A-level

## MATHEMATICS

## MFP4

UNIT FURTHER PURE 4
Mark scheme
June 2017

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

| Key to mark scheme abbreviations |  |
| :---: | :---: |
| M | mark is for method |
| m or dM | mark is dependent on one or more M marks and is for method |
| A | mark is dependent on M or m marks and is for accuracy |
| B | mark is independent of $M$ or $m$ marks and is for method and accuracy |
| E | mark is for explanation |
| $\checkmark$ orft or F | follow through from previous incorrect result |
| CAO | correct answer only |
| CSO | correct solution only |
| AWFW | anything which falls within |
| AWRT | anything which rounds to |
| ACF | any correct form |
| AG | answer given |
| SC | special case |
| OE | or equivalent |
| A2,1 | 2 or 1 (or 0) accuracy marks |
| -x EE | deduct $x$ marks for each error |
| NMS | no method shown |
| PI | possibly implied |
| SCA | substantially correct approach |
| c | candidate |
| sf | significant figure(s) |
| dp | decimal place(s) |

## No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award full marks. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn no marks.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.




| Q3 | Solution | Mark | Total | Comment |
| :---: | :---: | :---: | :---: | :---: |
| (a) | $\mathbf{a} \times \mathbf{b}=\left[\begin{array}{c} 2 \\ -p \\ -1 \end{array}\right] \times\left[\begin{array}{c} 0 \\ 2 p+1 \\ -1 \end{array}\right]=\left[\begin{array}{c} 3 p+1 \\ 2 \\ 4 p+2 \end{array}\right]$ | M1 |  | Vector product attempted - all components correct (could be unsimplified). |
|  | $\begin{aligned} & (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}=\left[\begin{array}{c} 3 p+1 \\ 2 \\ 4 p+2 \end{array}\right] \cdot\left[\begin{array}{c} p-1 \\ 4 \\ 3 \end{array}\right] \\ & =(3 p+1)(p-1)+8+3(4 p+2) \\ & =3 p^{2}+10 p+13 \end{aligned}$ | A1 A1 | 3 | Correct scalar product expansion. <br> CAO |
|  | ALTERNATIVE $\left\lvert\, \begin{aligned} & \left\|\begin{array}{ccc} 2 & 0 & p-1 \\ -p & 2 p+1 & 4 \\ -1 & -1 & 3 \end{array}\right\| \\ & =2\left\|\begin{array}{cc} 2 p+1 & 4 \\ -1 & 3 \end{array}\right\|+(p-1)\left\|\begin{array}{cc} -p & 2 p+1 \\ -1 & -1 \end{array}\right\| \\ & =2(6 p+3+4)+(p-1)(p+2 p+1) \\ & =3 p^{2}+10 p+13 \end{aligned}\right.$ | (M1) <br> (A1) <br> (A1) | (3) | Correct expansion of determinant by row or column. <br> Correct expansion of $2 \times 2$ determinants. CAO |
| (b) | Either $\begin{aligned} & 3 p^{2}+10 p+13=13 \\ & p(3 p+10)=0 \\ & p=0 \text { or }-\frac{10}{3} \end{aligned}$ <br> Or | $\begin{gathered} \text { M1 } \\ \text { A1 } \end{gathered}$ |  | Solving their quadratic to find two solutions. <br> Both required . |
|  | $3 p^{2}+10 p+13=-13$ <br> Gives $3 p^{2}+10 p+26=0$ <br> No further (real) solutions as $b^{2}-4 a c=-212<0$ | M1 <br> A1 | 4 | Considering negative value of given volume. <br> Correctly justified conclusion using $b^{2}-4 a c$ <br> or finding correct complex roots of $-\frac{5}{3} \pm \frac{\sqrt{53}}{3} i$ |
|  | Total |  | 7 |  |


