GCE MARKING SCHEME

MATHEMATICS - M1-M3 & S1-S3 AS/Advanced

SUMMER 2013

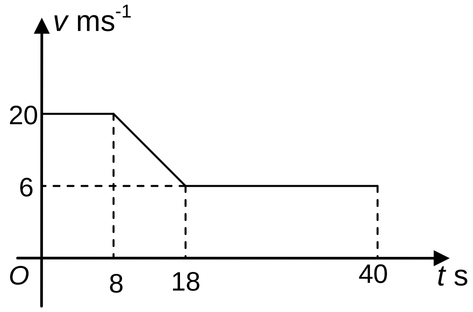
INTRODUCTION

The marking schemes which follow were those used by WJEC for the Summer 2013 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

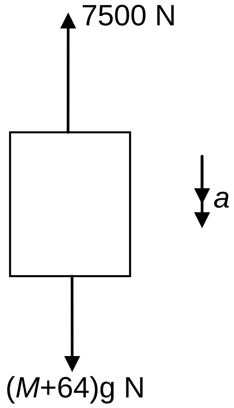
It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

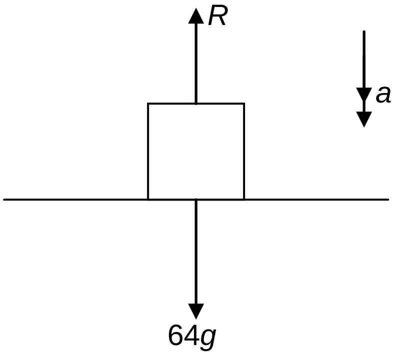
M1

Q	Solution	Mark	Notes
1(a)			
		B1	(0, 20) to (8, 20)
			Or (18, 6) to (40, 6)
		B1	(8, 20) to (18, 6)
		B1	completely correct with all units and labels.
1(b)	Deceleration = gradient of graph	M1	any correct method
	D = $\frac{20-6}{18-8}$	A1	ft graph +/-
	D = <u>1.4 ms^{-2}</u>	A1	cao
	OR		
	Use of v = u + at, v=6, u=20, t=10	M1	
	6 = 20 + 10a	A1	allow –a
	a = -1.4 ms^{-2}		
	Magnitude of acceleration = <u>1.4 ms^{-2}</u>	A1	cao
1(c)	Distance AB = Area under graph	M1	used. Oe
	= (8 × 20) + 0.5(20 + 6)×10 + (22 × 6)	B1	any correct area, ft graph
	= 160 + 130 + 132		
	= <u>422 m</u>	A1	cao

Q	Solution	Mark	Notes



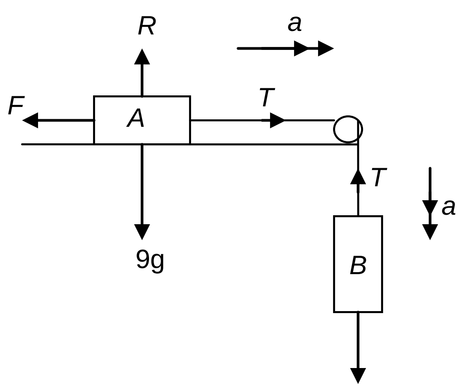
2(a)			
	N2L applied to lift and person	M1	dim correct equation, forces opposing
	$(M + 64)g - 7500 = (M+64) \times 0.425$	A1	correct equation
	M = <u>736</u>	A1	



2(b)			
	N2L applied to person	M1	64g and R opposing Dim correct equation
	$64g - R = 64a$	A1	correct equation
	$R = 64 \times 9.8 - 64 \times 0.425$		
	R = <u>600 N</u>	A1	

Q	Solution	Mark	Notes
3(a)	$v^2 = u^2 + 2as$, v=0, a=(±)9.8, s=18.225	M1	oe used
	$0 = u^2 - 2 \times 9.8 \times 18.225$	A1	
	u = $\underline{18.9}$	A1	convincing
3(b)	Use of $s = ut + 0.5at^2$, s=(±)2.8, a=(±)9.8, u=18.9	M1	oe
	$-2.8 = 18.9t + 0.5 \times (-9.8)t^2$	A1	
	$4.9t^2 - 18.9t - 2.8 = 0$		
	$7t^2 - 27t - 4 = 0$		
	$(7t + 1)(t - 4) = 0$	m1	correct method for solving quad equ seen
	t = $\underline{4\text{s}}$	A1	cao

Q	Solution	Mark	Notes

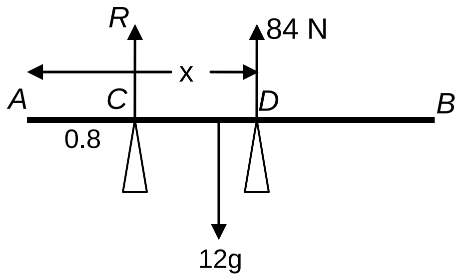


4

5

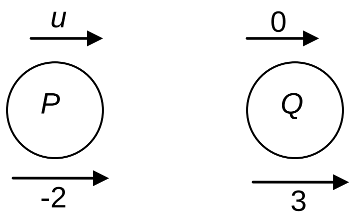
Q	Solution	Mark	Notes
4(a)	N2L applied to B		
	$5g - T = 5a$	M1	dim correct equation 5g and T opposing.
	$T = 5 \times 9.8 - 5 \times 1.61$	A1	
	$T = \underline{40.95 \text{ N}}$	A1	cao
	$R = 9g = (88.2 \text{ N})$	B1	si
	$F = 9\mu g = (88.2\mu)$	B1	si
	N2L applied to A	M1	dim correct equation T and F opposing
	$T - F = 9a$	A1	
	$T - 88.2\mu = 9 \times 1.61$		
	$\mu = \underline{0.3}$	A1	cao
4(b)	limiting friction = $9\mu g = 9 \times 0.6g = 5.4g$	B1	
	Limiting friction > $5g$		
	Particle will remain at rest	R1	oe
	$T = 5g = \underline{49 \text{ N}}$	B1	

Q	Solution	Mark	Notes



5			
5(a)(i)	Resolve vertically	M1	all forces, no extras
	$R + 84 = 12g$	A1	
	$R = \underline{33.6}$	A1	cao
5(a)(ii)	Moments about C	M1	equation, no extra force oe
	$12g \times 0.2 = 84(x - 0.8)$	B1	any correct moment
		A1	correct equation
	$84x = 12g \times 0.2 + 84 \times 0.8$		
	$x = \underline{1.08}$	A1	cao
5(b)	When about to tilt about C, $R_D = 0$	M1	si
	Moments about C	m1	equation, no extra force
	$Mg \times 0.8 = 12g \times 0.2$		
	$M = \underline{3}$	A1	

Q	Solution	Mark	Notes



6.

