GCE MARKING SCHEME

SUMMER 2017

MATHEMATICS - FP3
0979-01

INTRODUCTION

This marking scheme was used by WJEC for the 2017 examination. It was finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conference was held shortly after the paper was taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conference was to ensure that the marking scheme was interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conference, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about this marking scheme.

FP3 – June 2017 - Mark Scheme

Ques	Solution	Mark	Notes
1	EITHER Rewrite the equation in the form $2\left(\frac{e^{\theta}-e^{-\theta}}{2}\right)+\frac{e^{\theta}+e^{-\theta}}{2}=2$	M1A1	
	$3e^{\theta}-4-e^{-\theta}=0$	A1	
	$3e^{2\theta}-4e^{\theta}-1=0$	A1	
	$e^{\theta}=\frac{4\pm\sqrt{16+12}}{6}$	M1	
	$= 1.548...,(-0.215...)$	A1	
	$\theta = 0.437$	A1	
	OR Let $2\sinh\theta + \cosh\theta = r\sinh(\theta + \alpha)$	(M1)	
	$= r\sinh\theta\cosh\alpha + r\cosh\theta\sinh\alpha$	(A1)	
	Equating coefficients, $r\cosh\alpha = 2$; $r\sinh\alpha = 1$	(M1)	
	Solving, $r=\sqrt{3}$; $\alpha=\tanh^{-1}(0.5)$ $(= 0.54930..)$	(A1)	
	Consider $\sqrt{3}\sinh(\theta + \alpha) = 2$	(M1)	
	$\theta + \alpha = \sinh^{-1}(2/\sqrt{3})$ $(= 0.98664...)$	(A1)	
	$\theta = 0.98664 - 0.54930 = 0.437$	(A1)	
2	Putting $t=\tan\left(\frac{x}{2}\right)$		
	$[0,\pi/2]$ becomes $[0,1]$	B1	
	$dx=\frac{2dt}{1+t^2}$	B1	
	$I=2\int_0^1\frac{2dt/(1+t^2)}{1+2t/(1+t^2)+2(1-t^2)/(1+t^2)}$	M1A1	M0 no working
	$=4\int_0^1\frac{dt}{3+2t-t^2}$	A1	Accept
	$=4\int_0^1\frac{dt}{4-(t-1)^2}$	m1	$=\int_0^1\left(\frac{1}{3-t}+\frac{1}{1+t}\right)dt$
	$=\left[\ln\left(\frac{2+t-1}{2-t+1}\right)\right]_0^1$	A1	$=\left[-\ln(3-t)+\ln(1+t)\right]_0^1$
	$=\ln 3$	A1	$=\ln 3$

Ques	Solution	Mark	Notes
3	$y = x^3, \dfrac{dy}{dx} = 3x^2$	B1	
	CSA $= 2\pi\int_0^1 y\sqrt{1+\left(\dfrac{dy}{dx}\right)^2}\,dx$	M1	
	$= 2\pi\int_0^1 x^3\sqrt{1+9x^4}\,dx$	A1	
	Put $u = 1+9x^4$	M1	
	$du = 36x^3dx, [0,1] \to [1,10]$	A1	
	CSA $= 2\pi\int_1^{10} u^{1/2}\dfrac{du}{36}$	M1	
	$= \left[2\pi \times \dfrac{u^{3/2}}{54}\right]_1^{10}$	A1	
	$= \dfrac{\pi}{27}\left(10^{3/2} - 1\right)$	A1	
	= 3.56	A1	

Ques	Solution	Mark	Notes
4(a)	EITHER $f(x) = \cos\ln(1+x)$ $f'(x) = -\sin\ln(1+x) \times \dfrac{1}{1+x}$ $(1+x)f'(x) = -\sin\ln(1+x)$ $(1+x)f''(x) + f'(x) = -\cos\ln(1+x) \times \dfrac{1}{1+x}$ $(1+x)^2 f''(x) + (1+x)f'(x) + f(x) = 0$	 B1 B1 M1 A1	Convincing
	OR $f(x) = \cos\ln(1+x)$ $f'(x) = -\sin\ln(1+x) \times \dfrac{1}{1+x}$ $f''(x) = -\cos\ln(1+x) \times \dfrac{1}{(1+x)^2} + \sin\ln(1+x) \times \dfrac{1}{(1+x)^2}$ $(1+x)^2 f''(x) + (1+x)f'(x) + f(x)$ $= -\cos\ln(1+x) + \sin\ln(1+x) - \sin\ln(1+x) + \cos\ln(1+x) = 0$	 (B1) (B1) (M1) (A1)	Convincing
(b)	Using the above results, $f(0) = 1, f'(0) = 0, f''(0) = -1$ Differentiating again, $2(1+x)f''(x) + (1+x)^2 f'''(x) + f'(x)$ $+ (1+x)f''(x) + f'(x) = 0$ Therefore $f'''(0) = 3$ The Maclaurin series is $1 - \dfrac{1}{2}x^2 + \dfrac{3}{6}x^3 + ..$ giving $1 - \dfrac{1}{2}x^2 + \dfrac{1}{2}x^3 + ...$	B2 M1 A1 A1	Award B1 for two correct values convincing
(c)	Differentiating, $-\sin\ln(1+x) \times \dfrac{1}{1+x} = -x + \dfrac{3}{2}x^2 + \ldots$ $\sin\ln(1+x) = -(1+x)(-x + \dfrac{3}{2}x^2 + \ldots))$ $= x - \dfrac{3}{2}x^2 + x^2 + \ldots$ $= x - \dfrac{1}{2}x^2 + \ldots$	 M1 A1 M1 A1	

