

A-level MATHEMATICS MFP4

UNIT FURTHER PURE 4

Mark scheme

June 2017

Version: 1.0 Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M m or dM	mark is for method mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
\checkmark or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
С	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q1	Solution	Mark	Total	Comment
(a)	$\mathbf{B}\mathbf{A} = \begin{bmatrix} 0 & p^2 & -4p \\ 4 & -2p-2 & 10 \end{bmatrix}$	M1 A1		BA a 3x3 matrix with 3 elements correct and simplified. At least 6 elements correct and simplified.
	2 -3p - 1 13	A1	3	All correct and simplified.
(b)	$\begin{vmatrix} -4 \\ p^2 & -4p \\ -3p-1 & 13 \end{vmatrix} + 2 \begin{vmatrix} p^2 & -4p \\ -2p-2 & 10 \end{vmatrix}$	M1		Correct expansion of their determinant in (a) by row or column (must be 3x3)
	$= -4(13p^2 - 12p^2 - 4p) + 2(10p^2 - 8p^2 - 8p)$	A1		2x2 determinants correctly expanded – CAO
	$= -4p^{2} + 16p + 4p^{2} - 16p = 0$ (so BA is always) singular for all <i>p</i> .	A1	3	Fully correct cancelling and final comment made. Must reference singular and p .
	Total		6	

Q2	Solution	Mark	Total	Comment
(a)	5 2 11			
	$\begin{vmatrix} 2 & -1 & 5 \end{vmatrix}$			
	-3 3 a			
	$= -3\begin{vmatrix} 2 & 11 \\ -1 & 5 \end{vmatrix} - 3\begin{vmatrix} 5 & 11 \\ 2 & 5 \end{vmatrix} + a\begin{vmatrix} 5 & 2 \\ 2 & -1 \end{vmatrix}$	M1		Correct expansion by row or column
	=-9a-72	A1		Correctly simplified
	a = -8		•	
		Al	3	CAO
(b)	5 . 0 . 11 . 15			
	5x + 2y + 11z = 45			
	2x - y + 5z = 15			
	-3x + 3y + az = b			
	a_{1}			
	9r + 21z - 75			
	3x + 212 = 75 3xequation 2 + equation 3 gives			Elimination of same variable in two
	9r + 21z - 3b + 135	M1		equations, using their value of "a".
	5x + 212 = 50 + 155	A1		Fully correct, same coefficients on the
	Hence intersect in a line/sheaf			two variables.
	3b + 135 = 75			
	h = -20	A 1		Configuration correct and correct
				value of b stated.
	Form a prism			
	$3b+135 \neq 75$			
	$b \neq -20$			Configuration correct and correct
		AI	4	value of b stated.

ALTERNATIVE – 1			
Putting a value of x, y or z into their equations and attempt to solve.	(M1)		Must have correct value of " a " for any
Correct values of other two variables.	(A1)		of the A marks.
Line/sheaf $b = -20$	(A1)		x = 0, y = 20/7, z = 25/7
Prism $b \neq -20$	(A1)	(4)	x = 20, y = 0, z = -5 x = 25/3, y = 5/3, z = 0
ALTERNATVE – 2 $\begin{bmatrix} 5 & 2 & 11 & 45 \\ 2 & -1 & 5 & 15 \\ -3 & 3 & -8 & b \end{bmatrix}$ $\begin{bmatrix} 5 & 2 & 11 & 45 \end{bmatrix}$			Elimination of one variable in order to
$\begin{bmatrix} 0 & -9 & 3 & -15 \\ 0 & 21 & -7 & 5b + 135 \end{bmatrix}$	(M1) (A1)		consider constant term, using their value of " <i>a</i> ". Fully correct, same coefficients on the two variables or line of three zeros.
or 5 2 11 45			$\begin{bmatrix} 5 & 2 & 11 & 45 \\ 0 & -9 & 3 & -15 \\ 0 & 0 & 0 & 15b + 300 \end{bmatrix}$ Leading to
$\begin{bmatrix} 9 & 0 & 21 & 75 \\ -21 & 0 & -49 & 2b - 135 \end{bmatrix}$			$\begin{bmatrix} 5 & 2 & 11 & 45 \\ 9 & 0 & 21 & 75 \\ 0 & 0 & 0 & 6b+120 \end{bmatrix}$
$\begin{bmatrix} 5 & 2 & 11 & 45 \\ -3 & -21 & 0 & -60 \\ 7 & 49 & 0 & 11b + 360 \end{bmatrix}$			Leading to $\begin{bmatrix} 5 & 2 & 11 & 45 \\ -3 & -21 & 0 & -60 \\ 0 & 0 & 0 & 33b + 660 \end{bmatrix}$
Intersect in a line/sheaf $15b + 300 = 0$			
<i>b</i> = -20	(A1)		Configuration correct and correct value of <i>b</i> stated.
Form a prism $15b + 300 \neq 0$ $b \neq -20$	(A1)	(4)	Configuration correct and correct value of <i>b</i> stated.
Total		7	
ivtai		1	

