Please write clearly in block capitals.

Centre number [ ][ ][ ][ ][ ] Candidate number [ ][ ][ ][ ]

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Forename(s) ____________________

Candidate signature ____________________

# A-level
# MATHEMATICS

## Unit Further Pure 4

Wednesday 24 May 2017 Morning Time allowed: 1 hour 30 minutes

**Materials**

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

**Instructions**
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

**Information**
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

**Advice**
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

| For Examiner's Use | |
|---|---|
| Question | Mark |
| 1 | |
| 2 | |
| 3 | |
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Answer **all** questions.

Answer each question in the space provided for that question.

**1** The matrices $\mathbf{A}$ and $\mathbf{B}$ are given by

$$\mathbf{A} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & p & -4 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 0 & p \\ 2 & -2 \\ 1 & -3 \end{bmatrix}$$

where $p$ is a constant.

**(a)** Find $\mathbf{BA}$ in terms of $p$.

**[3 marks]**

**(b)** Show that $\mathbf{BA}$ is a singular matrix for all values of $p$.

**[3 marks]**

QUESTION PART REFERENCE

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**2** Three planes have equations

$$\begin{aligned} 5x + 2y + 11z &= 45 \\ 2x - \phantom{2}y + \phantom{1}5z &= 15 \\ -3x + 3y + az &= b \end{aligned}$$

where $a$ and $b$ are constants. The planes **do not** meet at a unique point.

**(a)** Find the value of $a$.

**[3 marks]**

**(b)** There are two possible geometrical configurations for the planes. Identify each configuration and find the corresponding values of $b$.

**[4 marks]**

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**3** The points $A$, $B$ and $C$ have position vectors

$$\mathbf{a} = \begin{bmatrix} 2 \\ -p \\ -1 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 0 \\ 2p+1 \\ -1 \end{bmatrix} \text{ and } \mathbf{c} = \begin{bmatrix} p-1 \\ 4 \\ 3 \end{bmatrix}$$

respectively, relative to the origin $O$ where $p$ is a constant.

**(a)** Find $(\mathbf{a} \times \mathbf{b}).\mathbf{c}$ in terms of $p$.

**[3 marks]**

**(b)** These three position vectors define the edges of a parallelepiped, with volume $13$ cubic units. Find all the possible values of $p$.

**[4 marks]**

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**4** The transformation T maps $(x, y)$ to $(x', y')$ such that $\begin{bmatrix} x' \\ y' \end{bmatrix} = \mathbf{M}\begin{bmatrix} x \\ y \end{bmatrix}$.

Given that $\mathbf{M} = \begin{bmatrix} 3 & -2 \\ 4 & a \end{bmatrix}$ where $a$ is a constant and $\det \mathbf{M}^{-1} = -\dfrac{1}{10}$:

**(a)** find the value of $a$;

**[3 marks]**

**(b)** find the equations of all the invariant lines of T.

**[6 marks]**

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**5** A 3 by 3 matrix $\mathbf{N}$ has characteristic equation $2\lambda^3 + \lambda^2 + k\lambda + 6 = 0$, where $k$ is a constant. One of the eigenvalues of $\mathbf{N}$ is $-3$.

**(a) (i)** Find the value of $k$.

**[2 marks]**

**(ii)** Find the other two eigenvalues, showing all your working.

**[2 marks]**

**(b)** An eigenvector corresponding to the eigenvalue $-3$ is $\begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$.

**(i)** Find $\mathbf{N}^2 \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$.

**[1 mark]**

**(ii)** Find the values of $x$, $y$ and $z$ if $\mathbf{N}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$.

**[1 mark]**

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**6** Let $\Delta(x) = \begin{vmatrix} a-1 & b+1 & x-1 \\ x^2-b^2 & x^2-a^2 & a^2-b^2 \\ 2 & -2 & 2 \end{vmatrix}$

**(a)** Factorise $\Delta(x)$ as fully as possible.

**[6 marks]**

**(b)** Solve $\Delta(x) = 0$.

**[2 marks]**

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**7** A plane transformation T is defined by

$$\mathrm{T}: \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \mathbf{M} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

where $\mathbf{M} = \begin{bmatrix} 2 & 4 & 3 \\ 1 & 1 & 1 \\ 0 & 3 & k+1 \end{bmatrix}$ and $k$ is a constant.

**(a)** In the case when $\mathbf{M}$ is a singular matrix, show that the image of every point under T lies in the plane $3x - 6y - 2z = 0$.

**[4 marks]**

**(b)** In the case when $\mathbf{M}$ is a non-singular matrix:

**(i)** find $\mathbf{M}^{-1}$, in terms of $k$;

**[5 marks]**

**(ii)** show that there is one value of $k$ for which T has a line of invariant points and find the Cartesian equations of this line.

**[5 marks]**

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**8** The lines $L_1$ and $L_2$ have equations

$$\left(\mathbf{r}-\begin{bmatrix}2\\0\\-1\end{bmatrix}\right)\times\begin{bmatrix}0\\3\\1\end{bmatrix}=\mathbf{0} \text{ and } \left(\mathbf{r}-\begin{bmatrix}5\\1\\-1\end{bmatrix}\right)\times\begin{bmatrix}1\\2\\-2\end{bmatrix}=\mathbf{0} \text{ respectively.}$$

**(a)** Find the direction cosines of a line which is perpendicular to both $L_1$ and $L_2$ .

**[4 marks]**

**(b)** The plane $\Pi_1$ has equation $\mathbf{r}.\begin{bmatrix}1\\b\\-3\end{bmatrix}=d$ and contains the line $L_1$ . Find the value of $b$ and the value of $d$.

**[4 marks]**

**(c)** The plane $\Pi_2$ has equation $\mathbf{r}.\begin{bmatrix}p\\4\\0\end{bmatrix}=-1$ . Given that the acute angle between $\Pi_2$ and $L_2$ is $\theta$, where $\cos\theta=\dfrac{\sqrt{8}}{3}$, find the value of $p$.

**[5 marks]**

**(d)** By using your answers to parts **(b)** and **(c)**, find the line of intersection of $\Pi_1$ and $\Pi_2$ in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$.

**[5 marks]**

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