6(a)	Conservation of momentum	M1	equation required, only 1 sign error.
	$2u + 5 \times 0 = 2 \times (-2) + 5 \times 3$	A1	correct equation
	$u = \underline{5.5}$	A1	
6(b)	Restitution	M1	only 1 sign error
	$3 - (-2) = -e(0 - 5.5)$	A1	ft u
	$e = \frac{10}{11} = 0.909$	A1	cao
6(c)	Impulse = change of momentum	M1	for P or Q
	$I = 5(3 - 0)$		
	$I = \underline{15\text{ (Ns)}}$	A1	+ required
6(d)	$v' = ev$	M1	used
	$v' = 0.25 \times 3$		
	$v' = \underline{0.75\text{ ms}^{-1}}$	A1	+ required

Q	Solution	Mark	Notes
7.(a)	Resolve	M1	attempted
	$X = 85 - 40 + 75\cos\alpha$	B1	any correct resolution
	$X = 85 - 40 + 75 \times 0.8$	A1	all correct accept cos36.9
	$X = 105$		
	Resolve	M1	attempted
	$Y = 60 - 75\sin\alpha$		
	$Y = 60 - 75 \times 0.6$	A1	all correct, accept sin 36.9
	$Y = 15$		
	$R = \sqrt{105^2 + 15^2}$	M1	
	$R = 75\sqrt{2} = \underline{106.066\text{ N}}$	A1	cao
	$\theta = \tan^{-1}\left(\frac{15}{105}\right)$	M1	allow reciprocal
	$\theta = \underline{8.13°}$	A1	cao
7(b)	N2L applied to particle	M1	dim correct equation
	$75\sqrt{2} = 5a$		
	$a = 15\sqrt{2} = \underline{21.21\text{ ms}^{-2}}$	A1	ft R if first 2 M's gained.

Q	Solution				Mark	Notes
8.		Area	from *AD*	from *AB*		
	APCD	48	3	4	B1	
	PBC	24	8	8/3	B1	
	Circle	4π	3	3	B1	
	Lamina	$(72-4\pi)$	x	y	B1	areas
8(a)	Moments about *AD*				M1	equation
	$48 \times 3 + 24 \times 8 = 4\pi \times 3 + (72 - 4\pi)x$				A1	ft table
	x = <u>5.02 cm</u>				A1	cao
	Moments about *AB*				M1	equation
	$48 \times 4 + 24 \times 8/3 = 4\pi \times 3 + (72 - 4\pi)y$				A1	ft table
	y = <u>3.67 cm</u>				A1	cao
8(b)	*AQ* = <u>3.67 cm</u>				B1	ft y

Q	Solution	Mark	Notes
1(a)	Loss in KE $= 0.5mv^2$		
	$= 0.5 \times 8 \times 7^2$	M1	Corr use of KE formula
	$= \underline{196\text{J}}$	A1	
1(b)	Work energy principle	M1	correct use
	$196 = F \times 15$	A1	ft loss in KE
	$F = \mu R$		
	$= 8g\mu = (78.4\mu)$	B1	
	Therefore $196 = 78.4\mu \times 15$		
	$\mu = \frac{1}{6}$	A1	ft loss in KE. Isw
	OR		
	Use of $v^2=u^2+2as$		
	$0=7^2 + 2a\times 15$	(M1)	
	$a = -1.633$		
	Use $F = ma$		
	$-F = 8\times -1.633$	(M1)	
	$F = 8\mu g$	(B1)	
	$\mu = \frac{13\cdot 067}{8g} = \frac{1}{6}$	(A1)p	

Q	Solution	Mark	Notes
2(a)	$\mathbf{r} = \int v \, \mathrm{d}t$	M1	use of integration
	$\mathbf{r} = \int (13t - 3)\mathbf{i} + (2 + 3t^2)\mathrm{j} \, \mathrm{d}t$		
	$\mathbf{r} = \left(\frac{13}{2}t^2 - 3t\right)\mathrm{i} + (2t + t^3)\mathrm{j} + (\underline{c})$	A1 A1	one for each coefficient
	When t = 0,	m1	use of initial conditions
	$\mathbf{c} = 2\mathbf{i} + 7\mathbf{j}$	A1	ft **r**
	$\mathbf{r} = (6.5t^2 - 3t + 2)\mathbf{i} + (2t + t^3 + 7)\mathbf{j}$		
2(b)	$\mathbf{a} = \frac{\mathrm{d}v}{\mathrm{d}t}$		M1 use of differentiation
	$= 13\mathbf{i} + 6t\mathbf{j}$	A1	
2(c)	We require $\mathbf{v}.(\mathbf{i} - 2\mathbf{j}) = 0$	M1	used
	$\mathbf{v}.(\mathbf{i} - 2\mathbf{j}) = (13t - 3) - 2(2 + 3t^2)$	M1	allow sign errors
	$= -6t^2 + 13t - 7$	A1	any form
	$6t^2 - 13t + 7 = 0$		
	$(6t - 7)(t - 1) = 0$	m1	method for quad equation Depends on both M's
	$t = \underline{1, 7/6}$	A1	