Q3	Solution	Mark	Total	Comment
(a)	$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 2\\-p\\-1 \end{bmatrix} \times \begin{bmatrix} 0\\2p+1\\-1 \end{bmatrix} = \begin{bmatrix} 3p+1\\2\\4p+2 \end{bmatrix}$	M1		Vector product attempted – all components correct (could be unsimplified).
	$(\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c} = \begin{bmatrix} 3p+1\\2\\4p+2 \end{bmatrix} \cdot \begin{bmatrix} p-1\\4\\3 \end{bmatrix}$ $= (3p+1)(p-1) + 8 + 3(4p+2)$ $= 3p^2 + 10p + 13$	A1 A1	3	Correct scalar product expansion.
	$\begin{vmatrix} 2 & 0 & p-1 \\ -p & 2p+1 & 4 \\ -1 & -1 & 3 \end{vmatrix}$ $= 2\begin{vmatrix} 2p+1 & 4 \\ -1 & 3 \end{vmatrix} + (p-1)\begin{vmatrix} -p & 2p+1 \\ -1 & -1 \end{vmatrix}$ $= 2(6p+3+4) + (p-1)(p+2p+1)$ $= 3p^{2} + 10p + 13$	(M1) (A1) (A1)	(3)	Correct expansion of determinant by row or column. Correct expansion of 2x2 determinants. CAO
(b)	Either $3p^{2} + 10p + 13 = 13$ p(3p+10) = 0 $p = 0 \text{ or } -\frac{10}{3}$	M1 A1		Solving their quadratic to find two solutions. Both required .
	Or			
	$3p^2 + 10p + 13 = -13$ Gives $3p^2 + 10p + 26 = 0$	M1		Considering negative value of given volume.
	No further (real) solutions as $b^2 - 4ac = -212 < 0$	A1	4	Correctly justified conclusion using $b^2 - 4ac$ or finding correct complex roots of $-\frac{5}{3} \pm \frac{\sqrt{53}}{3}i$
	Total		7	

Q4	Solution	Mark	Total	Comment
(a)		M1		Set up equation one error only.
()	3a + 8 = -10	A1		Set up equation correct.
	a = -6	A1	3	Correct single value stated.
(b)	Using $y = mx + c$ gives x' = 3x - 2(mx + c) y' = 4x - 6(mx + c) Then using $y' = mx' + c$ gives 4x - 6y = m[3x - 2y] + c 4x - 6(mx + c) = m[3x - 2(mx + c)] + c $(2m^2 - 9m + 4)x + c(2m - 7) = 0$ (2m - 1)(m - 4)x + c(2m - 7) = 0	B1F M1 * A1		Correct substitution of their $y = mx + c$ (Need to see x', y' or substituted correctly to get *). Substitution of their $y' = mx' + c$ Fully correct simplification – collecting x and non x terms appropriately.
	Hence $m = \frac{1}{2}$ m = 4 When $m = \frac{1}{2}$ c - 7c = 0 c = 0	A1		Factorising and solving to find two values for <i>m</i> .(Not 3 values)
	When m = 4 8c - 7c = 0 c = 0 Hence only invariant lines are $y = \frac{1}{r}$	dM1		Clear and justified working to determine c in each case.
	y = 4x	A1	6	CSO – both equations correctly stated (must have dM1).