| Q4 | Solution | Mark | Total | Comment |
| :---: | :---: | :---: | :---: | :---: |
| (a) |  | M1 |  | Set up equation one error only. |
|  | $3 a+8=-10$ | A1 |  | Set up equation correct. |
|  | $a=-6$ | A1 | 3 | Correct single value stated. |
| (b) | Using $y=m x+c$ gives |  |  |  |
|  | $x^{\prime}=3 x-2(m x+c)$ |  |  | Correct substitution of their $y=m x+c$ <br> (Need to see $x^{\prime}, y^{\prime}$ or substituted |
|  | $y^{\prime}=4 x-6(m x+c)$ <br> Then using $y^{\prime}=m x^{\prime}+c$ gives | B1F |  | correctly to get *). |
|  | $4 x-6 y=m[3 x-2 y]+c$ | M1 |  | Substitution of their $y^{\prime}=m x^{\prime}+c$ |
|  | $\begin{aligned} & 4 x-6(m x+c)=m[3 x-2(m x+c)]+\mathrm{c} \\ & \left(2 m^{2}-9 m+4\right) x+c(2 m-7)=0 \end{aligned}$ | $\begin{gathered} * \\ \text { A1 } \end{gathered}$ |  |  |
|  | $(2 m-1)(m-4) x+c(2 m-7)=0$ |  |  | Fully correct simplification - collecting $x$ and non $x$ terms appropriately. |
|  | Hence $\begin{aligned} & m=\frac{1}{2} \\ & m=4 \end{aligned}$ | A1 |  | Factorising and solving to find two values for $m$.(Not 3 values) |
|  | When $\begin{aligned} & m=\frac{1}{2} \\ & c-7 c=0 \\ & c=0 \end{aligned}$ |  |  |  |
|  | When $\begin{aligned} & m=4 \\ & 8 c-7 c=0 \\ & c=0 \end{aligned}$ | dM1 |  | Clear and justified working to determine $c$ in each case. |
|  | Hence only invariant lines are $\begin{aligned} & y=\frac{1}{2} x \\ & y=4 x \end{aligned}$ | A1 | 6 | CSO - both equations correctly stated (must have dM1). |




\begin{tabular}{|c|c|c|c|c|}
\hline Q6 \& Solution \& Mark \& Total \& Comment <br>
\hline \multirow[t]{4}{*}{(a)} \& $$
\left|\begin{array}{ccc}
a-1 & b+1 & x-1 \\
x^{2}-b^{2} & x^{2}-a^{2} & a^{2}-b^{2} \\
2 & -2 & 2
\end{array}\right|
$$
$$
\left\lvert\, \begin{gathered}
\mathrm{c}_{1} \text { replaced by } \mathrm{c}_{1}-\mathrm{c}_{3} \\
\left|\begin{array}{ccc}
a-x & b+1 & x-1 \\
x^{2}-a^{2} & x^{2}-a^{2} & a^{2}-b^{2} \\
0 & -2 & 2
\end{array}\right|
\end{gathered}\right.
$$ \& M1 \& \& Correct use of column operations to obtain first linear factor. <br>
\hline \& $$
(x-a)\left|\begin{array}{ccc}
-1 & b+1 & x-1 \\
x+a & x^{2}-a^{2} & a^{2}-b^{2} \\
0 & -2 & 2
\end{array}\right|
$$ \& A1 \& \& Correct extraction of linear factor. (Condone missing brackets, but penalise in final A1 CSO, even if recovered). <br>
\hline \& $$
\begin{aligned}
& c_{2} \text { replaced by } c_{2}+c_{3} \\
& (x-a)\left|\begin{array}{ccc}
-1 & x+b & x-1 \\
x+a & x^{2}-b^{2} & a^{2}-b^{2} \\
0 & 0 & 2
\end{array}\right| \\
& (x-a)(x+b)\left|\begin{array}{ccc}
-1 & 1 & x-1 \\
x+a & x-b & a^{2}-b^{2} \\
0 & 0 & 2
\end{array}\right|
\end{aligned}
$$ \& M1

A1 \& \& | Correct use of column operations to obtain second linear factor. |
| :--- |
| Correct extraction of second linear factor. (Condone missing brackets, but penalise in final A1 CSO, even if recovered). | <br>

\hline \& $$
\Delta(x)=2(x-a)(x+b)(b-a-2 x)
$$ \& dM1

A1 \& 6 \& | Correct expansion of their resulting determinant to find final factor. |
| :--- |
| Fully correct - must extract the "2" for final |
| A1. CSO | <br>

\hline \multirow[t]{3}{*}{(b)} \& $$
2(x-a)(x+b)(b-a-2 x)=0
$$ \& M1 \& \& Sets their determinant equal to 0 and obtains one correct value of $x(\mathrm{PI})$. <br>

\hline \& \[
(x=) a,-b, \frac{b-a}{2}

\] \& A1 \& 2 \& | All three values obtained CSO - must have scored 6 marks in (a). |
| :--- |
| SC - if 5 in (a) due to " 2 " not extracted can get M1A1. | <br>

\hline \& Total \& \& 8 \& <br>
\hline
\end{tabular}