Q	Solution	Mark	Notes
3(a)(i)	Initial horizontal speed = $15\cos\alpha$	B1	
	$= 15 \times 0.8$		
	$= 12\ \text{ms}^{-1}$		
	Time of flight = 9/12	M1	
	= <u>0.75s</u>	A1	any correct form
3(a)(ii)	Initial vertical speed = $15\sin\alpha$	B1	
	$= 15 \times 0.6$		
	$= 9\ \text{ms}^{-1}$		
	Use of $s = ut + 0.5at^2$, u=9(c), a=(±)9.8, t=0.75(c)	M1	
	$s = 9 \times 0.75 - 0.5 \times 9.8 \times 0.75^2$		
	$s = 3.99375$ m	A1	si
	Height of B above ground = <u>4.99375 m</u>	A1	ft s
3(b)	use of $v^2 = u^2 + 2as$, u=9, a=(±)9.8, s=-1	M1	allow sign errors
	$v^2 = 9^2 + 2(-9.8)(-1)$	A1	
	$v^2 = 100.6$		
	$u_H = 12$	B1	ft candidate's value
	Speed = $\sqrt{12^2 + 100.6}$	m1	
	Speed = <u>15.64 ms^{-1}</u>	A1	cao

Q	Solution	Mark	Notes
4(a)	Resolve vertically	M1	dim correct
	$R\sin\theta = Mg$	A1	
	$\sin\theta = \frac{3}{5}$	B1	
	$R = Mg \times \frac{5}{3}$		
	$R = 5Mg/3$	A1	answer given, convincing.
4(b)	N2L towards centre	M1	dim correct
	$R\cos\theta = Ma$	A1	
	$\frac{5Mg}{3} \times \frac{4}{5} = M \times \frac{8g}{3r}$		
	$CP = r = 2$	A1	
	$\frac{\text{Height}}{r} = \frac{4}{3}$	M1	use of similar triangles
	Height = $\frac{8}{3}$ m	A1	ft candidate's r if first M1 given.

Q	Solution	Mark	Notes
5(a)	$0 < t < 6$	B1 B1	
5(b)	Distance t = 6 to t = 9 = $\int_6^9 2t^2 - 12t \, dt$	M1	use of integration Limits not required
	Distance = $[\,2t^3/3 - 6t^2]_6^9$ = 72	A1	correct integration
	Distance t = 0 to t = 6 = $-\int_0^6 2t^2 - 12t \, dt$ Distance = $-[\,2t^3/3 - 6t^2]_0^6$ = $-[-72]$ = 72	A1	or for the other integral
	Required distance = 72 + 72	m1	
	= <u>144</u>	A1	cao

Q	Solution	Mark	Notes
6(a)	$T = P/v$	M1	used
	$T = \dfrac{60\times1000}{20}$		
	$T = \underline{3000\text{ N}}$	A1	
6(b)	Apply N2L to car and trailer	M1	dim correct equation All forces present
	$T - (1500+500)g\sin\alpha - (170+30) = 2000a$	A2	-1 each error
	$3000 - 2000 \times 9.8 \times \dfrac{1}{14} - 200 = 2000a$		
	$a = \underline{0.7\text{ ms}^{-2}}$	A1	convincing
6(c)	N2L applied to trailer	M1	dim correct, all forces
	$T - 500g\sin\alpha - 30 = 500a$	A2	-1 each error
	$T = 500 \times 9.8 \times \dfrac{1}{14} + 30 + 500 \times 0.7$		
	$T = \underline{730\text{ N}}$	A1	
	OR		
	N2L applied to car	(M1)	dim correct, all forces
	$3000 - 1500g\sin\alpha - 170 - T = 1500 \times 0.7$	(A2)	-1 each error
	$T = 3000 - 1500\times9.8\times\dfrac{1}{14} - 170 - 1500 \times 0.7$		
	$T = \underline{730\text{ N}}$	(A1)	

Q	Solution	Mark	Notes
7(a)	PE at start = $- 2 \times 9.8 \times 0.7$	M1	mgh used
		A1	allow 0.7, (1.2+x), (0.5+x), 1.2, 0.5, x.
	$= -13.72$ J		
	PE at end = $-2 \times 9.8 \times (1.2 + x)$		
	$= -23.52 - 19.6x$		
	EE at end = $\frac{1}{2} \times \frac{360}{1 \cdot 2} x^2$	M1	use of formula
		A1	
	EE at end $= 150x^2$		
	Conservation of energy	M1	equation, all energies
	$150x^2 - 19.6x - 23.52 = -13.72$	A1	correct equation any form
	$150x^2 - 19.6x - 9.8 = 0$		
	$x = \underline{0.33}$	A1	cao
7(b)	KE at end $= 0.5 \times 2v^2$	B1	
	$= v^2$		
	PE at end $= -2 \times 9.8 \times 1.2$		
	$= -23.52$		
	Conservation of energy	M1	equation, no EE
	$v^2 - 23.52 = -13.72$	A1	correct equation, any form
	$v^2 = 9.8$		
	$v = \underline{3.13 \text{ ms}^{-1}}$	A1	

Q	Solution	Mark	Notes
8(a)	Conservation of energy	M1	equation required
	$0.5mu^2 + mgr\cos\alpha = 0.5mv^2 + mgr\cos\theta$	A1	KE
		A1	PE
	$0.5 \times 3 \times 5^2 + 3 \times 9.8 \times 4 \times 0.8 =$ $0.5 \times 3 \times v^2 + 3 \times 9.8 \times 4 \times \cos\theta$		
	$75 + 188.16 = 3v^2 + 235.2\cos\theta$		
	$v^2 = 87.72 - 78.4\cos\theta$		
	$v = \sqrt{(87.72 - 78.4\cos\theta)}$	A1	cao
8(b)	N2L towards centre	M1	dim correct, all forces
	$mg\cos\theta - R = ma$	A1	
	$R = 3 \times 9.8\cos\theta - \frac{3}{4}(87.72 - 78.4\cos\theta)$	m1	substitute, v^2/r
	$R = 29.4\cos\theta - 65.79 + 58.8\cos\theta$		
	$R = \underline{88.2\cos\theta - 65.79}$		