ALTERNATIVE			
$0 = \begin{vmatrix} 3 - \lambda & -2 \\ 4 & -6 - \lambda \end{vmatrix}$ $= \lambda^2 + 3\lambda - 10$ $= (\lambda - 2)(\lambda + 5)$ $\lambda = 2 \text{ or } \lambda = -5$	(B1F) (M1)		For solving their equation correctly.
$\frac{\lambda = 2}{\begin{pmatrix} 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 1 & -2 \\ 4 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \rightarrow y = \frac{1}{2}x$	(A1)		OE
$\frac{\lambda = -5}{\begin{pmatrix} 0 \\ 0 \end{pmatrix}} = \begin{pmatrix} 8 & -2 \\ 4 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} \longrightarrow y = 4x$	(A1)		OE
As $\lambda \neq 1$ no LoIP's \therefore no Inv. Lines not through origin.	(E1) (E1)	(6)	
Total		9	

Q5	Solution	Mark	Total	Comment
(a)(i)	$\lambda = -3$ gives $2(-3)^3 + (-3)^2 + k(-3) + 6 = 0$ k = -13	M1 A1	2	Correct substitution of $\lambda = -3$
	(ALTERNATIVE)			
	Factorising gives $(\lambda + 3)(2\lambda^2 - 5\lambda + 2) = 0$ Comparing coefficients gives k = -15 + 2 k = -13	(M1) (A1)	(2)	Correct factorisation and comparison of coefficients.
(a)(ii)	$2\lambda^{3} + \lambda^{2} - 13\lambda + 6$ = $(\lambda + 3)(2\lambda^{2} - 5\lambda + 2)$ = $(\lambda + 3)(\lambda - 2)(2\lambda - 1)$	M1		Attempt at factorisation, either of these two lines.
	Hence $\lambda = \frac{1}{2}$ $\lambda = 2$ $(\lambda = -3)$	A1	2	Both other eigenvalues found. NMS = 0 marks
(b)(i)	$(-3)^{2} \begin{bmatrix} -4\\3\\1 \end{bmatrix} = 9 \begin{bmatrix} -4\\3\\1 \end{bmatrix} = \begin{bmatrix} -36\\27\\9 \end{bmatrix}$	B1	1	Correct evaluation – with or without factor.
(ii)	x = 4/3, y = -1, z = -1/3	B1	1	Can have $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{4}{3} \\ -1 \\ -\frac{1}{3} \end{bmatrix}$
	Total		6	

Q6	Solution	Mark	Total	Comment
(a)	$\begin{vmatrix} a-1 & b+1 & x-1 \\ x^2-b^2 & x^2-a^2 & a^2-b^2 \\ 2 & -2 & 2 \end{vmatrix}$			
	$\begin{vmatrix} c_1 \text{ replaced by } c_1 - c_3 \\ a - x & b + 1 & x - 1 \\ x^2 - a^2 & x^2 - a^2 & a^2 - b^2 \\ 0 & -2 & 2 \end{vmatrix}$	M1		Correct use of column operations to obtain first linear factor.
	$ (x-a) \begin{vmatrix} -1 & b+1 & x-1 \\ x+a & x^2-a^2 & a^2-b^2 \\ 0 & -2 & 2 \end{vmatrix} $	A1		Correct extraction of linear factor. (Condone missing brackets, but penalise in final A1 CSO, even if recovered).
	$\begin{vmatrix} c_{2} \text{ replaced by } c_{2} + c_{3} \\ (x-a) \begin{vmatrix} -1 & x+b & x-1 \\ x+a & x^{2}-b^{2} & a^{2}-b^{2} \\ 0 & 0 & 2 \end{vmatrix}$	M1		Correct use of column operations to obtain second linear factor.
	$ \begin{vmatrix} (x-a)(x+b) \\ x+a & x-b & a^2-b^2 \\ 0 & 0 & 2 \end{vmatrix} $	A1		Correct extraction of second linear factor. (Condone missing brackets, but penalise in final A1 CSO, even if recovered).
	$\Delta(x) = 2(x-a)(x+b)(b-a-2x)$	dM1 A1	6	Correct expansion of their resulting determinant to find final factor. Fully correct – must extract the "2" for final A1 . CSO
(b)	2(x-a)(x+b)(b-a-2x) = 0	M1		Sets their determinant equal to 0 and obtains one correct value of x (PI).
	$(x =) a, -b, \frac{b-a}{2}$	A1	2	All three values obtained CSO – must have scored 6 marks in (a) . SC – if 5 in (a) due to "2" not extracted can get M1A1 .
	Total		8	

