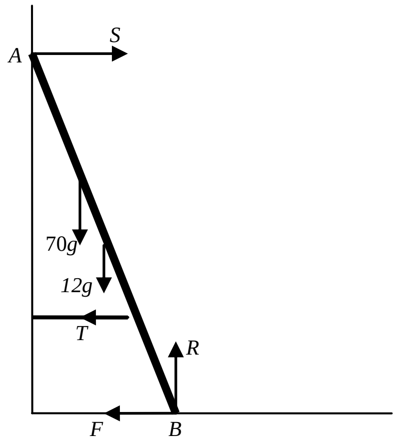


	Impulse = change in momentum	M1	used
	$J = 2u\cos30 - 2v$	A1	
	$J = 3v$	B1	
	Eliminating J	m1	one variable eliminated
	$3v = 2u\cos30 - 2v$		
	$5v = 2u\cos30$		
	$v = 0.4u\cos30$		
	$v = 2.77$ (ms^{-1})(speed of A)	A1	cao
	$J = 1.2\,u\cos30 = 8.31$ (Ns)	A1	ft 3 x c's v.
	$u_B = u\sin30 = 4$ (ms^{-1})	B1	
	Speed of $B = \sqrt{(2.77^2 + 4^2)}$	m1	
	Speed of $B = 4.87$ (ms^{-1})	A1	cao

Q	Solution	Mark	Notes
4(a)	Auxiliary equation		
	$2m^2 + 6m + 5 = 0$	B1	
	$m = -1.5 \pm 0.5i$	B1	
	C.F. is $x = e^{-1.5t}(A\sin 0.5t + B\cos 0.5t)$	B1	ft complex roots
	For PI, try $x = a$		
	$5a = 1$		
	$a = 0.2$	B1	ft CF + a
	GS is $x = e^{-1.5t}(A\sin 0.5t + B\cos 0.5t) + 0.2$	B1	
4(b)	$e^{-1.5t} \to 0$ as $t \to \infty$	M1	si
	x tends to 0.2 as t tends to infinity		
	Limiting value = 0.2	A1	ft similar expression
4(c)(i)	$x = 0.5$ and $\frac{dx}{dt} = 0$ when $t = 0$	M1	used
	$B + 0.2 = 0.5$		
	$B = 0.3$	A1	cao
	$\frac{dx}{dt} = -1.5e^{-1.5t}(A\sin 0.5t + B\cos 0.5t)$		
	$+ e^{-1.5t}(0.5A\cos 0.5t - 0.5B\sin 0.5t)$	B1	ft similar expressions
	$0 = -1.5B + 0.5A$		
	$A = 3B = 0.9$	A1	cao
	$x = e^{-1.5t}(0.9\sin 0.5t + 0.3\cos 0.5t) + 0.2$		
4(c)(ii)	When $t = \frac{\pi}{3}$		
	$x = e^{-\pi/2}(0.9\sin\frac{\pi}{6} + 0.3\cos\frac{\pi}{6}) + 0.2$		
	$x = 0.348$	A1	cao

Q	Solution	Mark	Notes
5(a)	Using F = ma		
	$1200(v+3)^{-1} = 800\,a$	M1	
	$2v\dfrac{dv}{dx} = \dfrac{3}{v+3}$	A1	convincing
5(b)	$\int 3dx = \int 2v(v+3)dv$	M1	separate variables
	$3x = \dfrac{2v^3}{3} + 3v^2$ + (C)	A1	correct integration
	$x = 0$, $v = 0$, hence C = 0	B1	
	When $v = 3$, $3x = 18 + 27$	m1	
	$x = 15$	A1	convincing
5(c)	$\dfrac{dv}{dt} = \dfrac{3}{2(v+3)}$		
	$\int 2(v+3)dv = \int 3dt$	M1	
	$v^2 + 6v = 3t$ + (C)	A1	
	$t = 0$, $v = 0$, hence C = 0	B1	
	When $v = 3$		
	$3t = 9 + 18 = 27$		
	$t = 9$	A1	cao
5(d)(i)	$v^2 + 6v - 3t = 0$	M1	recognition of quadratic And attempt to solve
	$v = 0.5(-6 \pm \sqrt{(6^2 - 4 \times -3t)})$	A1	si
	$v = -3 + \sqrt{(9 + 3t)}$	A1	
(ii)	$\dfrac{dx}{dt} = -3 + (9+3t)^{\frac{1}{2}}$	M1	
	$x = -3t + \dfrac{2}{9}(9+3t)^{\frac{3}{2}}$ + (C)	A1	correct integration
	$x = 0$, $t = 0$, (hence C = -6)	m1	
	$x = -3t + \dfrac{2}{9}(9+3t)^{\frac{3}{2}}$ + (-6)		
	When $t = 7$		
	$x = -21 - 6 + 2 \times 30^{1.5}/9 = 9.5148$	A1	cao
	x is approximately 9.5		

