



**GCE AS/A level**

0979/01



S15-0979-01

**MATHEMATICS – FP3**  
**Further Pure Mathematics**

A.M. WEDNESDAY, 24 June 2015

1 hour 30 minutes

### **ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

### **INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

### **INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Express  $5 \cosh \theta + 3 \sinh \theta$  in the form  $r \cosh(\theta + \alpha)$ ,  $r > 0$ , where the values of  $r$  and  $\alpha$  are to be found. [4]

- (b) Hence solve the equation

$$5 \cosh \theta + 3 \sinh \theta = 10. \quad [4]$$

2. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} e^{2x} \cos x \, dx,$$

giving your answer in the form  $\frac{ae^{\pi} + b}{5}$ , where  $a$  and  $b$  are integers to be found. [7]

3. The function  $f$  is defined by

$$f(x) = 3x^4 - 4x^3 - 3x^2 - 6x + 4.$$

You are given that the graph of  $f$  has exactly one stationary point whose  $x$ -coordinate is denoted by  $\alpha$ .

- (a) Show that

- (i)  $\alpha$  lies between 1.4 and 1.6,

(ii)  $\alpha = \left( \frac{2\alpha^2 + \alpha + 1}{2} \right)^{\frac{1}{3}}.$  [5]

- (b) It is suggested that the following sequence could be used to determine the value of  $\alpha$ .

$$x_{n+1} = \left( \frac{2x_n^2 + x_n + 1}{2} \right)^{\frac{1}{3}}; \quad x_0 = 1.5$$

- (i) By considering an appropriate derivative, show that this sequence is convergent.  
 (ii) Use this sequence to find the value of  $\alpha$  correct to three decimal places. [8]

4. The function  $f$  is defined by

$$f(x) = \ln(1 + \cosh x).$$

- (a) Show that

$$f''(x) = \frac{1}{1 + \cosh x}. \quad [3]$$

- (b) Determine the Maclaurin series for  $f(x)$  as far as the term in  $x^4$ . [6]

5. The curve  $C$  has parametric equations

$$x = t + \sin t, \quad y = 1 - \cos t \quad (0 \leq t \leq \pi)$$

(a) Show that

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = 4\cos^2 \frac{1}{2}t. \quad [3]$$

(b) (i) Find the arc length of  $C$ .

(ii) Find the curved surface area of the solid generated when  $C$  is rotated through an angle  $2\pi$  about the  $x$ -axis. [8]

6. (a) Show that

$$\frac{d}{dx}\left((4-x^2)^{\frac{3}{2}}\right) = -3x(4-x^2)^{\frac{1}{2}}. \quad [1]$$

The integral  $I_n$  is defined, for  $n \geq 0$ , by

$$I_n = \int_0^2 x^n \sqrt{4-x^2} \, dx.$$

(b) Show that, for  $n \geq 2$ ,

$$I_n = \left(\frac{4(n-1)}{n+2}\right)I_{n-2}. \quad [5]$$

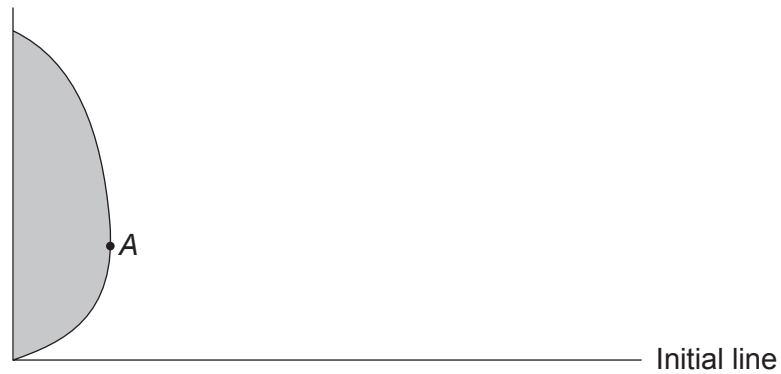
(c) (i) Show that

$$I_0 = \pi.$$

(ii) Evaluate  $I_4$ , giving your answer in the form  $p\pi$  where  $p$  is a positive integer. [8]

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7.



The above diagram shows the curve  $C$  with polar equation

$$r = \tan\left(\frac{\theta}{2}\right), \quad 0 \leq \theta \leq \frac{\pi}{2}.$$

- (a) Show that the  $\theta$ -coordinate of the point  $A$  at which the tangent to  $C$  is perpendicular to the initial line satisfies the equation

$$2 \tan \theta \tan\left(\frac{\theta}{2}\right) = 1 + \tan^2\left(\frac{\theta}{2}\right).$$

Hence find the polar coordinates of  $A$ . [9]

- (b) Find the area of the shaded region enclosed between  $C$  and the line  $\theta = \frac{\pi}{2}$ . [4]

**END OF PAPER**