AQA

**General Certificate of Education (A-level)
January 2012**

Mathematics   **MM1B**

(Specification 6360)

Mechanics 1B

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown
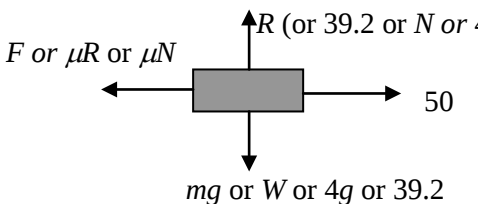
Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1	$7(3\mathbf{i} + 8\mathbf{j}) + 3(6\mathbf{i} - 5\mathbf{j}) = 10\mathbf{v}$ $\mathbf{v} = 3.9\mathbf{i} + 4.1\mathbf{j}$	M1A1 A1	 3	M1: Three term equation for conservation of momentum with addition of terms and total mass of 10. Allow one error, for example switching masses or omitting negative sign in velocity. A1: Correct equation for velocity. A1: Correct velocity. Accept $\begin{bmatrix} 3.9 \\ 4.1 \end{bmatrix}$ Finding speed as 5.66 without showing velocity scores M1 A0 A0 Finding speed after having correct velocity should be considered as further work and not penalised. Note: For consistent use of weight deduct one mark.
	Total		**3**	
2(a)	 F or μR or μN R (or 39.2 or N or $4g$) 50 mg or W or $4g$ or 39.2	B1	1	B1: Correct force diagram with four forces with arrows and labels. Accept words eg friction instead of letters Ignore negative signs in labels. Do not accept 4 kg for the weight. Award marks if forces are drawn on the diagram in the question.
(b)	39.2 N	B1	1	B1: Correct reaction force. Accept $4g$. Do not accept 39.
(c)	$50 - F = 4 \times 3$ $F = 38$	M1A1 A1	 3	M1: Three term equation of motion with the correct terms. A1: Correct equation with correct signs. A1: Correct friction.
(d)	$38 = \mu \times 39.2$ $\mu = \dfrac{38}{39.2} = 0.969$	M1 A1F	 2	M1: Use of $F = \mu R$ with their answers to (b) and (c). A1F: Correct μ based on their answers to (b) and (c). Accept AWRT 0.969. Note: $F = 12$ leads to 0.306 and award M1 A1F Condone 0.97 or FT to 2sf Condone use of inequalities.
(e)	Less friction, so a smaller coefficient of friction.	B1 B1	 2	B1: Less friction. B1: Smaller μ. Note: More friction anywhere scores B0 B0 Less friction, greater μ scores B1 B0 Smaller μ with no/inexact reason B0 B1
	Total		**9**	

Q	Solution	Marks	Total	Comments
3(a)	$s_1 = \dfrac{1}{2} \times 5 \times 28 = 70$ m	M1A1	2	M1: For $\dfrac{1}{2} \times 5 \times 28$ or equivalent. A1: Correct distance.
(b)	$s = 70 + \dfrac{1}{2} \times 5 \times 22$ $= 70 + 55$ $= 125$ m	B1M1 A1F	3	B1: For $\pm\dfrac{1}{2} \times 5 \times 22$ or equivalent. M1: For adding the distances. A1F: Correct distance. Follow through their answer from part (a) only.
(c)	Average speed $= \dfrac{125}{50} = 2.5$ m s^{-1}	M1 A1F	2	M1: For their answer to (b) divided by 50. A1F: Correct average speed. Follow through answers from part (b).
(d)	Displacement from $O = 70 - 55$ $= 15$ m	B1	1	B1: Correct displacement.
(e)	Average velocity $= \dfrac{15}{50} = 0.3$ m s^{-1}	M1 A1F	2	M1: For their answer to (d) divided by 50, provided they have subtracted in (d). A1F: Correct average velocity. Follow through answers from part (d) Award no marks if the final answer is 0.
(f)	$a = \dfrac{5}{18} = 0.278$ m s^{-2}	B1	1	B1: Correct acceleration. Accept $\dfrac{5}{18}$ or equivalent fraction or 0.277 or AWRT 0.278. Condone 0.28
	Total		**11**	