Q	Solution	Mark	Notes
5(d)(ii)	$v = -3 + \sqrt{(9 + 3t)}$		
	When t=7, $v = -3+\sqrt{(9+21)}$	M1	
	$v = -3+\sqrt{30}$	A1	si
	$v = 2.4723$		
	$x = \frac{2}{9}(-2.4723)^3 + (2.4723)^2$	m1	
	x = <u>9.51 (m)</u>	A1	cao

Q	Solution	Mark	Notes
6(a)			
		B2 B0	B1 if one error. more than one error.
6(b)	Resolve vertically $R = 12g + 70g = 82g$	M1 A1	all forces
6(c)	Moments about B	M1	dim correct equation All terms
	$3T\sin75 + 12g\times4\cos75 + 70gx\times\cos75$ $= 8S\sin75$	A4	-1 each incorrect term Accept T=100.
	Resolve horizontally $T + F = S$		
	$F = 0.1R = 8.2g$	B1	ft R
	$S = T + 8.2g$	B1	ft F
	$8(8.2g+T)\sin75 - 3T\sin75 - 48g\cos75$ $= 70gx\cos75$ $5T\sin75 =$ $48g\cos75 - 65.6g\sin75 + 70gx\cos75$ $T = 100$		
	$x = 5.53$ m	A1	cao

Q	Solution	Mark	Notes
	<u>OR</u>		
	Moments about A	M1	dim correct equation All terms
	$5T\sin75 + 12g\times4\cos75 + 70g(8-x)\times\cos75$ $+8F\sin75 = 8R\cos75$	A5	-1 each incorrect term Accept T=100.
	$F = 0.1R = 80.36$ N	B1	Ft R
	$T = 100$ $x = 5.53$ m	A1	cao
6(d)	Ladder modelled as a rigid rod.	B1	

Ques	Solution	Mark	Notes
1(a)	EITHER $P(A\cap B) = P(A) + P(B) - P(A\cup B)$ $= 0.2$	**M1** **A1**	Award M1 for using formula
(b)	This is not equal to $P(A) \times P(B)$ therefore not independent.	**A1**	
	OR Assume A,B are independent so that $P(A\cap B) = P(A) + P(B) - P(A)P(B)$ $= 0.58$	**M1** **A1**	Award M1 for using formula
	Since $P(A\cup B) \neq 0.58$, A,B are not independent.	**A1**	
	$P(A\mid B') = \dfrac{P(A\cap B')}{P(B')}$	**M1**	Award M1 for using formula
	$= \dfrac{0.3-0.2}{0.6}$	**A1**	FT their $P(A\cap B)$ if independence not assumed
	$= \dfrac{1}{6}$	**A1**	Accept Venn diagram
2	$np = 0.9,\quad npq = 0.81$ Dividing, $q = 0.9$, $p = 0.1$ $n = 9$	**B1B1** **M1A1** **A1**	
3(a)	P(1 of each) = $\frac{3}{9}\times\frac{3}{8}\times\frac{3}{7}\times 6$ or $\binom{3}{1}\times\binom{3}{1}\times\binom{3}{1}\div\binom{9}{3}$	**M1A1**	M1A0 if 6 omitted
	$= \dfrac{9}{28}$	**A1**	
(b)	P(2 particular colour and 1 different) = $\frac{3}{9}\times\frac{2}{8}\times\frac{6}{7}\times 3$ or $\binom{3}{2}\times\binom{6}{1}\div\binom{9}{3}$	**M1A1**	M1A0 if 3 omitted
	$= \dfrac{3}{14}$	**A1**	Allow 3/28
	P(2 of any colour and 1 different) = $\dfrac{9}{14}$	**B1**	FT previous line
4(a)	Let X denote the number of goals scored in the first 15 minutes so that X is Po(1.5) si	**B1**	
	$P(X = 2) = \dfrac{e^{-1.5}\times 1.5^2}{2!}$ $= 0.251$	**M1** **A1**	Award M0 if no working seen
(b)	$P(X > 2) = 1 - e^{-1.5}\left(1 + 1.5 + \dfrac{1.5^2}{2!}\right)$	**M1A1**	
	$= 0.191$	**A1**	