Q	Solution	Mark	Notes
1(a)(i)	Apply N2L to particle	M1	dim correct equation
	ma = -mg – 3v	A1	
	$2\frac{dv}{dt} = -19.6 - v$		
1(a)(ii)	$\int \frac{2dv}{19.6+v} = -\int dt$	M1	sep. of variables
	$2\ln\lvert 19.6+v\rvert = -t + (C)$	A1	correct integration
	$t = 0$, $v = 24.5$	m1	use of initial conditions
	$C = 2\ln\lvert 44.1\rvert$	A1	ft no 2,1/2.
	$-t = 2\ln\left\lvert\frac{19.6+v}{44.1}\right\rvert$		
	$e^{-t/2} = \frac{19.6+v}{44.1}$	m1	inversion ln to e
	$\underline{v = 44.1e^{-t/2} - 19.6}$	A1	cao
1(b)	At maximum height, $v = 0$	M1	si
	$t = -2\ln\left\lvert\frac{19.6}{44.1}\right\rvert$		
	$= \underline{2\ln(2.25) = 1.62\text{ s}}$	A1	ft similar expression
1(c)	$\frac{dx}{dt} = 44.1e^{-t/2} - 19.6$	M1	$v = \frac{dx}{dt}$ used
	$x = -88.2e^{-t/2} - 19.6t\ (+ C)$	A1	ft correct integration
	When $t = 0$, $x = 0$	m1	use of initial conditions
	$C = 88.2$		
	$x = \underline{88.2 - 88.2e^{-t/2} - 19.6t}$	A1	ft one slip

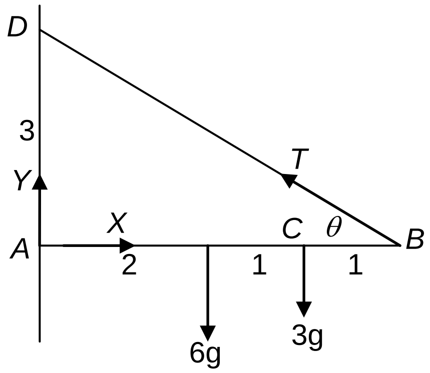
Q	Solution	Mark	Notes
2(a)	Amplitude $a = 0.5$	B1	
2(b)	Period $= \dfrac{2\pi}{\omega} = 2$	M1	si
	$\omega = \pi$	A1	
	Maximum acceleration $= a\omega^2 = 0.5 \times \pi^2$	B1	ft amplitude a.
	Occurs at end points of motion	B1	
2(c)	Let $x = a\cos(\omega t)$	M1	
	$-0.25 = 0.5\cos(\pi t)$	m1	
	$\cos(\pi t) = -0.5$		
	$\pi t = \dfrac{2\pi}{3}$		
	$t = \dfrac{2}{3}$	A1	cao
2(d)	$v^2 = \omega^2(a^2 - x^2)$, $x = 0.3$, $\omega = \pi$	M1	
	$v^2 = \pi^2(0.5^2 - 0.3^2)$	A1	ft
	$v^2 = \pi^2 \times 0.4^2$		
	$v = (\pm)0.4\pi$		
	speed $= 0.4\pi$	A1	cao

Q	Solution	Mark	Notes
3(a)(i)	Apply N2L to P	M1	
	$2a = -8x - 10v$	A1	
	$\frac{d^2x}{dt^2} = -4x - 5\frac{dx}{dt}$		
3(a)(ii)	$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 4x = 0$		
	Auxiliary equation $m^2 + 5m + 4 = 0$	B1	
	$(m + 4)(m + 1) = 0$		
	$m = -4, -1$	B1	
	CF $x = Ae^{-t} + Be^{-4t}$	B1	ft values of roots
	When $t = 0$, $x = 2$, $\frac{dx}{dt} = 3$	M1	use of initial conditions
	$2 = A + B$		
	$\frac{dx}{dt} = -Ae^{-t} - 4Be^{-4t}$	B1	
	$3 = -A - 4B$	A1	both equations correct
	Adding	m1	solving simultaneously
	$5 = -3B$		
	$B = -\frac{5}{3}$		
	$A = 2 + -\frac{5}{3} = \frac{11}{3}$		
	$x = \frac{11}{3}e^{-t} - \frac{5}{3}e^{-4t}$	A1	cao
3(b)	Try $x = at + b$	M1	
	$\frac{dx}{dt} = a$		
	$5a + 4(at + b) = 12t - 3$	A1	
	$4a = 12$	m1	comparing coefficients
	$a = 3$		
	$5a + 4b = -3$		
	$15 + 4b = -3$		
	$4b = -18$		
	$b = -\frac{9}{2}$		
	General solution $x = Ae^{-t} + Be^{-4t} + 3t - \frac{9}{2}$	A1	cao

Q	Solution	Mark	Notes
4	Initial speed of A just before impact = v		
	$v^2 = u^2 + 2as$, $u=0$, $a=(\pm)9.8$, $s=(1.8-0.2)$	M1	
	$v^2 = 0 + 2 \times 9.8 \times 1.6$	A1	
	$v = \underline{5.6 \text{ ms}^{-1}}$	A1	cao
	Impulse = Change in momentum	M1	used
	Applied to B		
	$J = 3v$	B1	
	Applied to A		
	$J = 5 \times 5.6 - 5v$	A1	ft c's answer in (a)
	Solving		
	$3v = 28 - 5v$	m1	
	$8v = 28$		
	$v = \underline{3.5 \text{ ms}^{-1}}$	A1	cao
	$J = \underline{10.5 \text{ Ns}}$	A1	cao