Q	Solution	Marks	Total	Comments
4(a)	$V \sin 30° = 3$ $$V = \frac{3}{\sin 30°} = 6$$	M1A1 A1	3	M1: Resolving parallel to the bank. Accept $V \cos 30° = 3$. A1: Correct equation. A1: Correct V.
(b)	$$t = \frac{200}{6 \sin 60°} = 38 \text{ (seconds)}$$	M1 A1F A1F	3	M1: 200 divided by $V \sin 60°$ or $V \sin 30°$ or equivalent with their value for V from (a). A1F: Correct expression for t. A1F: Correct value for t to nearest second. Follow through their answer to part (a)
	OR $$h = \frac{200}{\sin 60°} = 230.94$$ $$t = \frac{230.94}{6} = 38 \text{ (seconds)}$$ OR	(M1) (A1F) (A1F)		M1:Distance divided by corresponding velocity. A1F: Correct expression for t A1F: Correct value for t to nearest second. Follow through their answer to part (a)
	resultant velocity $= \sqrt{27}$ $$t = \frac{200}{\sqrt{27}} = 38 \text{ (seconds)}$$	(M1) (A1F) (A1F)		Do not accept 38.5
	Total		**6**	
5(a)	$4720 - 3R = 2200 \times 1.6$ $$R = \frac{4720 - 3520}{3} = 400$$	M1A1 A1 A1	4	M1: Three term horizontal equation of motion with mass of 2200 kg and $3R$ (or $2R$ and R). A1: All terms correct (4720, $3R$ and 2200×1.6). A1: Correct signs. A1: Correct value for R.
	OR $4720 - R - T = 1200 \times 1.6$ $T - 2R = 1000 \times 1.6$ $4720 - 3R = 3520$ $R = 400$	(M1A1) (A1) (A1)		M1: Forming an equation for each body and adding to eliminate T. A1: Two correct equations. A1: Correct equation in R. A1: Correct value for R.
(b)	$T - 2 \times 400 = 1000 \times 1.6$ $T = 800 + 1600 = 2400 \text{ N}$ OR	M1A1F A1F	3	M1: Three term equation of motion for caravan with T, $2R$ and 1000×1.6. A1F: Correct equation, with their value for R from part (a). A1F: Correct tension. Follow through from part (a) using $T = 1600 + 2R$
	$4720 - T - 400 = 1200 \times 1.6$ $T = 4720 - 400 - 1920 = 2400 \text{ N}$	(M1) (A1F) (A1F)		M1: Four term equation of motion for car with 4720, T, R and 1200×1.6. A1F: Correct equation, with their value for R from part (a) A1F: Correct tension. Follow through from part (a) using $T = 2800 - R$ Note: do not follow through if a negative value is used for R.
	Total		**7**	

Q	Solution	Marks	Total	Comments
6(a)(i)	$10^2 = 4^2 + 2 \times a \times 50$ $a = \dfrac{100-16}{100} = 0.84 \text{ m s}^{-2}$	M1A1 A1	3	M1: Use of a constant acceleration equation to find a, with v and u substituted correctly. For example $4^2 = 10^2 + 100a$ scores M0A0A0. A1: Correct constant acceleration equation. A1: Correct a. Note if t found first award M1 for use of $v = u + at$ or $s = ut + \dfrac{1}{2}at^2$.
(ii)	$50 = \dfrac{1}{2}(4+10)t$ $t = \dfrac{50}{7} = 7.14 \text{ s}$ OR $10 = 4 + 0.84t$ $t = \dfrac{6}{0.84} = 7.14 \text{ s}$ OR $50 = 4t + \dfrac{1}{2} \times 0.84t^2$ $0.42t^2 + 4t - 50 = 0$ $t = 7.14$ (or $t = -16.6$)	M1A1 A1 (M1A1F) (A1) (M1A1F) (A1)	3	M1: Use of a constant acceleration equation to find t. A1F: Correct constant acceleration equation with their acceleration from (a)(i) seen. A1: Correct t. Accept $\dfrac{50}{7}$ or $7\dfrac{1}{7}$ or AWRT 7.14. If t has been found in part (a) the working does not have to be repeated, but value of t must be stated. Do not follow through incorrect values of a.
(b)	$70 \times 0.84 = 58.8 \text{ N}$	M1A1F	2	M1: Use of $F = ma$ with $m = 70$ and their a from (a)(i). A1F: Correct F. Follow through their value of a from part (a)(i).
(c)(i)	$58.8 = 70 \times 9.8 \sin\alpha$ $\sin\alpha = \dfrac{58.8}{70 \times 9.8} = 0.08571$ $\alpha = 4.92°$	M1A1F A1F	3	M1: Resolving parallel to the slope must see $70g$ or mg OE with $\sin\alpha$ or $\cos\alpha$ and their answer to part (b). A1F: Correct equation. Follow through their answer to part (b) provided $\sin\alpha < 1$ A1F: Correct angle. Follow through their answer to part (b). Accept 4.91° provided $\sin\alpha < 1$.