Ques	Solution	Mark	Notes
5(a) (i)	Let X = number of female dogs so X is B(20,0.55) $P(X=12)=\binom{20}{12}\times 0.55^{12}\times 0.45^{8}$ $= 0.162$	**B1** **M1** **A1**	si Accept 0.4143 – 0.2520 or 0.7480 – 0.5857
(ii)	Let Y = number of male dogs so Y is B(20,0.45) $P(8 \le X \le 16) = P(4 \le Y \le 12)$ $= 0.9420 - 0.0049$ or $0.9951 - 0.0580$ $= 0.9371$	**M1** **A1** **A1A1** **A1**	Award M0 if no working seen
(b)	Let U = number of yellow dogs so U is B(60,0.05) $\approx$ Po(3) $P(U < 5) = 0.8153$	**M1** **m1A1**	
6(a)	P(head) = $\frac{3}{4}\times\frac{1}{2}+\frac{1}{4}\times 1$ $= \frac{5}{8}$	**M1A1** **A1**	M1 Use of Law of Total Prob (Accept tree diagram)
(b)(i)	P(DH|head) = $\frac{1/4}{5/8}$ $= \frac{2}{5}$ cao	**B1B1** **B1**	B1 num, B1 denom FT denominator from (a)
(ii)	EITHER P(head) = $\frac{3}{5}\times\frac{1}{2}+\frac{2}{5}\times 1$ $= \frac{7}{10}$ OR P(Head) $= \dfrac{3/4 \times 1/2 \times 1/2 + 1/4 \times 1}{5/8}$ $= \frac{7}{10}$	**M1A1** **A1** **B1B1** **B1**	M1 Use of Law of Total Prob (Accept tree diagram) B1 num, B1 denom FT denominator from (a)

Ques	Solution	Mark	Notes
7(a)	[0,0.4]	**B1**	Allow(0,0.4)
(b)	$E(X) = 0.1 + 0.6 + 3\theta + 0.8 + 5(0.4 - \theta)$	**M1**	
	$= 3.5 - 2\theta$	**A1**	
	The range is [2.7,3.5]	**A1**	FT the range from (a)
(c)	$E(X^2) = 0.1 + 1.2 + 9\theta + 3.2 + 25(0.4 - \theta)$	**M1A1**	Must be in terms of θ
	Var(X) = $0.1 + 1.2 + 9\theta + 3.2 + 25(0.4 - \theta) - (3.5 - 2\theta)^2$	**M1**	
	$= 2.25 - 2\theta - 4\theta^2$	**A1**	
	Var(X) = 1.5 gives	**M1**	
	$4\theta^2 + 2\theta - 0.75 = 0$	**A1**	
	$16\theta^2 + 8\theta - 3 = 0$		
	$(4\theta + 3)(4\theta - 1) = 0$	**M1**	Allow use of formula
	$\theta = 0.25$	**A1**	
8(a)	EITHER the sample space contains 64 pairs of which 8 are equal OR whatever number one of them obtains, 1 number out of 8 obtained by the other one gives equality.	**M1**	
	P(equal numbers) = $\frac{1}{8}$	**A1**	
(b)	The possible pairs are (4,8);(5,7);(6,6);(7,5);(8,4)	**B1**	
	EITHER the sample space contains 64 pairs of which 5 give a sum of 12 OR each pair has probability 1/64.	**M1**	
	P(sum = 12) = $\frac{5}{64}$	**A1**	
(c)	EITHER reduce the sample space to (4,8);(5,7);(6,6);(7,5);(8,4) OR $P(\text{equal numbers}) = \frac{P(6,6)}{P(\text{sum}=12)} = \frac{1/64}{5/64}$	**M1**	
	Therefore P(equal numbers) = $\frac{1}{5}$	**A1**	

