GCE AS/A Level

0979/01

S17-0979-01-R1

MATHEMATICS – FP3
Further Pure Mathematics

WEDNESDAY, 28 JUNE 2017 – MORNING

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:
- a WJEC pink 16-page answer booklet;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Solve the equation

$$2\sinh\theta + \cosh\theta = 2.$$

Give your answer correct to three significant figures. [7]

2. By putting $t = \tan\left(\frac{x}{2}\right)$, determine the value of the integral

$$\int_0^{\frac{\pi}{2}} \frac{2}{1+\sin x + 2\cos x}\,\mathrm{d}x\,.$$

Give your answer in the form $\ln N$, where N is a positive integer. [8]

3. The curve *C* has equation $y = x^3$. The arc joining the points (0, 0) and (1, 1) on *C* is rotated through an angle 2π about the x-axis. Calculate the curved surface area of the solid generated, giving your answer correct to three significant figures. [9]

4. The function f is defined by

$$f(x) = \cos(\ln(1+x)).$$

(a) Show that

$$(1+x)^2 f''(x) + (1+x)f'(x) + f(x) = 0.$$ [4]

(b) Hence, or otherwise, show that the Maclaurin series for $f(x)$ is

$$1 - \frac{1}{2}x^2 + \frac{1}{2}x^3 + \ldots$$ [5]

(c) Deduce the Maclaurin series for $\sin(\ln(1+x))$ as far as the term in x^2. [4]

5. *(a)* Show that the equation $\tan\theta\tanh\theta = 1$ has a root, α, between 0·9 and 1·1. [3]

(b) Consider the sequence defined by

$$\theta_{n+1} = \tan^{-1}\left(\frac{1}{\tanh\theta_n}\right) \qquad \text{with } \theta_0 = 1\,.$$

(i) Show that

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left(\tan^{-1}\left(\frac{1}{\tanh\theta}\right)\right) = -\left(\frac{1-\tanh^2\theta}{1+\tanh^2\theta}\right)\,.$$

(ii) Hence show that the sequence defined above is convergent. [5]

(c) Using this sequence, with $\theta_0 = 1$,

(i) write down the value of θ_1,

(ii) write down the value of α correct to three decimal places. [3]

6. The integral I_n is given, for $n \geqslant 0$, by

$$I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx.$$

(a) Show that, for $n \geqslant 2$,

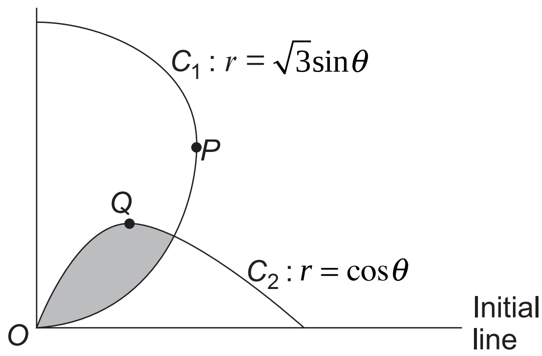
$$I_n = \frac{1}{n-1} - I_{n-2} \ .$$ [5]

(b) Hence determine the value of the integral

$$\int_0^{\frac{\pi}{4}} \left(3 + \tan^2 x\right)^2 dx,$$

leaving your answer in terms of π. [7]

7.



The diagram shows a sketch of the curve C_1 with polar equation $r = \sqrt{3}\sin\theta$ and a sketch of the curve C_2 with polar equation $r = \cos\theta$, both defined for $0 \leqslant \theta \leqslant \frac{\pi}{2}$.

(a) The point at which the tangent to C_1 is perpendicular to the initial line is denoted by *P* and the point at which the tangent to C_2 is parallel to the initial line is denoted by *Q*. Show that the origin *O* and the points *P* and *Q* lie on a straight line. [5]

(b) (i) Show that the polar coordinates of the point of intersection of C_1 and C_2 are $\left(\frac{\sqrt{3}}{2}, \frac{\pi}{6}\right)$.

(ii) Find the area of the shaded region enclosed by C_1 and C_2. [10]

END OF PAPER