Colution	Ivial K	Total	Comment
$\begin{vmatrix} 2 & 4 & 3 \\ 1 & 1 & 1 \\ 0 & 3 & k+1 \end{vmatrix} = -3 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + (k+1) \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix}$	M1		Correct expansion by row or column.
$=1-2k$ $1-2k = 0$ $k = \frac{1}{2}$ $x' = 2x + 4y + 3z$ $y' = x + y + z$ $z' = 3y + 1.5z$	A1		Correct simplification and sets equal to zero – correct value found.
3x'-6y'-2z' =3(2x+4y+3z)-6(x+y+z)-2(3y+1.5z) =0 Therefore each point must lie on the plane 3x'-6y'-2z'=0	dM1 A1	4	Substitutes their expressions in given plane equation, must use their value for <i>k</i> . Fully shown – AG . Must see either first three lines or concluding statement when the top line is missing.
$\begin{bmatrix} k-2 & -k-1 & 3\\ 5-4k & 2k+2 & -6\\ 1 & 1 & -2 \end{bmatrix}$ $\mathbf{M}^{-1} = \frac{1}{1-2k} \begin{bmatrix} k-2 & 5-4k & 1\\ -k-1 & 2k+2 & 1\\ 3 & -6 & -2 \end{bmatrix}$	M1 A2 dM1 A1	5	M1 one full row or column correct. A1 at least 6 entries correct. A2 all entries correct. Transpose of their cofactors (with one further error at most) and dividing by their determinant. Fully correct – CAO
	$\begin{vmatrix} 2 & 4 & 3 \\ 1 & 1 & 1 \\ 0 & 3 & k+1 \end{vmatrix} = -3 \begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} + (k+1) \begin{vmatrix} 2 & 4 \\ 1 & 1 \end{vmatrix}$ $= 1 - 2k$ $1 - 2k = 0$ $k = \frac{1}{2}$ $x' = 2x + 4y + 3z$ $y' = x + y + z$ $z' = 3y + 1.5z$ $3x' - 6y' - 2z' = 3(2x + 4y + 3z) - 6(x + y + z) - 2(3y + 1.5z)$ $= 0$ Therefore each point must lie on the plane $3x' - 6y' - 2z' = 0$ $\begin{bmatrix} k - 2 & -k - 1 & 3 \\ 5 - 4k & 2k + 2 & -6 \\ 1 & 1 & -2 \end{bmatrix}$ $M^{-1} = \frac{1}{1 - 2k} \begin{bmatrix} k - 2 & 5 - 4k & 1 \\ -k - 1 & 2k + 2 & 1 \\ 3 & -6 & -2 \end{bmatrix}$	$\begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 1 \\ 0 & 3 & k+1 \end{bmatrix} = -3 \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} + (k+1) \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix}$ $= 1 - 2k$ $1 - 2k = 0$ $k = \frac{1}{2}$ $x' = 2x + 4y + 3z$ $y' = x + y + z$ $z' = 3y + 1.5z$ $3x' - 6y' - 2z' = 3(2x + 4y + 3z) - 6(x + y + z) - 2(3y + 1.5z)$ $= 0$ Therefore each point must lie on the plane $3x' - 6y' - 2z' = 0$ $A1$ $\begin{bmatrix} k - 2 & -k - 1 & 3 \\ 5 - 4k & 2k + 2 & -6 \\ 1 & 1 & -2 \end{bmatrix}$ $M1$ $A2$ $M^{-1} = \frac{1}{1 - 2k} \begin{bmatrix} k - 2 & 5 - 4k & 1 \\ -k - 1 & 2k + 2 & 1 \\ 3 & -6 & -2 \end{bmatrix}$ $M1$	$\begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 1 \\ 0 & 3 & k+1 \end{bmatrix} = -3 \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix} + (k+1) \begin{bmatrix} 2 & 4 \\ 1 & 1 \end{bmatrix} $ M1 =1-2k 1-2k = 0 $k = \frac{1}{2}$ A1 x' = 2x + 4y + 3z y' = x + y + z z' = 3y + 1.5z 3 3x' - 6y' - 2z' = 3(2x + 4y + 3z) - 6(x + y + z) - 2(3y + 1.5z) dM1 =0 Therefore each point must lie on the plane 3x' - 6y' - 2z' = 0 A1 4 $\begin{bmatrix} k - 2 & -k - 1 & 3 \\ 5 - 4k & 2k + 2 & -6 \\ 1 & 1 & -2 \end{bmatrix}$ M1 A2 M1 ¹ = $\frac{1}{1 - 2k} \begin{bmatrix} k - 2 & 5 - 4k & 1 \\ -k - 1 & 2k + 2 & 1 \\ 3 & -6 & -2 \end{bmatrix}$ M1 A1 5

