

Friday 16 June 2017 – Afternoon

A2 GCE MATHEMATICS

4727/01 Further Pure Mathematics 3

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4727/01
- List of Formulae (MF1)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Answer **all** the questions.

- 1 Solve the differential equation

$$\frac{dy}{dx} + y \cot x = 9 \operatorname{cosec} x$$

to find y in terms of x subject to the condition $y = \pi$ when $x = \frac{1}{6}\pi$. [8]

- 2 The group G consists of the set $\{1, 5, 7, 11\}$ combined under multiplication modulo 12.

(i) Draw the group table for G . [2]

The group H consists of the set $\{1, 3, 5, 7\}$ combined under multiplication modulo 8.

(ii) Determine whether G and H are isomorphic. [3]

- 3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 25 \sin x. \quad [8]$$

- 4 A plane Π_1 passes through the points $(1, 2, -1)$, $(2, -3, 1)$ and $(-1, 0, 2)$.

(i) Show that the plane Π_1 has equation $11x + 7y + 12z = 13$. [4]

The plane Π_2 has equation $3x + y + z = 4$.

(ii) Find a vector equation of the line of intersection of Π_1 and Π_2 . [4]

(iii) Find the acute angle between Π_1 and Π_2 . [2]

- 5 In an Argand diagram the points O , A and B are represented by the complex numbers 0 , z and $2e^{\frac{1}{3}\pi i}z$ respectively, where z is a complex number with modulus 5.

(i) Calculate the exact area of the triangle OAB . [3]

The numbers $-1 + i$ and $3 + 3i$ are represented by the points P and Q respectively. The complex number w is represented by the point R , such that $PQ = PR$ and angle $QPR = \frac{1}{4}\pi$.

(ii) Sketch an Argand diagram showing P , Q and the two possible positions of R . Calculate the possible values of w , giving your answers in the form $a + bi$. [5]

- 6 The plane Π and the line l have equations

$$\mathbf{r} \cdot \begin{pmatrix} 2 \\ -3 \\ 5 \end{pmatrix} = 7 \text{ and } \mathbf{r} = \lambda \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

respectively. The point A has coordinates $(1, 2, -4)$.

- (i) Find the shortest distance from the point A to the plane Π . [3]

- (ii) Find the acute angle between Π and l . [3]

- (iii) Find the point where the line parallel to l passing through A intersects the plane Π . [4]

- 7 (i) By expressing $\cos \theta$ in terms of $e^{i\theta}$ show that

$$\cos^6 \theta = \frac{1}{32}(\cos 6\theta + 6 \cos 4\theta + 15 \cos 2\theta + 10). \quad [4]$$

- (ii) Hence solve, for $0 \leq \theta \leq \pi$,

$$\cos 6\theta + 6 \cos 4\theta + 2 \cos 2\theta = 3. \quad [5]$$

- 8 A group G has the elements $\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix}$ where $a, b \in \{1, -1, i, -i\}$. The group operation is matrix multiplication. The subset H consists of the matrices with $a = 1$.

- (i) State the order of G . [1]

- (ii) Show that H is a subgroup of G . [3]

K is a proper subgroup of G such that H is a proper subgroup of K .

- (iii) Show that K must have order 8. [4]

- (iv) Show that there is only one such subgroup K and identify its elements. [6]

END OF QUESTION PAPER

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