Q	Solution	Mark	Notes
5(a)	N2L applied to particle		
	$0.25\,a = \dfrac{5}{2x+1}$	M1	
	$v\dfrac{dv}{dx} = \dfrac{20}{2x+1}$	M1	$a = v\dfrac{dv}{dx}$
	$\int v\,dv = 10\int \dfrac{2}{2x+1}dx$	M1	separating variables
	$\frac{1}{2}v^2 = 10\ln\|2x+1\| + C$	A1	correct integration ln
		A1	LHS correct
	When $x = 0$, $v = 4$	m1	use of boundary cond. All 3 M's awarded
	$8 = 10\ln(1) + C$		
	$C = 8$		
	$v^2 = 20\ln\|2x+1\| + 16$		
	$\ln\|2x+1\| = \frac{1}{20}(v^2 - 16)$		
	$2x + 1 = e^{0.05(v^2-16)}$	m1	inversion, 3 M's awarded
	$x = 0.5(e^{0.05(v^2-16)} - 1)$	A1	cao any equivalent exp.
5(b)	$v = 6$		
	$x = 0.5(e^{0.05(36-16)} - 1)$	M1	exp. with v^2 needed
	$x = 0.5(e - 1)$		
	$x = \underline{0.86\text{ m}}$	A1	cao
5(c)	$a = 5$		
	$\dfrac{20}{2x+1} = 5$	M1	
	$20 = 10x + 5$		
	$x = 1.5$	A1	
	$v^2 = 20\ln(3 + 1) + 16$	m1	substitution in expression with v^2.
	$= 20\ln 4 + 16$		
	$v = \underline{6.61\text{ ms}^{-1}}$	A1	cao

Q	Solution	Mark	Notes



Q	Solution	Mark	Notes
6			
6(a)	Moments about A	M1	equation, no extra forces No missing forces
	$6g \times 2 + 3g \times 3 = T \times 4\sin\theta$	A2	-1 each error
	$4 \times \frac{3}{5}T = 21g$		
	$T = \frac{35}{4}g = 85.75$ N	A1	cao
6(b)	Resolve vertically	M1	equation, all forces, no extra force
	$T\sin\theta + Y = 9g$	A1	
	$Y = 9g - \frac{35}{4}g \times \frac{3}{5}$		
	$Y = \frac{15}{4}g = 36.75$ N	A1	cao
	Resolve horizontally		
	$T\cos\theta = X$	M1	equation, all forces, No extra force
	$X = \frac{35}{4}g \times \frac{4}{5}$		
	$X = 7g = 68.6$ N	A1	cao
6(b)(i)	Magnitude of reaction at wall		
	$= \sqrt{68\cdot6^2 + 36\cdot75^2}$	M1	
	$= 77.82$ N	A1	ft X and Y
6(b)(ii)	$\mu = \frac{Y}{X}$	M1	used
	$\mu = \frac{15}{4\times7} = \frac{15}{28}$	A1	ft X and Y if answer<1.

Ques	Solution	Mark	Notes
1(a)	$P(A\cup B) = P(A) + P(B)$	**M1**	Award M1 for using formula
	$P(B) = 0.4 – 0.25 = 0.15$	**A1**	
(b)	$P(A\cup B) = P(A) + P(B) – P(A)P(B)$	**M1**	Award M1 for using formula
	$0.4 = 0.25 + P(B) – 0.25P(B)$	**A1**	
	$P(B) = 0.15/0.75 = 0.2$	**A1**	
2(a)	P(1 of each) =		
	$\frac{5}{10}\times\frac{3}{9}\times\frac{2}{8}\times 6$ or $\binom{5}{1}\times\binom{3}{1}\times\binom{2}{1}\div\binom{10}{3}$	**M1A1**	M1A0A0 if 6 omitted Special case : if they use an incorrect total, eg 9 or 11, FT their incorrect total but subtract 2 marks at the end
	$=\frac{1}{4}$	**A1**	
(b)	P(3 war) = $\frac{5}{10}\times\frac{4}{9}\times\frac{3}{8}$ or $\binom{5}{3}\div\binom{10}{3}$	**M1**	
	$=\frac{1}{12}$	**A1**	
(c)	P(3 cowboy) = $\frac{3}{10}\times\frac{2}{9}\times\frac{1}{8}$ or $\binom{3}{3}\div\binom{10}{3}$		
	$=\frac{1}{120}$	**B1**	
	P(3 the same) = $\frac{1}{12}+\frac{1}{120}=\frac{11}{120}$	**M1A1**	FT previous values
3	$E(X) = 20$	**B1**	
	$\mathrm{Var}(X) = 4$ (SD = 2)	**B1**	
	$E(Y) = 20a + b = 65$	**B1**	
	$\mathrm{Var}(Y) = 4a^2 = 36$	**B1**	Accept SD(Y) = $2a = 6$
	$a = 3$	**B1**	Must be justified by solving the two equations
	$b = 5$	**B1**	
4(a)(i)	B(20,0.25)	**B1**	B must be mentioned and the parameters n and p must be seen or implied somewhere in the question
(ii)	$P(3 \le X \le 9) = 0.9087 – 0.0139$ or $0.9861 – 0.0913$	**B1B1**	
	$= 0.8948$	**B1**	
(iii)	$P(X = 6) = \binom{20}{6}\times 0.25^6\times 0.75^{14}$	**M1**	FT an incorrect p except for the last three marks
	$= 0.169$	**A1**	M0 if no working seen
(b)(i)	Let Y denote the number of throws giving ‘8’		
	Then Y is B(160,0.0625) ≈ Poi(10).	**B1**	
	$P(Y = 12) = e^{-10}\times\frac{10^{12}}{12!}$	**M1**	M0 if no working seen
	$= 0.0948$	**A1**	Accept the use of tables
(ii)	$P(6\le Y\le 14) = 0.9165 – 0.0671$ or $0.9329 – 0.0835$	**B1B1**	Correct values only (no FT)
	$= 0.8494$ cao	**B1**	