(ii)	For invariant points $\begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 1 \\ 0 & 3 & k+1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$			
	2x + 4y + 3z = x x + y + z = y 3y + (k+1)z = z Gives x + 4y + 3z = 0 x + z = 0	M1		Use of Mv = v to give three correct equations.
	3y + kz = 0 eg $x + z = 0$, $2y = -z$	A1		Reduces first two equations to only two variables in each (PI correct equation in k & one variable).
	-3z - 4kz + 9z = 0	A1		Elimination to form equation in k & one variable (PI).eg $3y-2ky=0,1.5x-kx=0$
	k = 1.5 Line is $-x = -2y = z$	A1 B1	5	Correct value of <i>k</i> . Correct equation OE.
	ALTERNATIVEEigen value $\lambda = 1$ $\begin{vmatrix} 1 & 4 & 3 \\ 1 & 0 & 1 \\ 1 & 3 & k \end{vmatrix} = 0$	(M1)		Fully correct equation with $\lambda = 1$ substituted in.
	$\begin{vmatrix} 1 \begin{vmatrix} 0 & 1 \\ 3 & k \end{vmatrix} - 1 \begin{vmatrix} 4 & 3 \\ 3 & k \end{vmatrix} = 0$	(A1)		Correct expansion by row or column of 3x3 determinant.
	6 - 4k = 0	(A1)		Correct equation in <i>k</i> .
	$k = \frac{3}{2}$	(A1)		Correct value of k
	Line is $\frac{x}{2} = y = \frac{z}{-2}$	(B1)	(5)	Correct equation OE.
	Total		14	

Q8	Solution	Mark	Total	Comment
(a)	$\begin{bmatrix} 0\\3\\1 \end{bmatrix} \times \begin{bmatrix} 1\\2\\-2 \end{bmatrix} = \begin{bmatrix} -8\\1\\-3 \end{bmatrix}$	M1 A1		Attempt at vector product with direction vectors – two components correct. All correct.
	$\sqrt{8^2 + 1^2 + 3^2} = \sqrt{74}$	dM1		Finds modulus of their perpendicular vector.
	$(\cos \alpha =) \frac{1}{\sqrt{74}}$ $(\cos \beta =) \frac{1}{\sqrt{74}}$			or $\begin{bmatrix} 1\\2\\-2 \end{bmatrix} \times \begin{bmatrix} 0\\3\\1 \end{bmatrix} = \begin{bmatrix} 8\\-1\\3 \end{bmatrix}$ leading to $(\cos \beta =) \frac{-1}{\sqrt{74}}$
	$(\cos\gamma =)\frac{\sqrt{74}}{\sqrt{74}}$	Δ1	4	$(\cos \gamma =)\frac{1}{\sqrt{74}}$ All three correct.
			-	clearly stated as directional cosines then A0. No ISW here.
(b)	pt (2,0,-1) is common to line & plane			
	$\begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ b \\ -3 \end{pmatrix} = d \text{ which gives } d = 5$	M1A1		Must use a correct common point and correct directional vector for M1. Must have " $d=5$ " for A1.
	Direction vectors			
	$ \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} \begin{pmatrix} 1 \\ b \\ -3 \end{pmatrix} = 0 \text{ which gives } b = 1 $	M1A1	4	Uses scalar product of correct vectors or uses another correct common point for M1. Must have " $b=1$ " for A1.
	ALTERNATIVE			
	Substituting $\begin{pmatrix} 2 \\ 3t \\ -1+t \end{pmatrix} \cdot \begin{pmatrix} 1 \\ b \\ -3 \end{pmatrix} = d$	(M1)		Substitutes parametric form of line into correct plane equation or substitutes two points to obtain two equations.
	5 + (3b - 3)t = d	(dM1)		Expands scalar product and collects terms or solves simultaneous equations.
	b = 1 $d = 5$	(A1) (A1)	(4)	b correct, must have " $b=1$ " d correct, must have " $d=5$ "

