GCE AS/A level

0979/01

MATHEMATICS – FP3
Further Pure Mathematics

A.M. TUESDAY, 24 June 2014

1 hour 30 minutes

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Starting with the exponential definition of $\sinh x$, show that

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right).$$ [4]

(b) Solve the equation

$$\cosh 2x = 2\sinh x + 5,$$

giving your answers in the form $\ln\left(a + \sqrt{b}\right)$ where a, b are integers. [5]

2. The equation $x^3 + x = 3$ has a root α between 1·2 and 1·3.

(a) Alun suggests the following iterative sequence for finding the value of α based on rearranging the equation

$$x_{n+1} = \sqrt[3]{3 - x_n} \text{ with } x_0 = 1{\cdot}25.$$

By evaluating an appropriate derivative, show that this sequence is convergent.
Use it to find the value of α correct to 4 decimal places. [8]

(b) Starting with $x_0 = 1{\cdot}25$, use the Newton-Raphson method to find the value of α correct to 6 decimal places. [6]

3. *(a)* Assuming the derivative of $\cosh x$, show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\operatorname{sech} x\right) = -\operatorname{sech} x \tanh x.$$ [1]

(b) Determine the Maclaurin series for $\tanh x$ as far as the term in x^3. [6]

(c) Hence find an approximate value for the integral

$$\int_0^{0\cdot5} (1 + x)\tanh x \,\mathrm{d}x.$$

Give your answer correct to three significant figures. [4]

4. Using the substitution $t = \tan\left(\frac{x}{2}\right)$, determine the value of the integral

$$\int_0^{\frac{\pi}{2}} \frac{1}{2 - \cos x}\,\mathrm{d}x.$$ [8]

5. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^1 x^n e^{-x^2} dx.$$

(a) Show that, for $n \geqslant 2$,

$$I_n = \left(\frac{n-1}{2}\right) I_{n-2} - \frac{e^{-1}}{2}.$$ [3]

(b) Evaluate I_5, giving your answer in the form $a - be^{-1}$, where a, b are positive constants to be determined. [6]

6. The curve *C* has polar equation

$$r = \sin\theta + \cos\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

(a) Find the polar coordinates of the point at which the tangent is parallel to the initial line. [8]

(b) Find the area of the region enclosed between *C*, the initial line and the line $\theta = \frac{\pi}{2}$. [5]

7. *(a)* Using the substitution $x = a\sinh\theta$, show that

$$\int \sqrt{x^2 + a^2}\, dx = \frac{a^2}{2}\left(\sinh^{-1}\left(\frac{x}{a}\right) + \frac{x\sqrt{x^2 + a^2}}{a^2} \right) + \text{constant}.$$ [5]

(b) The equation of the curve *C* is

$$y = x^2, \quad 0 \leqslant x \leqslant 1.$$

Find the arc length of *C*. [6]

END OF PAPER