Ques	Solution	Mark	Notes
5(a)	tan(0.9)tanh(0.9) – 1 = – 0.0973... tan(1.1)tanh(1.1) – 1 = 0.572... The change of sign indicates a root between 0.9 and 1.1	B1 B1 B1	
(b)(i)	$\frac{d}{d\theta}\left(\tan^{-1}\left(\frac{1}{\tanh\theta}\right)\right) = \frac{1}{1+\frac{1}{\tanh^2\theta}} \times -\frac{1}{\tanh^2\theta} \times \operatorname{sech}^2\theta$ $= -\frac{1-\tanh^2\theta}{1+\tanh^2\theta}$	**M1A1A1**	Do not award the second A1 if the required result is not derived
(ii)	EITHER For $\theta > 0$, tanhθ lies between 0 and 1. Therefore $1-\tanh^2\theta < 1+\tanh^2\theta$ so that the modulus of the above derivative is less than 1 therefore convergent. OR For $\theta = 1$, $\left\lvert -\left(\frac{1-\tanh^2\theta}{1+\tanh^2\theta}\right)\right\rvert = 0.266$ This is less than 1 therefore convergent.	B1 B1 (B1) (B1)	
(c)(i)	Successive iterations give 1 0.9199161588 ... etc	**M1A1**	
(ii)	The value of α is 0.938 correct to 3 decimal places.	A1	

Ques	Solution	Mark	Notes
6(a)	$I_n = \int_0^{\pi/4} \tan^{n-2} x \tan^2 x \mathrm{d}x$	M1	
	$I_n = \int_0^{\pi/4} \tan^{n-2} x (\sec^2 x - 1)\mathrm{d}x$	A1	
	$= \left[\frac{\tan^{n-1} x}{n-1}\right]_0^{\pi/4} - I_{n-2}$	M1A1A1	convincing
	$= \frac{1}{n-1} - I_{n-2}$		
(b)	$\int_0^{\pi/4}(3+\tan^2 x)^2 \mathrm{d}x = \int_0^{\pi/4} 9\mathrm{d}x + \int_0^{\pi/4} 6\tan^2 x\mathrm{d}x + \int_0^{\pi/4}\tan^4 x\mathrm{d}x$		
	$= 9I_0 + 6I_2 + I_4$	M1	
		A1	
	$I_0 = \frac{\pi}{4}$	B1	
	$I_2 = 1 - I_0 = 1 - \frac{\pi}{4}$	B1	
	$I_4 = \frac{1}{3} - I_2 = \frac{\pi}{4} - \frac{2}{3}$	B1	
	Substituting above,		
	$\int_0^{\pi/4}(3+\tan^2 x)^2 \mathrm{d}\theta = 9\frac{\pi}{4} + 6(1-\frac{\pi}{4}) + \left(\frac{\pi}{4} - \frac{2}{3}\right)$	M1	
	$= \frac{16}{3} + \pi$	A1	

Ques	Solution	Mark	Notes
7(a)	For C_1 consider		
	$x = r\cos\theta = \sqrt{3}\sin\theta\cos\theta$	M1	
	$= \frac{\sqrt{3}}{2}\sin 2\theta$		
	It follows that x is maximised at P when $\theta = \frac{\pi}{4}$.	A1	
	For C_2 consider		
	$y = r\sin\theta = \sin\theta\cos\theta$	M1	
	$= \frac{1}{2}\sin 2\theta$		
	It follows that y is maximised at Q when $\theta = \frac{\pi}{4}$	A1	
	Therefore O, P and Q lie on the line $\theta = \frac{\pi}{4}$. oe	A1	
(b)(i)	The graphs intersect where		
	$\sqrt{3}\sin\theta = \cos\theta$	M1	
	$\tan\theta = \frac{1}{\sqrt{3}}$	A1	
	$\theta = \frac{\pi}{6}, r = \sqrt{3}\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$	A1	Convincing
(ii)	Area of region = $\frac{1}{2}\int_0^{\pi/6} 3\sin^2\theta \mathrm{d}\theta + \frac{1}{2}\int_{\pi/6}^{\pi/2} \cos^2\theta \mathrm{d}\theta$	M1M1	M1 the integrals, M1 for addition
	$= \frac{3}{4}\int_0^{\pi/6}(1-\cos 2\theta)\mathrm{d}\theta + \frac{1}{4}\int_{\pi/6}^{\pi/2}(1+\cos 2\theta)\mathrm{d}\theta$ oe	A1A1	Limits si
	$= \frac{3}{4}\left[\theta - \frac{1}{2}\sin 2\theta\right]_0^{\pi/6} + \frac{1}{4}\left[\theta + \frac{1}{2}\sin 2\theta\right]_{\pi/6}^{\pi/2}$	A1A1	Award A1 for one correct integration, A1 for fully correct line
	$= 0.221 \quad \left(\frac{5\pi}{24} - \frac{\sqrt{3}}{4}\right)$	A1	