Q	Solution	Marks	Total	Comments
6(c)(ii)	$70 \times 9.8 \sin \alpha - 30 = 58.8$ $\sin \alpha = 0.12945$ $\alpha = 7.44°$	M1A1F A1F	3	M1: Three term equation of motion. must see $70g$ or mg OE with $\sin \alpha$ or $\cos \alpha$. A1F: Correct equation. Follow through their answer to part (b) provided $\sin \alpha < 1$ A1F: Correct angle. Follow through their answer to part (b) provided $\sin \alpha < 1$. Accept 7.43°. Accept 7.41° from 0.129.
(d)	The air resistance force will increase (vary or change) with speed.	B1	1	B1: Correct statement.
	Total		**15**	

Q	Solution	Marks	Total	Comments
7(a)	$h = \dfrac{1}{2} \times 2.5 \times 20^2$ $= 500$ m	M1 A1A1	3	M1: Expression for height or position vector at $t = 20$. A1: Correct expression for height or position vector with correct **j** component (…**i** + 500 **j**) A1: Correct height stated. Condone 500 **j**.
(b)	$\mathbf{v}(20) = (4.2\mathbf{i} + 2.5\mathbf{j}) \times 20$ $= 84\mathbf{i} + 50\mathbf{j}$	M1 A1	2	M1: Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to find the velocity at $t = 20$ with $\mathbf{u} = 0\mathbf{i} + 0\mathbf{j}$. A1: Correct velocity.
(c)	$1.25t^2 = 180$ $t = \sqrt{\dfrac{180}{1.25}} = 12$ s $\mathbf{v} = (4.2\mathbf{i} + 2.5\mathbf{j}) \times 12$ $= 50.4\mathbf{i} + 30\mathbf{j}$ $v = \sqrt{50.4^2 + 30^2} = 58.7 \text{ m s}^{-1}$ OR $a = \sqrt{4.2^2 + 2.5^2} = 4.89$ $v = 4.89 \times 12 = 58.7 \,\text{m s}^{-1}$ OR $x = 4.2 \times \dfrac{180}{2.5} = 302.4$ $v_x = \sqrt{2 \times 4.2 \times 302.4} = 50.4$ $v_y = \sqrt{2 \times 2.5 \times 180} = 30$ $v = \sqrt{50.4^2 + 30^2} = 58.7$	M1A1 A1 dM1 A1 dM1A1 (dM1A1) (dM1A1) (M1A1) (dM1A1) (A1) (dM1A1)	7	M1: Equation based on height of 180 to find t. A1: Correct equation. A1: Correct t. dM1: Using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ to find the velocity at their time with $\mathbf{u} = 0\mathbf{i} + 0\mathbf{j}$. A1: Correct velocity. dM1: Finding speed from their velocity. A1: Correct speed. Accept 58.6 or AWRT 58.7. dM1: finding magnitude of acceleration. A1: correct magnitude. dM1: acceleration × 12. A1: correct speed. M1: finding horizontal displacement when height is 180. Must see 4.2, 2.5 and 180. May be implied by seeing 302.4. A1: Seeing 302.4 dM1: Finding both components of velocity. A1: Seeing 50.4. A1: Seeing 30. dM1: Finding the speed. A1: Final answer of 58.7