5(a) **(b)**	P(1) = $\frac{1}{3}\times\frac{1}{4}+\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{1}{2}$ = $\frac{13}{36}$ (0.361) P(A|1) = $\frac{1/12}{13/36}$ = $\frac{3}{13}$ cao (0.231)	**M1A1** **A1** **B1B1** **B1**	M1 Use of Law of Total Prob (Accept tree diagram) FT denominator from (a) B1 num, B1 denom
6(a) **(b)**	The sequence is MMMH si Prob = 0.3×0.3×0.3×0.7 = 0.0189 The sequence is MHH or HMH si Prob = 0.3×0.7×0.7 + 0.7×0.3×0.7 = 0.294	**B1** **M1A1** **B1** **M1A1**	 Award B1 for 0.147
7(a) **(b)** **(c)(i)** **(ii)**	$\sum p_x = k\left(1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}\right) = 1$ $k\left(\frac{8+4+2+1}{8}\right)=1\rightarrow k=\frac{8}{15}$ $E(X)=\frac{8}{15}\times1+\frac{4}{15}\times2+\frac{2}{15}\times4+\frac{1}{15}\times8$ $=\frac{32}{15}$ (2.13) $\text{E}(X^2)=\frac{8}{15}\times1+\frac{4}{15}\times4+\frac{2}{15}\times16+\frac{1}{15}\times64$ (8) Var(X) = $8-\left(\frac{32}{15}\right)^2=3.45$ (776/225) The possibilities are (1,1); (2,2); (4,4); (8,8) si $\text{P}\left(X_1=X_2\right)=\left(\frac{8}{15}\right)^2+\left(\frac{4}{15}\right)^2+\left(\frac{2}{15}\right)^2+\left(\frac{1}{15}\right)^2$ $=\frac{17}{45}$ (0.378) It follows that $\text{P}\left(X_1\neq X_2\right)=\frac{28}{45}$ And therefore by symmetry $\text{P}\left(X_1>X_2\right)=\frac{14}{45}$	**M1** **A1** **M1** **A1** **M1A1** **A1** **B1** **M1** **A1** **M1** **A1**	 Convincing Accept 3.46 FT their answer from (c)(i) Do not accept any other method.

8(a)	Let X denote the number of calls between 9am and 10 am so that X is Po(5)	**B1**	
	$P(X = 7) = \dfrac{e^{-5} \times 5^7}{7!}$	**M1**	M0 no working
	$= 0.104$	**A1**	
(b)	We require P(calls betw 9 and 10=7|calls betw 9 and 11=10)		
	$= \dfrac{P(\text{c b 9 and 10} = 7 \text{ AND c b 9 and 11} = 10)}{P(\text{calls between 9 and 11} = 10)}$	**M1**	
	$= \dfrac{P(\text{c b 9 and 10} = 7) \times P(\text{c b 10 and 11} = 3)}{P(\text{calls between 9 and 11} = 10)}$	**A1**	
	$= \dfrac{e^{-5} \times 5^7}{7!} \times \dfrac{e^{-5} \times 5^3}{3!} \div \dfrac{e^{-10} \times 10^{10}}{10!}$ (denom = 0.125)	**A1A1**	A1 numerator, A1 denominator The denominator A1 can be
	$= 0.117$	**A1**	awarded if the M1 is awarded
9(a)	$\int_0^2 k\left(1 - \dfrac{x^2}{4}\right)dx = 1$	**M1**	M1 for $\int f(x)dx$, limits not required until next line
	$k\left[x - \dfrac{x^3}{12}\right]_0^2 = 1$	**A1**	
	$k\left(2 - \dfrac{8}{12}\right) = 1$	**A1**	
	$k = \dfrac{3}{4}$		
(b)	$E(X) = \int_0^2 x\left(\dfrac{3}{4} - \dfrac{3x^2}{16}\right)dx$	**M1A1**	M1 for the integral of $xf(x)$, A1 for completely correct although
	$= \left[\dfrac{3x^2}{8} - \dfrac{3x^4}{64}\right]_0^2$	**A1**	limits may be left until 2nd line.
	$= 0.75$	**A1**	
(c)(i)	$F(x) = \int_0^x \left(\dfrac{3}{4} - \dfrac{3t^2}{16}\right)dt$	**M1**	M1 for $\int f(x)dx$
	$= \left[\dfrac{3t}{4} - \dfrac{t^3}{16}\right]_0^x$	**A1**	A1 for performing the integration
	$= \dfrac{3x}{4} - \dfrac{x^3}{16}$	**A1**	A1 for dealing with the limits
(ii)	$P(0.5 \le X \le 1.5) = F(1.5) - F(0.5)$	**M1**	FT their F(x)
	$= 0.547$	**A1**	

Ques	Solution	Mark	Notes
1(a)(i)	$z = \frac{10.5-10}{2} = 0.25$ P($X \le 10.5$) = 0.5987	**M1A1** **A1**	M0 for 2^2 or $\sqrt{2}$ M1A0 for – 0.25 if final answer incorrect M0 no working
(ii)	$x = \frac{x-\mu}{\sigma} = 1.282$ = 12.564	**M1** **A1**	M1 for 2.326, 1.645, 2.576 Accept 12.6
(b)(i)	E(X + 2Y) = 34 Var(X + 2Y) = Var(X) + 4Var(Y) = 40 We require P(X + 2Y < 36) $z = \frac{36-34}{\sqrt{40}} = 0.32$ Prob = 0.6255	**B1** **B1** **M1A1** **A1**	FT their mean and variance M0 no working
(ii)	Consider $U = X_1 + X_2 + X_3 - Y_1 - Y_2$ E(U) = 3 × 10 – 2 × 12 = 6 Var(U) = 3 × 4 + 2 × 9 = 30 We require P(U < 0) $z = \frac{0-6}{\sqrt{30}} = -1.10$ Prob = 0.136	**B1** **M1A1** **m1A1** **A1**	Do not FT their mean and variance
2(a)	$\bar{x} = \frac{9980}{50}$ (= 199.6) SE of $\bar{X} = \frac{4}{\sqrt{50}}$ (= 0.5656…) 95% conf limits are 199.6 ± 1.96 × 0.5656… giving [198.5, 200.7] cao	**B1** **B1** **M1A1** **A1**	M1 correct form, A1 correct z. M0 no working
(b)	Width of 95% CI = $3.92 \times \frac{4}{\sqrt{n}}$ si We require $3.92 \times \frac{4}{\sqrt{n}} < 1$ $n >$ 245.86… Minimum n = 246	**B1** **M1** **A1** **A1**	FT their z from (a) Award M1A0A0 for 1.96 instead of 3.92 FT from line above if n > 50