Ques	Solution	Mark	Notes
9(a)(i)	$P(0.4 \le X \le 0.6) = F(0.6) - F(0.4)$ $= 0.261$	**M1** **A1**	
(ii)	The median m satisfies $2m^3 - m^6 = 0.5$ $2m^6 - 4m^3 + 1 = 0$ $m^3 = \frac{4 \pm \sqrt{8}}{4}$ (0.293) $m = 0.664$	 **B1** **M1A1** **A1**	Award M1 for a valid attempt to solve the equation Do not award A1 if both roots given
(b)(i)	Attempting to differentiate $F(x)$ $f(x) = 6x^2 - 6x^5$	**M1** **A1**	
(ii)	$E(X^3) = \int_0^1 x^3(6x^2 - 6x^5)\mathrm{d}x$ $= \left[\frac{6x^6}{6} - \frac{6x^9}{9}\right]_0^1$ $= 1/3$	**M1A1** **A1** **A1**	M1 for the integral of $x^3 f(x)$ A1 for completely correct although limits may be left until 2nd line. FT their $f(x)$ if M1 awarded in (i)

Ques	Solution	Mark	Notes
1	$\bar{x} = \frac{405.6}{8}$ (= 50.7)	**B1**	
	SE of $\bar{X} = \frac{4}{\sqrt{8}}$ (= 1.4142…)	**M1A1**	
	90% conf limits are 50.7 ± 1.645 × 1.4142… giving [48.4, 53.0] cao	**M1A1** **A1**	M1 correct form, A1 correct z. Award M0 if no working seen
2(a)	Upper quartile = mean + 0.6745 × SD = 86.0	**M1** **A1**	
(b)	Let X=weight of an orange, Y=weight of a lemon $E(\Sigma X)$ = 1984 Var(ΣX) = 512	**B1** **B1**	
	$z = \frac{2000-1984}{\sqrt{512}} = 0.71$	**M1A1**	Award M0 if no working seen
	Prob = 0.7611 cao	**A1**	
(c)	Let $U = X - 3Y$ E(U) = – 7 Var(U) = 64 + 9 × 2.25 = 84.25 We require P(U >0)	**M1** **A1** **M1A1**	
	$z = \frac{0+7}{\sqrt{84.25}} = 0.76$	**m1A1**	Award m0 if no working seen
	Prob = 0.2236	**A1**	
3(a)	$H_0 : \mu_M = \mu_F ; H_1 : \mu_M \neq \mu_F$	**B1**	
(b)	Let X= male weight, Y=female weight ($\sum x = 39.2; \sum y = 46.6$) $\bar{x} = 4.9; \bar{y} = 4.66$	**B1B1**	
	SE of diff of means= $\sqrt{\frac{0.5^2}{8} + \frac{0.5^2}{10}}$ (0.237…)	**M1A1**	
	Test statistic = $\frac{4.9-4.66}{0.237...}$	**m1**	Award m0 if no working seen
	= 1.01 Prob from tables = 0.1562 p-value = 0.3124	**A1** **A1** **B1**	FT line above
	Insufficient evidence to conclude that there is a difference in mean weight between males and females.	**B1**	FT their p-value

