

2.

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 4 & 1 \\ 0 & 5 & 0 \end{pmatrix}$$

(a) Show that matrix \mathbf{M} is not orthogonal. (2)

(b) Using algebra, show that 1 is an eigenvalue of \mathbf{M} and find the other two eigenvalues of \mathbf{M} . (5)

(c) Find an eigenvector of \mathbf{M} which corresponds to the eigenvalue 1 (2)

The transformation $M : \mathbb{R}^3 \rightarrow \mathbb{R}^3$ is represented by the matrix \mathbf{M} .

(d) Find a cartesian equation of the image, under this transformation, of the line

$$x = \frac{y}{2} = \frac{z}{-1}$$
(4)

Question 3 continued

Lined writing area for the answer to Question 3 continued.

5. Given that $y = \operatorname{artanh} \frac{x}{\sqrt{1+x^2}}$

show that $\frac{dy}{dx} = \frac{1}{\sqrt{1+x^2}}$

(4)

6. [In this question you may use the appropriate trigonometric identities on page 6 of the pink Mathematical Formulae and Statistical Tables.]

The points $P(3\cos I, 2\sin I)$ and $Q(3\cos \bar{u}, 2\sin \bar{u})$, where $I \bar{u}$ lie on the ellipse with equation

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

- (a) Show the equation of the chord PQ is

$$\frac{x}{3} \cos \frac{(\alpha + \beta)}{2} + \frac{y}{2} \sin \frac{(\alpha + \beta)}{2} = \cos \frac{(\alpha - \beta)}{2} \tag{4}$$

- (b) Write down the coordinates of the mid-point of PQ . (1)

Given that the gradient, m , of the chord PQ is a constant,

- (c) show that the centre of the chord lies on a line

$$y = -kx$$

expressing k in terms of m . (5)

Question 6 continued

A series of horizontal lines for writing, continuing from the previous page.

7. A circle C with centre O and radius r has cartesian equation $x^2 + y^2 = r^2$ where r is a constant.

(a) Show that $1 + \left(\frac{dy}{dx}\right)^2 = \frac{r^2}{r^2 - x^2}$ (3)

(b) Show that the surface area of the sphere generated by rotating C through α radians about the x -axis is $4\alpha r^2$. (5)

(c) Write down the length of the arc of the curve $y = \sqrt{1 - x^2}$ from $x = 0$ to $x = 1$ (1)

8. The position vectors of the points A, B and C from a fixed origin O are

$$\mathbf{a} = \mathbf{i} - \mathbf{j}, \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}, \mathbf{c} = 2\mathbf{j} + \mathbf{k}$$

respectively.

(a) Using vector products, find the area of the triangle ABC (4)

(b) Show that $\frac{1}{6} \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0$ (3)

(c) Hence or otherwise, state what can be deduced about the vectors \mathbf{a} , \mathbf{b} and \mathbf{c} . (1)