3(a) **(b)**	$H_0: \mu_B = \mu_G; H_1: \mu_B \neq \mu_G$ $\bar{x}_B = \frac{482}{8} = 60.25; \bar{x}_G = \frac{430}{8} = 53.75$ SE of diff of means= $\sqrt{\frac{7.5^2}{8} + \frac{7.5^2}{8}}$ (3.75) Test statistic (z) = $\frac{60.25 - 53.75}{3.75}$ = 1.73 Prob from tables = 0.0418 p-value = 0.0836 Insufficient evidence to conclude that there is a difference in performance between boys and girls.	**B1** **B1B1** **M1A1** **m1A1** **A1** **A1** **B1** **B1**	 FT their z if M marks gained FT on line above FT their p-value
4(a) **(b)** **(c)**	$H_0: p = 0.4; H_1: p > 0.4$ Let X = No. supporting politician so that X is B(50,0.4) (under H_0) si p-value = P($X \geq 25$|X is B(50,0.4)) = 0.0978 Insufficient evidence to conclude that the support is greater than 40%. X is now B(400,0.4) (under H_0) ≈ N(160,96) p-value = P($X \geq 181$|X is N(160,96)) $z = \frac{180.5 - 160}{\sqrt{96}}$ = 2.09 p-value = 0.0183 Strong evidence to conclude that the support is greater than 40%.	**B1** **B1** **M1** **A1** **B1** **B1** **M1** **m1A1** **A1** **A1** **B1**	 M0 for P(X = 25) or P(X > 25) M0 normal or Poisson approx FT on p-value Award m1A0A1A1 for incorrect or no continuity correction 181.5 → z = 2.19 → p = 0.01426 181 → z = 2.14 → p = 0.01618 FT on p-value
5(a) **(b)(i)** **(ii)**	$H_0: \mu = 1.2$: $H_1: \mu < 1.2$ Let X = number of accidents in 60 days Then X is Poi(72) (under H_0) ≈ N(72,72) si Sig level = P($X \leq 58$| H_0) $z = \frac{58.5 - 72}{\sqrt{72}}$ = – 1.59 Sig level = 0.0559 X is now Poi(48) which is approx N(48,48) si P(wrong conclusion) = P($X \geq 59$| $\mu = 0.8$) $z = \frac{58.5 - 48}{\sqrt{48}}$ = 1.52 P(wrong conclusion) = 0.0643	**B1** **B1** **M1** **m1A1** **A1** **A1** **B1** **M1** **m1A1** **A1** **A1**	Must be μ Award m1A0A1A1 for incorrect or no continuity correction 57.5 → z = –1.71 → p = 0.0436 58 → z = –1.65 → p = 0.0495 Award m1A0A1A1 for incorrect or no continuity correction 59.5 → z = 1.66 → p = 0.0485 59 → z = 1.59 → p = 0.0559

6(a)(i)	$E(C) = 2\pi E(R)$ $= 2\pi \times 7 = 14\pi$ (43.98) $\text{Var}(C) = 4\pi^2 \text{Var}(R)$ $= \dfrac{4\pi^2}{3}$ (13.16)	**M1** **A1** **M1** **A1**	Accept the use of integration, M1 for a correct integral and A1 for the correct answer
(ii)	$P(C \le 45) = P(R \le 45/2\pi)$ $= \dfrac{(45/2\pi - 6)}{8-6}$ $= 0.581$	**M1** **A1** **A1**	
(b)(i)	$A = \pi R^2$ $P(A \ge 150) = P\left(R \ge \sqrt{150/\pi}\right)$ $= \dfrac{8-\sqrt{150/\pi}}{8-6}$ $= 0.545$	**M1A1** **A1** **A1**	
(ii)	EITHER $E(A) = \displaystyle\int_6^8 \pi r^2 \times \frac{1}{2}\,\mathrm{d}r$ $= \dfrac{\pi}{6}\left[r^3\right]_6^8$ $= \dfrac{148\pi}{3}$ (155) OR $E(A) = \pi E(R^2) = \pi\left(\text{var}(R) + (E(R))^2\right)$ $= \pi\left(\dfrac{1}{3} + 7^2\right)$ $= \dfrac{148\pi}{3}$ (155)	**M1** **A1** **A1** **M1** **A1** **A1**	

Ques	Solution	Mark	Notes
1	$\hat{p} = 0.29$ si ESE = $\sqrt{\dfrac{0.29 \times 0.71}{300}}$ (= 0.02619..) si 95% confidence limits are $0.29 \pm 1.96 \times 0.02619..$ giving [0.24,0.34]	B1 M1A1 m1A1 A1	m1 correct form, A1 1.96
2	The possibilities are <u>3 red, 1 blue for which $\|X - Y\| = 2$</u> Therefore, P($X - Y = 2$) = $\dfrac{3}{10} \times \dfrac{2}{9} \times \dfrac{1}{8} \times \dfrac{7}{7} \times 4$ OR $\dfrac{\binom{3}{3} \times \binom{7}{1}}{\binom{10}{4}}$ = $\dfrac{1}{30}$ <u>2 red, 2 blue for which $\|X - Y\| = 0$</u> P($X - Y = 0$) = $\dfrac{3}{10} \times \dfrac{2}{9} \times \dfrac{7}{8} \times \dfrac{6}{7} \times 6$ OR $\dfrac{\binom{3}{2} \times \binom{7}{2}}{\binom{10}{4}}$ = $\dfrac{3}{10}$ <u>1 red, 3 blue for which $\|X - Y\| = 2$</u> P($X - Y = -2$) = $\dfrac{3}{10} \times \dfrac{7}{9} \times \dfrac{6}{8} \times \dfrac{5}{7} \times 4$ OR $\dfrac{\binom{3}{1} \times \binom{7}{3}}{\binom{10}{4}}$ = $\dfrac{1}{2}$ <u>0 red, 4 blue for which $\|X - Y\| = 4$</u> P($X - Y = -4$) = $\dfrac{7}{10} \times \dfrac{6}{9} \times \dfrac{5}{8} \times \dfrac{4}{7}$ OR $\dfrac{\binom{7}{4}}{\binom{10}{4}} = \dfrac{1}{6}$ The distribution of $\|X - Y\|$ is therefore $\|X - Y\|$: 0, 2, 4 Prob: 3/10, 8/15, 1/6	M1A1 A1 B1 B1 B1 M1A1	FT if found as 1 - Σprobs FT their probabilities

