Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level
MATHEMATICS

Unit Further Pure 4

Wednesday 25 May 2016 Morning Time allowed: 1 hour 30 minutes

Materials

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Do not write outside the box

Answer **all** questions.

Answer each question in the space provided for that question.

1 The three points A, B and C have coordinates $(2, -1, 1)$, $(4, 3, -2)$ and $(3, 0, -3)$ respectively.

(a) Find $\overrightarrow{AB} \times \overrightarrow{AC}$.

[3 marks]

(b) Hence find the exact value of the area of triangle ABC.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 1

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 1

Do not write outside the box

2 The matrices $\mathbf{A}$ and $\mathbf{B}$ are such that

$$(\mathbf{AB})^{-1} = \begin{bmatrix} 10 & 3 \\ 2 & -1 \end{bmatrix} \quad \text{and} \quad \det(\mathbf{A}^{-1}) = \frac{1}{2}$$

Find the determinant of $\mathbf{B}$.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 2

Do not write outside the box

3 Let $\Delta = \begin{vmatrix} a & -12 & 1 \\ 4 & -3a & a-3 \\ -3 & 4a & a+4 \end{vmatrix}$.

(a) (i) Use a row operation to show that $a - 4$ is a factor of Δ.

[2 marks]

(ii) Factorise Δ as fully as possible.

[4 marks]

(b) The vectors $\begin{bmatrix} a \\ 4 \\ -3 \end{bmatrix}$, $\begin{bmatrix} -12 \\ -3a \\ 4a \end{bmatrix}$ and $\begin{bmatrix} 1 \\ a-3 \\ a+4 \end{bmatrix}$ are linearly dependent.

Find the possible values of a.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 3

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 3

Do not write outside the box

4 The matrix $\mathbf{A} = \begin{bmatrix} 5 & 2 & 4 \\ 7 & 4 & 6 \\ 6 & k-2 & k \end{bmatrix}$ is non-singular.

(a) Show that $k \neq 5$.

[3 marks]

(b) Find $\mathbf{A}^{-1}$ in terms of k.

[5 marks]

(c) Use your result from part **(b)** to solve the equations

$$\begin{aligned} 5x + 2y + 4z &= a \\ 7x + 4y + 6z &= -a \\ 6x - 3y - z &= -5a \end{aligned}$$

[4 marks]

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 4

Do not write outside the box

5 The matrix $\mathbf{M} = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 5 & -1 \\ 2 & p & 1 \end{bmatrix}$, where p is a constant, has three distinct eigenvalues, λ_1, λ_2 and λ_3.

(a) Given that $\lambda_1 = 2$, find the value of p.

[4 marks]

(b) Given that $\lambda_2 < 0$, find the values of λ_2 and λ_3.

[5 marks]

(c) The transformation T has matrix $\mathbf{M}$. Find a vector equation for any one of the invariant lines of T.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 5

Do not write outside the box

6 The transformation T maps (x, y) to (x', y') where

$$x' = -2x + 2y + 4$$
$$y' = 3x - y - 4$$

(a) Explain what is meant by a line of invariant points.

[1 mark]

(b) Find an equation of the line of invariant points of T.

[3 marks]

(c) Show that T has an infinite number of invariant lines.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 6

Do not write outside the box

7 The linear transformation T is represented by the matrix $\mathbf{M} = \begin{bmatrix} -k\sqrt{3} & 0 & -k \\ 0 & 2k & 0 \\ k & 0 & -k\sqrt{3} \end{bmatrix}$.

(a) Find the determinant of $\mathbf{M}$ in terms of k.

[1 mark]

(b) The transformation T is the composition of two transformations. The first transformation, T_1, is an enlargement centre O and scale factor $2k$ $(k > 0)$, and the second is a transformation T_2. When T is applied to a solid shape of volume $48\,\mathrm{cm}^3$, the volume of the image is $0.75\,\mathrm{cm}^3$.

(i) Find the value of k and hence state the scale factor of the enlargement.

[3 marks]

(ii) Give a full geometrical description of T_2.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 7

Do not write outside the box

8 (a) The plane Π_1 is perpendicular to the line $\left(\mathbf{r} - \begin{bmatrix} 2 \\ 4 \\ -3 \end{bmatrix}\right) \times \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

The point $(3, -1, 2)$ lies on Π_1. Find an equation for Π_1 in the form $\mathbf{r}.\mathbf{n} = c$.

[2 marks]

(b) The plane Π_2 contains the points $(2, 3, 2)$, $(4, 1, -1)$ and $(0, -1, 2)$.

Show that an equation for Π_2 is $2x - y + 2z = d$, where d is an integer to be found.

[5 marks]

(c) Show that the acute angle, θ, between the planes Π_1 and Π_2 is such that $\cos\theta = \dfrac{\sqrt{30}}{15}$.

[3 marks]

(d) The plane Π_3 has the equation $4(k-2)x + (k+1)y - 4k^2z = 8$, where k is an integer. The three planes Π_1, Π_2 and Π_3 have no point in common.

(i) Show that one possible value of k is 1 and find the other possible value.

[4 marks]

(ii) State the geometrical configuration of the three planes for each of the two cases in part **(d)(i)**, giving reasons for your answers.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8

END OF QUESTIONS

Do not write outside the box

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

Copyright information

For confidentiality purposes, from the November 2015 examination series, acknowledgements of third party copyright material will be published in a separate booklet rather than including them on the examination paper or support materials. This booklet is published after each examination series and is available for free download from www.aqa.org.uk after the live examination series.

Permission to reproduce all copyright material has been applied for. In some cases, efforts to contact copyright-holders may have been unsuccessful and AQA will be happy to rectify any omissions of acknowledgements. If you have any queries please contact the Copyright Team, AQA, Stag Hill House, Guildford, GU2 7XJ.

Copyright © 2016 AQA and its licensors. All rights reserved.