Q	Solution	Marks	Total	Comments
7(c) cont	OR $v_y = \sqrt{2 \times 2.5 \times 180} = 30$ $30 = 0 + 2.5t$ $t = \dfrac{30}{2.5} = 12$ $v_x = 0 + 4.2 \times 12 = 50.4$ $v = \sqrt{50.4^2 + 30^2} = 58.7 \text{ m s}^{-1}$ OR $\tan^{-1}\left(\dfrac{2.5}{4.2}\right) = 30.76°$ $v_v = \sqrt{2 \times 2.5 \times 180} = 30$ $v = \dfrac{30}{\sin 30.76} = 58.7 \text{ m s}^{-1}$	(M1A1) (A1) (dM1A1) (dM1A1) (M1A1) (A1) (dM1A1) (dM1A1)		M1: Equations based on height of 180 to find v and then t. A1: Correct equation for t. A1: Correct t. dM1: Using $v = u + at$ to find the **j** component of velocity at their time with $u = 0$. A1: Correct velocity. dM1: Finding speed from their velocity. A1: Correct speed. Accept 58.6 or AWRT 58.7. M1: Finding angle using acceleration components. A1: Correct expression for acceleration components A1: Correct angle. dM1: Finding v_y at height of 180 A1: Correct speed of 30. dM1: Using trig to get v. A1: Correct speed. Accept 58.6 or AWRT 58.7.
	Total		**12**	

Q	Solution	Marks	Total	Comments
8(a)	$\tan\alpha = \dfrac{6}{10}$ $\alpha = 31.0°$	M1 A1	 2	M1: Using tan with 10 and 5 or 6, OR sin or cos with $\sqrt{136}$ and 6 or 10, OR sin or cos with $\sqrt{125}$ and 5 or 10. Note: $\sin\alpha = \dfrac{6}{\sqrt{136}}$ and $\cos\alpha = \dfrac{10}{\sqrt{136}}$ A1: Correct angle. Accept 30.9° or AWRT 31°
(b)	$8\sin\alpha t + 4.9t^2 = 6$ $4.9t^2 + 4.116t - 6 = 0$ $t = 0.76359$ or $t = -1.60\,\text{s}$ $t = 0.764$	M1 A1FA1F A1 dM1 A1	 6	M1: equation for the vertical motion containing ± 6 or ± 5, $\pm 4.9t^2$ and $\pm 8\sin\alpha$ or $\pm 8\cos\alpha$, where α has a value related to their answer to part (a) (May be a negative angle). A1F: Correct terms. A1F: Correct signs and terms Follow through angle from part (a). A1: Correct equation rearranged equal to zero, but may be implied by subsequent working. dM1: Attempting to solve their quadratic equation. Only award method mark if method seen or correct answers obtained or –0.764 with +1.60. A1: Correct solution obtained. Accept 0.763 or AWRT 0.764.
	OR $v = \sqrt{(8\sin 31.0°)^2 + 2 \times 9.8 \times 6} = 11.60$ $11.60 = 8\sin 31° + 9.8t$ $t = \dfrac{11.60 - 8\sin 31°}{9.8} = 0.763$	 (M1) (A1FA1F) (dM1) (A1) (A1)		 M1: Use a constant acceleration equation $v^2 = u^2 + 2as$ to find v. A1F: Correct equation. A1F: Correct v. dM1: Use of $v = u + at$ to find t A1: Correct equation. A1: Correct t (0.763)
(c)	$d = 10 - 8\cos\alpha \times 0.764$ $\quad = 10 - 5.238$ $\quad = 4.76\,\text{m}$	M1dM1 A1 A1	 4	M1: Finding a horizontal distance using $8\cos\alpha$ or $8\sin\alpha$ multiplied by their time from part (b). dM1: For subtracting their distance from 10. A1: Seeing AWRT 5.24 or 5.23 from 0.763. A1: Correct final answer. Accept AWRT 4.76. Accept 4.77 from use of 0.763.
	Total		12	
	TOTAL		75	