Ques	Solution	Mark	Notes
4(a)(i)	$H_0: p = 0.6; H_1: p < 0.6$	B1	
(ii)	Let X = Number of games won Under H_0, X is B(20,0.6) si Let Y = Number of games lost Under H_0, Y is B(20,0.4) p-value = P($X \le 7$|(X is B(20,0.6))) = P($Y \ge 13$|Y is B(20,0.4)) = 0.021 Strong evidence to reject Gwilym's claim (or to accept Huw's claim).	B1 B1 M1 A1 A1 B1	 Award M0 if no working seen FT on p-value
(b)	X is now B(80,0.6) (under H_0) ≈ N(48,19.2) p-value = P($X \le 37$|X is N(48,19.2)) $z = \dfrac{37.5-48}{\sqrt{19.2}}$ = – 2.40 p-value = 0.0082 Very strong evidence to reject Gwilym's claim (or to accept Huw's claim).	B1B1 M1 A1 A1 A1 B1	 Award M0 if no working seen Award M1A0A1 for incorrect or no continuity correction No cc ; z = – 2.51, p = 0.00604 36.5 ; z = – 2.62, p = 0.0044 FT on p-value only if less than 0.01
5(a)	$E(X) = E(Y) = 1.2$ E(U) = E(X)E(Y) = 1.44 cao	B1 B1	
(b)	Var(X) = Var(Y) = 0.96 $E(X^2)(= E(Y^2)) = \text{Var}(X) + [E(X)]^2 = 2.4$ Var(U) = $E(X^2Y^2) - [E(XY)]^2$ = $E(X^2)E(Y^2) - [E(X)E(Y)]^2$ = 3.69 cao	B1 M1A1 M1 A1 A1	 FT their values from (a)
6(a)(i)	Under H_0, X is Po(15) si $P(X \le 10) = 0.1185; P(X \ge 20) = 0.1248$ Significance level = 0.2433	B1 B1 B1	 Award B1 for either correct
(ii)	X is now Poi(10) P(accept H_0) = $P(11 \le X \le 19)$ = $0.9965 - 0.5830$ or $0.4170 - 0.0035$ = 0.4135 cao	B1 M1 A1 A1	 Award M0 if no working seen
(b)	Under H_0, X is now Po(75) ≈ N(75,75) $z = \dfrac{91.5-75}{\sqrt{75}} = 1.91$ Prob from tables = 0.0281 p-value = 0.056 Insufficient evidence to reject H_0	B1 M1A1 A1 A1 B1	 Award M1A0 for incorrect or no continuity correction but FT further work. FT from line above FT from line above No cc gives z = 1.96, p = .05 92.5 gives z = 2.02, p = 0.0434

Ques	Solution	Mark	Notes
7(a)	$P(L \le 4) = P(A \le 4^2)$	**M1**	
	$= \dfrac{16-15}{20-15}$	**A1**	
	$= 0.2$	**A1**	
(b)	$E(L) = E(A^{1/2})$		
	$= \int_{15}^{20} a^{1/2} \times \dfrac{1}{5}\,\mathrm{d}a$	**M1A1**	Limits can be left until next line
	$= \dfrac{2}{15}\left[a^{3/2}\right]_{15}^{20}$	**A1**	
	$= 4.18$	**A1**	Do not accept $\sqrt{17.5} = 4.18$
(c)	$\text{Var}(L) = E(L^2) - [E(L)]^2$	**M1**	
	$= 17.5 - 4.18^2$	**A1**	FT their E(L)
	$= 0.03$	**A1**	

Ques	Solution	Mark	Notes
1	$\bar{x} = 52.0$ si Variance estimate = $\frac{162480}{59} - \frac{3120^2}{60 \times 59} = 4.068$ (Accept division by 60 which gives 4.0) 90% confidence limits are $52 \pm 1.645\sqrt{4.068/60}$ giving [51.6,52.4]	**B1** **M1A1** **M1A1** **A1**	
2(a) (b) (c)	$H_0 : \mu = 4.5; H_1 : \mu \neq 4.5$ $\sum x = 43.6; \sum x^2 = 190.3428$ UE of μ = 4.36 UE of $\sigma^2 = \frac{190.3428}{9} - \frac{43.6^2}{90}$ = 0.0274(22…) test-stat = $\frac{4.36 - 4.5}{\sqrt{0.0274222../10}}$ = – 2.67 (Accept +2.67) DF = 9 si Crit value = 3.25 This result suggests that we should accept H_0, ie that the mean weight is 4.5 kg because 2.67 < 3.25	**B1** **B1B1** **B1** **M1** **A1** **M1A1** **A1** **B1** **B1** **B1** **B1**	 No working need be seen Answer only no marks FT their values from (b) Answer only no marks FT their *t*-statistic
3(a) (b)	$\hat{p} = \frac{654}{1500} = 0.436$ si ESE = $\sqrt{\frac{0.436 \times 0.564}{1500}}$ = 0.0128.. si 95% confidence limits are $0.436 \pm 1.96 \times 0.0128..$ giving [0.41,0.46] $\hat{p} = \frac{0.4348 + 0.4852}{2} = 0.46$ Number of people = 0.46 × 1200 = 552 $0.4852 - 0.4348 = 2z\sqrt{\frac{0.46 \times 0.54}{1200}}$ z = 1.75 Prob from tables = 0.0401 or 0.9599 Confidence level = 92%	**B1** **M1A1** **M1** **A1** **A1** **B1** **B1** **M1A1** **A1** **A1** **B1**	 M1 correct form A1 correct *z* FT line above