(c)	$\left n\right = \sqrt{p^2 + 16} \qquad \left d\right = 3$			
	$\begin{bmatrix} p \\ 4 \\ 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} = p + 8$	B1		Correct expression for n .d
	Hence $\frac{p+8}{3\sqrt{p^2+16}} = k \text{OE}$	M1		Forms correct scalar product equation.
	$k = \frac{1}{3}$	B1		
	$\left(p+8\right)^2 = p^2 + 16$	dM1		Squaring both sides of their equation.
	16p = -48 $p = -3$	A1	5	Correct value obtained.
	ALTERNATIVE			
	$\mathbf{n} \times \mathbf{d} = \begin{bmatrix} -8\\2p\\2p-4 \end{bmatrix}$	(B 1)		Correct vector product obtained
	$\frac{\sqrt{8p^2 - 16p + 80}}{3\sqrt{p^2 + 16}} = \frac{\sqrt{8}}{3}$	(M1A1)		Correct LHS M1 Fully correct A1
	$p^2 + 16 = p^2 - 2p + 10$	(dM1)		Correctly squaring their fractions
	p = -3	(A1)	(5)	Correct value obtained

	Total		18	
	$\begin{pmatrix} \mathbf{r} - \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} \times \begin{bmatrix} 12 \\ 9 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$	(A1F)	(5)	Fully correct format – their point and direction in the correct places.
	$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1}{3} \\ 0 \\ -\frac{14}{9} \end{pmatrix} + t \begin{pmatrix} \frac{4}{3} \\ 1 \\ \frac{7}{9} \end{pmatrix}$	(dM1)		Rewriting to identify point and direction – can be implied.
	$z = \frac{9}{9}$			$x = \mu, y = (3\mu - 1)/4, z = (7\mu - 21)/12$ $z = \lambda x = (12\lambda + 21)/7, y = (9\lambda + 14)/7$
	5 - 7t - 14	(A1)		variables – alternatives are:
	$x = \frac{4t+1}{2}$	(A1)		A1 for each correct expression for other
	ALTERNATIVE $y = t$	(M1)		Set one variable to a parameter and attempt to find other variables (using their numerical values of b and p).
	$\left(\mathbf{r} - \begin{bmatrix} 3\\2\\0 \end{bmatrix}\right) \times \begin{bmatrix} 12\\9\\7 \end{bmatrix} = \begin{bmatrix} 0\\0\\0 \end{bmatrix}$	A1F	5	Fully correct format – their point and direction in the correct places. Can have 0 on RHS. Must have scored both M1's.
	<i>y</i> = 2, <i>x</i> = 3	A1		Correct common point, must have correct values of <i>b</i> and <i>p</i> – some alternatives are x = 0, y = -1/4, z = -7/4 x = 1/3, y = 0, z = -14/9
	For common point $z = 0$	M1		Finding common point from their numerical values of b and p – set one variable to a number and attempts to find both others.
	PART <i>p</i> & <i>b</i> MUST BOTH BE REAL. $\mathbf{b} = \begin{bmatrix} 1 \\ "1" \\ -3 \end{bmatrix} \times \begin{bmatrix} "-3" \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 12 \\ 9 \\ 7 \end{bmatrix}$	M1 A1		Vector product of their normals – at least two components correct from their numerical values of b and p . Fully correct, must have correct values of b and p .
(d)	TO GAIN ANY MARKS IN THIS			