3(a)	UE of $\mu = 34.3$ $\Sigma x^2 = 10609.43$ UE of $\sigma^2 = \frac{10609.43}{8} - \frac{9 \times 34.3^2}{8}$ $= 2.6275$	**B1** **B1** **M1** **A1**	No working need be seen M0 division by 9 Answer only no marks
(b)	DF = 8 si t-value = 1.86 90% confidence limits are $34.3 \pm 1.86\sqrt{\frac{2.6275}{9}}$ giving [33.3,35.3] cao	**B1** **B1** **M1A1** **A1**	 Answer only no marks
(c)	EITHER Width of interval = $2t\sqrt{\frac{2.6275}{9}} = 3.2$ So $t = 2.96$ For a 99% confidence interval, t = 3.355 Since 2.96 < 3.355, the confidence level is less than 99% OR For 99% confidence interval, $t = 3.355$ 99% confidence limits are $34.3 \pm 3.355\sqrt{\frac{2.6275}{9}}$ giving [32.5,36.1] The given confidence interval is narrower than this therefore its confidence level is less than 99%	 **M1** **A1** **B1** **A1** **B1** **M1** **A1** **A1**	
4(a)	The 5% critical value = 2000 + 1.645 × $\sqrt{\frac{2554}{120}}$ = 2007.6 The 10% critical value = 2000 + 1.282 × $\sqrt{\frac{2554}{120}}$ = 2005.9 The required range is therefore (2005.9,2007.6)	**M1** **A1** **M1** **A1** **A1**	M1A0 for – M1A0 for –
(b)	No because of the Central Limit Theorem AND THEN EITHER which ensures the normality of the sample mean OR which can be used because the sample is large	**B1** **B1**	

5(a) **(b)**	$H_0 : \mu_A = \mu_B ; H_1 : \mu_A \neq \mu_B$ $\bar{x} = 55.25; \bar{y} = 55.75$ si $s_x^2 = \frac{183345}{59} - \frac{3315^2}{59 \times 60} = 3.2415...$ $s_y^2 = \frac{186651}{59} - \frac{3345^2}{59 \times 60} = 2.8347...$ [Accept division by 60 giving 3.1875 and 2.7875] SE = $\sqrt{\frac{3.2415..}{60} + \frac{2.8347..}{60}}$ = (0.3182..,0.3155..) si Test stat = $\frac{55.75 - 55.25}{0.3182..}$ = 1.57 (1.58) p-value = 0.116 (0.114) cao Insufficient evidence for believing that the mean weights are unequal.	**B1** **B1** **M1A1** **A1** **M1** **A1** **m1** **A1** **A1** **B1**	 FT 1 error in the means Answer only no marks FT their p-value
6(a) **(b)**	$\sum x = 175, \sum x^2 = 5075, \sum y = 118.1, \sum xy = 3170$ $S_{xy} = 3170 - 175 \times 118.1/7 = 217.5$ $S_{xx} = 5075 - 175^2/7 = 700$ $b = \frac{217.5}{700} = 0.311$ $a = \frac{118.1 - 175 \times 0.311..}{7} = 9.10$ SE of a = $\sqrt{\frac{0.1^2 \times 5075}{7 \times 700}}$ (0.1017…) 95% confidence limits for α are $9.10 \pm 1.96 \times 0.1017...$ giving [8.9,9.3]	**B2** **B1** **B1** **M1** **A1** **M1** **A1** **M1A1** **m1A1** **A1**	Minus 1 each error FT 1 error in sums FT their value of a M1 correct form, A1 1.96

7(a)	$\mathrm{E}(\hat{p}) = \frac{E(X)}{n} = \frac{np}{n} = p$ Therefore unbiased. $\mathrm{SE}(\hat{p}) = \sqrt{\frac{\mathrm{Var}(X)}{n^2}} = \sqrt{\frac{np(1-p)}{n^2}} = \sqrt{\frac{p(1-p)}{n}}$	**M1** **A1** **M1** **A1**	 This line need not be seen Accept q for $1 - p$
(b)(i)	$\mathrm{E}(\hat{p}^2) = \frac{\mathrm{E}(X^2)}{n^2}$ $= \frac{\mathrm{Var}(X) + [\mathrm{E}(X)]^2}{n^2}$ $= \frac{np(1-p) + n^2p^2}{n^2}$ $(= p^2 + \frac{p(1-p)}{n})$ $\neq p^2$ therefore not unbiased	**M1** **m1** **A1** **A1**	 This line need not be seen
(ii)	$\mathrm{E}[X(X-1)] = \mathrm{E}(X^2) - \mathrm{E}(X)$ $= np(1-p) + n^2p^2 - np$ $= n(n-1)p^2$ It follows that $\frac{X(X-1)}{n(n-1)}$ is an unbiased estimator for p^2.	**M1** **A1** **A1** **A1**	
(c)(i)	EITHER By reversing the interpretation of success and failure, it follows that $\frac{(n-X)(n-X-1)}{n(n-1)}$ is an unbiased estimator for q^2. OR $q^2 = (1-p)^2 = 1 - 2p + p^2$ Therefore an unbiased estimator for q^2 is $1 - \frac{2X}{n} + \frac{X(X-1)}{n(n-1)}$	 **M1** **A1** **M1** **A1**	 This expression need not be simplified
(ii)	Since $pq = p(1-p) = p - p^2$ It follows that an unbiased estimator for pq $= \frac{X}{n} - \frac{X(X-1)}{n(n-1)}$ $= \frac{X(n-X)}{n(n-1)}$	**M1** **A1** **A1**	

WJEC
245 Western Avenue
Cardiff CF5 2YX
Tel No 029 2026 5000
Fax 029 2057 5994
E-mail: exams@wjec.co.uk
website: www.wjec.co.uk