Ques	Solution	Mark	Notes
4(a)	$H_0 : \mu_a = \mu_b ; H_1 : \mu_a \neq \mu_b$	B1	
(b)	SE = $\sqrt{\frac{0.115}{80} + \frac{0.096}{70}}$ (= 0.053)	M1A1	
	Test stat = $\frac{3.65 - 3.52}{0.053}$	M1A1	
	= 2.45 (Accept 2.46)	A1	
	Tabular value = 0.00714 (0.00695)	A1	
	p-value = 0.01428 (0.0139)	A1	
	Strong evidence to conclude that there is a difference in mean weight.	B1	FT their p-value Accept the conclusion that the Variety B mean is greater than the Variety A mean
(c)	Estimates of the variances of the sample means are used and not exact values.	B1	
	The sample means are assumed to be normally distributed (using the Central Limit Theorem).	B1	
5(a)	$\sum x = 42, \sum x^2 = 364, \sum y = 340.6, \sum xy = 2906.4$	B2	Minus 1 each error
	$S_{xy} = 2906.4 - 42 \times 340.6/6 = 522.2$	B1	
	$S_{xx} = 364 - 42^2/6 = 70$	B1	
	$b = \frac{522.2}{70} = 7.46$	M1 A1	Answers only no marks
	$a = \frac{340.6 - 7.46 \times 42}{6} = 4.55$	M1 A1	
(b)(i)	Unbiased estimate = $a + 5b$ = 41.85	B1	FT their values of and a,b if answer between 33.9 and 49.9
(ii)	SE of $a + 5b$ = $0.5\sqrt{\frac{1}{6} + \frac{(5-7)^2}{70}}$ (0.2365…)	M1A1	And FT their value of S_{xx}
	95% confidence limits for $\alpha + 5\beta$ are $41.85 \pm 1.96 \times 0.2365...$	m1A1	
	giving [41.4,42.3]	A1	
(iii)	Test stat = $\frac{7.6 - 7.46}{\sqrt{0.5^2/70}} = 2.34$	M1A1	FT their values of b and S_{xx} if possible.
	Critical value = 1.96 or p-value = 0.01928	A1	FT their test statistic
	We conclude that β = 7.6 is not consistent with the tabular values.	B1	FT the line above

Ques	Solution	Mark	Notes
6(a)(i)	$E(Y) = kE(\bar{X}) = kE(X) = \frac{k\theta}{2}$ For an unbiased estimator, k = 2.	M1A1 A1	
(ii)	$\text{Var}(Y) = 4\text{Var}(\bar{X})$	M1	FT their k
	$= \frac{4}{n}\text{Var}(X)$	A1	
	$= \frac{4}{n} \times \frac{\theta^2}{12}$	A1	
	$= \frac{\theta^2}{3n}$	A1	
	$\text{SE} = \frac{\theta}{\sqrt{3n}}$	A1	
(b)(i)	Using $\text{Var}(Y) = E(Y^2) - [E(Y)]^2$	M1	
	$E(Y^2) = \frac{\theta^2}{3n} + \theta^2$	A1	
	$\neq \theta^2$ therefore not unbiased	B1	FT the line above
(ii)	$E(Y^2) = \theta^2\left(\frac{3n+1}{3n}\right)$	M1	
	$E\left(\frac{3nY^2}{3n+1}\right) = \theta^2$	A1	
	Therefore $\frac{3nY^2}{3n+1}$ is an unbiased estimator for θ^2	A1	

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